A Companion Technical Report of "Joint Power and Admission Control based on Channel Distribution Information: A Novel Two-Timescale Approach"

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I. Proof of Theorem 1 in [1]

In this section, we prove Theorem 1 in [1], which says that, if

$$0 < \alpha < \alpha_1 := 1/\mathbf{e}^T\bar{\mathbf{p}}, \tag{1}$$

then the formulation

$$\begin{aligned} \max_{\mathcal{S},\,\mathbf{p}^\omega} \quad & |\mathcal{S}| - \alpha\mathbf{e}^T\mathbb{E}(\mathbf{p}^\omega) \\ \text{s.t.} \quad & \mathbb{P}\left(\frac{g_{k,k}^\omega p_k^\omega}{\eta_k + \sum\limits_{j\neq k} g_{k,j}^\omega p_j^\omega} \geq \gamma_k,\, k \in \mathcal{S} \subseteq \mathcal{K}\right) \geq 1-\epsilon, \\ & \mathbf{0} \leq \mathbf{p}^\omega \leq \bar{\mathbf{p}}, \forall\, \omega \in \Omega \end{aligned} \tag{2}$$

is able to find the maximum admissible set $\mathcal{S}$ with a minimum (expected) total transmission power. We introduce the following auxiliary problem:

$$\begin{aligned} \max_{\mathcal{S},\,\mathbf{p}^\omega} \quad & |\mathcal{S}| \\ \text{s.t.} \quad & \mathbb{P}\left(\frac{g_{k,k}^\omega p_k^\omega}{\eta_k + \sum\limits_{j\neq k} g_{k,j}^\omega p_j^\omega} \geq \gamma_k,\, k \in \mathcal{S} \subseteq \mathcal{K}\right) \geq 1-\epsilon, \\ & \mathbf{0} \leq \mathbf{p}^\omega \leq \bar{\mathbf{p}}, \forall\, \omega \in \Omega. \end{aligned} \tag{3}$$

To prove Theorem 1, it suffices to show the following statement: suppose $(\mathcal{S}^*, \mathbf{p}^{\omega *})$ is the solution of problem (2), then $\mathcal{S}^*$ is a maximum admissible set and $\mathbf{e}^T\mathbb{E}(\mathbf{p}^{\omega *})$ is the minimum total transmission power to support any maximum admissible set.

We first show that if the optimal value of problem (3) is M, then $|\mathcal{S}^*| = M$. We show the desired result by contradiction and suppose $|\mathcal{S}^*| < M$. Recall the fact $0 < \alpha < \alpha_1 = 1/\mathbf{e}^T\bar{\mathbf{p}}$ (cf. (1)) which implies

$$\alpha\, \mathbf{e}^T\mathbb{E}(\mathbf{p}^\omega) < 1, \quad \text{for any } \mathbf{0} \le \mathbf{p}^\omega \le \bar{\mathbf{p}}. \tag{4}$$

Note that the function $|\mathcal{S}|$ is discontinuous with an increment of 1. Therefore, for any solution $(\mathcal{S},\, \mathbf{p}^\omega)$ of problem (3) with $|\mathcal{S}| = M$, we have

$$|\mathcal{S}^*| - \alpha\mathbf{e}^T\mathbb{E}(\mathbf{p}^{\omega *}) \le |\mathcal{S}^*| \le M - 1 < |\mathcal{S}| - \alpha\mathbf{e}^T\mathbb{E}(\mathbf{p}^\omega),$$

where the last strict inequality is due to (4). This contradicts global optimality of $(\mathcal{S}^*,\, \mathbf{p}^{\omega *})$ to problem (2). Thus, the global maximum of (2) must be achieved at a power allocation such that M links are simultaneously supported. This shows that $|\mathcal{S}|$ is fully maximized by (2) and $|\mathcal{S}^*| = M$.

Now, we show that $\mathbf{e}^T\mathbb{E}(\mathbf{p}^{\omega *})$ is the minimum total transmission power to support any maximum admissible set. If there are multiple sets of $M := |\mathcal{S}^*|$ links that are simultaneously supportable, then they all induce the same objective value in the first term of the objective function in (2). In this case, the second term (i.e., $\mathbf{e}^T\mathbb{E}(\mathbf{p}^\omega)$) will play the role to select the one set of M links which requires the least amount of total transmission power. In other words, $\mathbf{e}^T\mathbb{E}(\mathbf{p}^{\omega *})$ is the minimum total power required to support any M links in the network. The proof is completed.

