A Companion Technical Report of "Sample Approximation-Based Deflation Approaches for Chance SINR Constrained Joint Power and Admission Control"

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I. Proof of Theorem 1 in [1]

In this section, we prove Theorem 1 in [1], which shows a relationship between the solutions of the following two problems:

$$\begin{aligned}\max_{\mathbf{p},\,\mathcal{S}} \quad & |\mathcal{S}| - \alpha \mathbf{e}^T\mathbf{p} \\ \text{s.t.} \quad & \Pr\left(\text{SINR}_k(\mathbf{p}) \geq \gamma_k\right) \geq 1-\epsilon,\ \ k \in \mathcal{S} \subseteq \mathcal{K}, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}},\end{aligned} \tag{1}$$

and

$$\begin{aligned}\max_{\mathbf{p},\,\mathcal{S}} \quad & |\mathcal{S}| \\ \text{s.t.} \quad & \Pr\left(\text{SINR}_k(\mathbf{p}) \geq \gamma_k\right) \geq 1-\epsilon,\ \ k \in \mathcal{S} \subseteq \mathcal{K}, \\ & \mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}.\end{aligned} \tag{2}$$

More specifically, Theorem 1 in [1] states: Suppose the parameter α satisfies

$$0 < \alpha < \alpha_1 := 1/\mathbf{e}^T\bar{\mathbf{p}}. \tag{3}$$

Then the optimal value of problem (2) is M if and only if the optimal value of problem (1) lies in $(M-1, M)$. Moreover, suppose $(\mathcal{S}^*, \mathbf{p}^*)$ is the solution of problem (1). Then, $\mathcal{S}^*$ is a maximum admissible set and $\mathbf{e}^T\mathbf{p}^*$ is the minimum total transmission power to support any maximum admissible set.

We first show that that if the optimal value of problem (1) lies in $(M-1, M)$, then the optimal value of problem (2) is M. Recall the fact $0 < \alpha < \alpha_1 = 1/\mathbf{e}^T\bar{\mathbf{p}}$ (cf. (3)) which implies

$$\alpha\,\mathbf{e}^T\mathbf{p} < 1, \quad \text{for any } \mathbf{0} \le \mathbf{p} \le \bar{\mathbf{p}}. \tag{4}$$

Consequently, the total contribution from the second term in the objective function of (1) cannot exceed 1, regardless of the power allocation $\mathbf{p}$. This immediately shows that if the optimal value of (1) lies in the interval $(M-1, M)$, then the optimal value of (2) is M.

Next, we show problem (1) is able to find the maximum admissible set, that is, if the optimal value of problem (2) is M, then $|\mathcal{S}^*| = M$ and the optimal value of (1) lies in the interval $(M-1, M)$. We show the desired result by contradiction and suppose $|\mathcal{S}^*| < M$. Note that the function $|\mathcal{S}|$ is discontinuous with an increment of 1. Therefore, for any solution ($\mathcal{S}$, $\mathbf{p}$) of problem (2) with $|\mathcal{S}| = M$, we have

$$|\mathcal{S}^*| - \alpha\mathbf{e}^T\mathbf{p}^* \le |\mathcal{S}^*| \le M - 1 < |\mathcal{S}| - \alpha\mathbf{e}^T\mathbf{p},$$

where the last strict inequality is due to (4). This contradicts global optimality of ($\mathcal{S}^*$, $\mathbf{p}^*$) to problem (1). Thus, the global maximum of (1) must be achieved at a power allocation such that M links are simultaneously supported. This shows that $|\mathcal{S}|$ is fully maximized by (1) and $|\mathcal{S}^*| = M$. Combining this with (4) further implies that the optimal value of (1) must lie in the interval $(M-1, M)$.

Finally, if there are multiple sets of $M := |\mathcal{S}^*|$ links that are simultaneously supportable, then they all induce the same objective value in the first term of the objective function in (1). In this case, the second term (i.e., $\mathbf{e}^T\mathbf{p}$) will play the role to select the one set of M links which requires the least amount of total transmission power. In other words, $\mathbf{e}^T\mathbf{p}^*$ is the minimum total power required to support any M links in the network.

II. Solution Convergence

In this section, we show that, as the parameter $\mu \downarrow 0$, the solution of the smoothing problem

$$\begin{aligned} \min_{\mathbf{q}} \quad & \tilde{f}(\mathbf{q}, \mu) = \sum_{k\in\mathcal{K}} \sqrt{\|\max\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}, \mathbf{0}\}\|_2^2 + \mu^2} + \alpha\bar{\mathbf{p}}^T\mathbf{q} \\ \text{s.t.} \quad & \mathbf{0} \le \mathbf{q} \le \mathbf{e} \end{aligned} \tag{5}$$

will converge to the one of problem

$$\begin{aligned} \min_{\mathbf{q}} \quad & f(\mathbf{q}) := \sum_{k\in\mathcal{K}} \left\|\max\left\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}, \mathbf{0}\right\}\right\|_2 + \alpha\bar{\mathbf{p}}^T\mathbf{q} \\ \text{s.t.} \quad & \mathbf{0} \le \mathbf{q} \le \mathbf{e}. \end{aligned} \tag{6}$$

We formally state the above convergence result in the following theorem.

