Math 231 Work Sheet 3 NAME:

Summary: This worksheet corresponds to sections 4.4, 4.5 in the textbook.

1. Find the solution to the initial value problem

$$y'' - y' = 2 - 2t \text{ with } y(1) = 2 \text{ and } y'(1) = -3$$

by following the steps:
(a) Eyeball a single solution $Y_P(t)$ to the differential equation. **Hint**: It's a simple polynomial with one term.

(b) Find a fundamental pair for the associated **homogeneous** differential equation.

(c) Write down the general solution for the given differential equation.

(d) Find the specific solution to the initial value problem.

2. (Method of Undetermined Coefficients). Using the Method of Undetermined Coefficients, write down the undetermined form of $Y_P(t)$ for each of the following. The first is done for you so you know how little you need to do!

(a) $y'' - 5y' + 6y = t + 1.$

Solution: $Y_P(t) = A_1 t + A_0$

(b) $y'' - 5y' + 6y = t^2$

(c) $y'' - 5y' + 6y = te^{2t}$

(d) $y'' - 5y' + 6y = e^{3t}$

(e) $y'' - 5y' + 6y = (3t^2 + 1)e^{3t}$

(f) $y'' - 4y' + 13y = e^{3t}\cos(t)$

(g) $y'' - 4y' + 13y = te^{2t}\sin(3t)$

(h) $y'' - 4y' = t - 2$

(i) $y'' - 4y' = \cos(t) - 2\sin(t)$

3. Find a particular solution to $y'' - 5y' + 6y = te^{2t}$ using the Method of Undetermined Coefficients. Note that you did part of this in 2(c).

4. (Method of Undetermined Coefficients, Superposition Principle). Follow the steps to find a solution to the IVP

$$y'' - y' - 2y = 10\sin(2t) + 10\sin(t) - 10\cos(2t), \qquad y(0) = 0, \quad y'(0) = -1.$$

(a) Find a fundamental pair for the associated **homogeneous** differential equation.

(b) Split the right hand side as: $f(t) = f_1(t) + f_2(t)$, where

$$f_1(t) = 10\sin(2t) - 10\cos(2t), \quad f_2(t) = 10\sin(t).$$

(c) Write down the undetermined form of $Y_{P1}(t)$ associated with $f_1(t)$, plug it into the ODE

$$y'' - y' - 2y = f_1(t)$$

and solve the coefficients to find $Y_{P1}(t)$.

(d) Redo part (c) to find $Y_{P2}(t)$. Notice that you need to use $f_2(t)$ instead of $f_1(t)$ here.

(e) The particular solution for the ODE with right hand $f(t)$ is given by $Y_P(t) = Y_{P1}(t) + Y_{P2}(t)$. Write down the general solution for the given differential equation using $Y_P(t)$ and the fundamental pair in (a).

(f) Find the specific solution to the initial value problem.