II. Proof of Theorem 2 in [1]

For ease of presentation, we first introduce some notation. For the *given* set $\mathcal{S}$, we define $\xi^\omega = 1$ if there exist $\{p_k^\omega\}$ such that

$$\frac{g_{k,k}^\omega p_k^\omega}{\eta_k + \sum\limits_{j\neq k} g_{k,j}^\omega p_j^\omega} \ge \gamma_k,\ \forall\ k \in \mathcal{S};$$

otherwise $\xi^\omega = 0$. By the above definition, we know that ξ^ω is a Bernoulli random variable with unknown parameter

$$p := E(\xi^\omega) = \mathbb{P}\left(\xi^\omega = 1\right) = \mathbb{P}\left(\frac{g_{k,k}^\omega p_k^\omega}{\eta_k + \sum\limits_{j\neq k} g_{k,j}^\omega p_j^\omega} \ge \gamma_k,\ k \in \mathcal{S}\right). \tag{5}$$

We use $\{\xi^n\}$ to denote independent samples of ξ^ω. Then, $\xi^n = 1$ if there exist $\{p_k^n\}$ such that

$$\frac{g_{k,k}^n p_k^n}{\eta_k + \sum\limits_{j \neq k} g_{k,j}^n p_j^n} \geq \gamma_k, \ \forall \ k \in \mathcal{S};$$

otherwise $\xi^n = 0$. To prove the theorem, it suffices to prove that p defined in (5) satisfies $p \geq 1 - \epsilon$ (with a significance level δ).

Suppose that the null hypothesis be

$$\mathbf{H_0} : p < 1 - \epsilon.$$

Moreover, let the test statistic be

$$\frac{\sum_{n=1}^{N} \xi^n}{N}.$$

By Hoeffding's inequality [2], we get

$$\mathbb{P}\left(\frac{\sum_{n=1}^{N} \xi^n}{N} > 1 - \epsilon + c\right) \leq \mathbb{P}\left(\frac{\sum_{n=1}^{N} \xi^n}{N} > p + c\right) \leq \exp\left(-2c^2 N\right),$$

where $c > 0$ is a constant. Substituting $c = \dfrac{\epsilon}{2}$ and $N = N_2^*$ (given in Theorem 2) in the above inequality yields

$$\mathbb{P}\left(\frac{\sum_{n=1}^{N_2^*} \xi^n}{N_2^*} > 1 - \frac{\epsilon}{2}\right) \leq \delta. \tag{6}$$

Define

$$\mathcal{W} = \left\{\left(\xi^1, \ldots, \xi^{N_2^*}\right) \mid \frac{\sum_{i=1}^{N_2^*} \xi^n}{N_2^*} > 1 - \frac{\epsilon}{2}\right\}.$$

Obviously, there holds $\mathbb{P}\left(\left(\xi^1, \ldots, \xi^{N_2^*}\right) \in \mathcal{W}\right) \leq \delta$, which implies that $\mathcal{W}$ is the region of rejection (critical region) with a significance level δ. Moreover, by the assumption of Theorem 2, we know that the samples satisfy $\xi^1 = \xi^2 = \cdots = \xi^{N_2^*} = 1$. Therefore, $\left(\xi^1, \xi^2, \ldots, \xi^{N_2^*}\right)$ lies in the region (of rejection) $\mathcal{W}$ and we must reject the null hypothesis $\mathbf{H_0}$, i.e., $p \geq 1 - \epsilon$ is satisfied (with a significance level δ). The proof is completed.

References

[1] Q. Chen, D. Kang, Y. He, T.-H. Chang, and Y.-F. Liu, "Joint Power and Admission Control based on Channel Distribution Information: A Novel Two-Timescale Approach," accepted for publication in *IEEE Signal Process. Lett.*

[2] W. Hoeffding, "Probability inequalities for sums of bounded random variables," *J. Am. Stat. Assoc.,* vol. 58, no. 301, pp. 13–30, Mar. 1963.