Theorem 1: Any accumulation point of the solution of the smoothing problem (5) with $\{\mu_n\} \downarrow 0$ is a solution to problem (6).

Proof: First of all, $f(\mathbf{q})$ in (6) is Lipschitz continuous [3]. By [2] and [3], to show Theorem 1, it suffices to show the gradient consistency

$$\lim_{\mathbf{q}_n\to\mathbf{q},\mu_n\downarrow 0} \nabla h_k(\mathbf{q}_n,\mu_n) \in \partial h_k(\mathbf{q}),\ \forall\ \mathbf{q},\ \forall\ k\in\mathcal{K} \tag{7}$$

between the subdifferential of

$$h_k(\mathbf{q}) = \|\max\left\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}, \mathbf{0}\right\}\|_2$$

and the gradient of the smoothing function

$$\tilde{h}_k(\mathbf{q},\mu) = \sqrt{\|\max\left\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}, \mathbf{0}\right\}\|_2^2 + \mu^2}.$$

Recall the definition of $\mathcal{N}_k^+$ in Proposition 2 in [1]. Clearly, (7) holds trivially for the case $\mathcal{N}_k^+ \neq \emptyset$. We now consider the case $\mathcal{N}_k^+ = \emptyset$, i.e., $\mathbf{c}_k - \mathbf{A}_k\mathbf{q} \le 0$. Without loss of generality, suppose $\mathbf{c}_k - \mathbf{A}_k\mathbf{q} = 0$. Then, for all $\mathbf{q}_n$ and $\mu_n > 0$, we have

$$\nabla\tilde{h}_k(\mathbf{q}_n,\mu_n) = \frac{-\mathbf{A}_k^T\max\left\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}_n, \mathbf{0}\right\}}{\sqrt{\left\|\max\left\{\mathbf{c}_k - \mathbf{A}_k\mathbf{q}_n, \mathbf{0}\right\}\right\|_2^2 + \mu_n^2}},\ \forall\ k\in\mathcal{K}.$$

By Proposition 2, we get

$$\lim_{\mathbf{q}_n\to\mathbf{q},\mu_n\downarrow 0} \nabla\tilde{h}_k(\mathbf{q}_n,\mu_n) \in \partial h_k(\mathbf{q}) = \left\{-\mathbf{A}_k^T\mathbf{s} \,|\, \mathbf{s} \ge \mathbf{0}, \|\mathbf{s}\|_2 \le 1\right\}$$

for all $k \in \mathcal{K}$. This completes the proof of Theorem 1. ■

III. Postprocessing Step in Algorithm 1 in [1]

The postprocessing step (Step 5) in Algorithm 1 in [1] aims at admitting the links removed in the preprocessing and admission control steps. Specifically, we enumerate all the removed links and admit one of them if it can be supported simultaneously with the already supported links. If there are more than one such candidate, we pick the one such that the minimum total transmission power is needed to simultaneously support it with the already supported links. The postprocessing step is terminated if no such candidate exists. The specification of the postprocessing step is described as follows.

Specification of Postprocessing

Step 1. Initialization: Input the set of supported links $\mathcal{S}$ and the set of removed links $\mathcal{R} = \mathcal{K} \setminus \mathcal{S}$.

Step 2. Termination: If $\mathcal{R} = \emptyset$, then terminate the algorithm.

Step 3. Iteration:

For $j \in \mathcal{R}$,

- Solve the linear program

$$\begin{aligned} \min_{\mathbf{q}} \quad & \bar{\mathbf{p}}^T \mathbf{q} \\ \text{s.t.} \quad & \mathbf{A}_k \mathbf{q} - \mathbf{c}_k \geq 0, \; k \in \mathcal{S} \bigcup \{j\}, \\ & \mathbf{0} \leq \mathbf{q} \leq \mathbf{e}, \end{aligned} \tag{8}$$

  and obtain its optimal value v_j. If problem (8) is infeasible, v_j is set to be $+\infty$.
- If $v_j = +\infty$, set $\mathcal{R} = \mathcal{R} \setminus \{j\}$.

End (For)

Step 4. Update of $\mathcal{S}$ and $\mathcal{R}$: If $\mathcal{R} = \emptyset$, then terminate the algorithm; else set $\mathcal{S} = \mathcal{S} \bigcup \{j^*\}$ and $\mathcal{R} = \mathcal{R} \setminus \{j^*\}$, where $j^* = \arg\min_{j \in \mathcal{R}} \{v_j\}$, and go to **Step 2**.

REFERENCES

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