Brief Table for Integrals:

$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x + \sqrt{x^2+a^2}|.$ $\quad$ $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x + \sqrt{x^2-a^2}|, \quad x^2 \geq a^2.$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin(\frac{x}{a}), \quad a^2 \geq x^2.$ $\quad$ $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan(\frac{x}{a}).$

Common trigonometric substitutions:

1. For integrand involving $\sqrt{a^2-x^2}$, $\quad$ set $x = a\sin(\theta)$,
2. For integrand involving $\sqrt{a^2+x^2}$, $\quad$ set $x = a\tan(\theta)$,
3. For integrand involving $\sqrt{x^2-a^2}$, $\quad$ set $x = a\sec(\theta)$,
4. For $\int \tan^n(x)\sec^{2m}(x)dx$, $\quad$ set $u = \tan(x)$,
5. For $\int \cot^n(x)\csc^{2m}(x)dx$, $\quad$ set $u = \cot(x)$.

Theorem for Special Integrating Factors. For equation

$$M(x,y)dx + N(x,y)dy = 0, \tag{1}$$

if $(\partial M/\partial y - \partial N/\partial x)/N$ is continuous and depends only on x, then

$$\mu(x) = \exp\left[\int\left(\frac{\partial M/\partial y - \partial N/\partial x}{N}\right)dx\right]$$

is an integrating factor for equation (1). If $(\partial N/\partial x - \partial M/\partial y)/M$ is continuous and depends only on y, then

$$\mu(y) = \exp\left[\int\left(\frac{\partial N/\partial x - \partial M/\partial y}{M}\right)dy\right]$$

is an integrating factor for equation (1).