Math 241 Calculus III

Explanations of the syllabus

- Communications : wli50@utk.edu . Canvas .
- Textbook : Calculus, Early Transcendentals (8th edition)
  by Stewart. eBook available on WebAssign .
- WebAssign : class key "utk 9758 2693"
- Grading : Homework + Quizzes + Midterm Exams + Final

  - Online homework on WebAssign : do it before the deadline!
  - Take-home quizzes : show your work .
  - Midterms : Sep 14 , Oct 12 , Nov 9 .

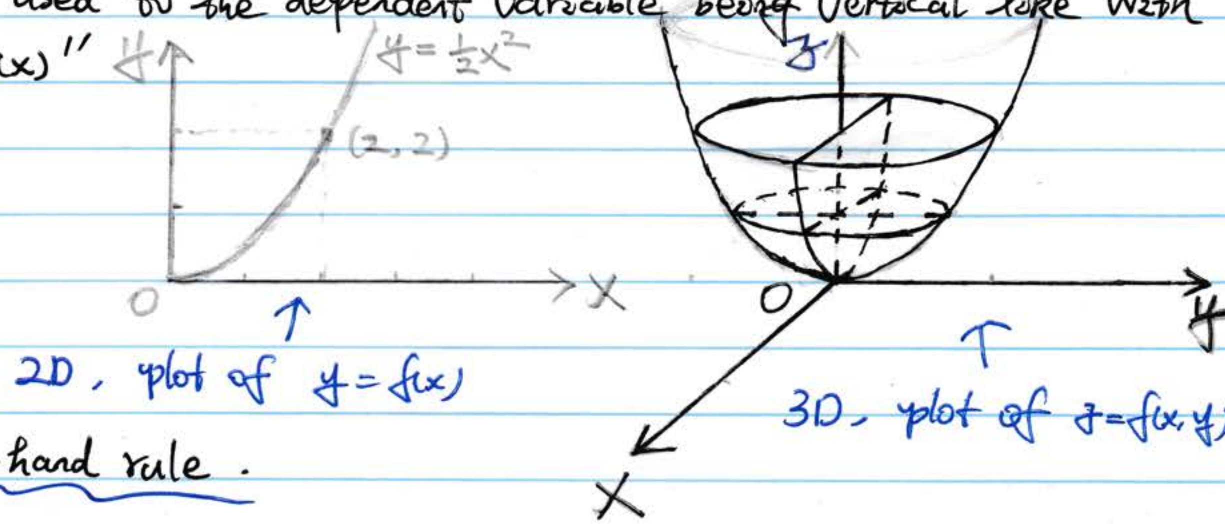
  - Final :

- Calculators : allowed for homework & quizzes , not allowed
  for all exams .
- Recitations : not always on Tuesday . See the schedule .
- Office hours : through zoom , also possible offline .
  Poll for the time .
- The Math Place Online : give it a try if you want .

12.1 Three-Dimensional Coordinate Systems

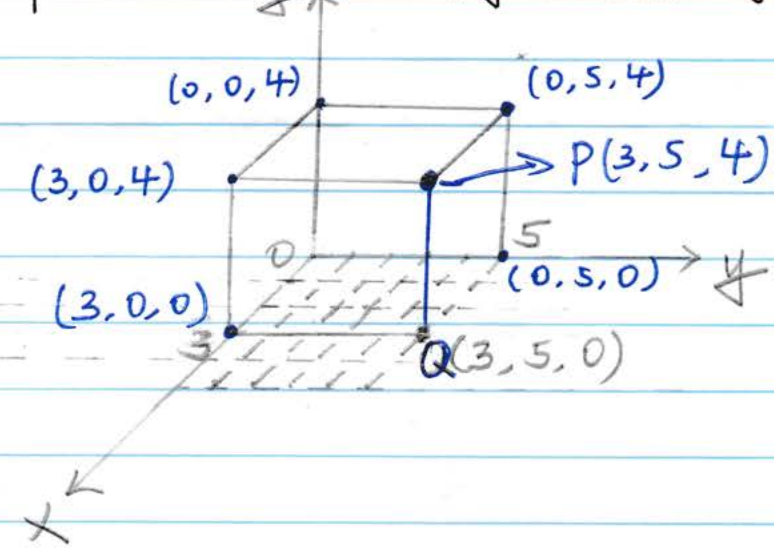
1. Goal: Most of Math 241 (Multivariable Calculus) is in 3D space so we need to understand how this works.

2. In addition to the x and y axis we add an extra axis, the z-axis. We rearrange so that the z-axis is pointing up. Reason: most of our functions are "$z = f(x, y)$", and we're used to the dependent variable being vertical like with "$y = f(x)$"



Picture: right-hand rule.

3. Plot points: locate x, y coordinates first and then z.



usually Capital letter $\qquad$ y- coordinate

<u>Notations</u>: $P(3, 5, 4)$

x- coordinate $\qquad$ z- coordinate

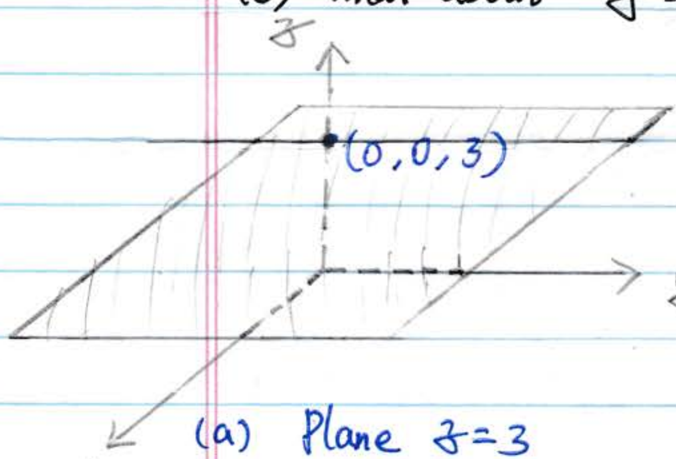
<u>Examples</u>: Plot $P_1 = (-2, 0, 0)$, $P_2 = (0, 0, 3)$
$P_3 = (-1, 3, -2)$

4. Three coordinate planes: xy-plane, yz-plane, xz-plane. These divide 3D space into 8 octants. The first octant is the one with $x, y, z > 0$.

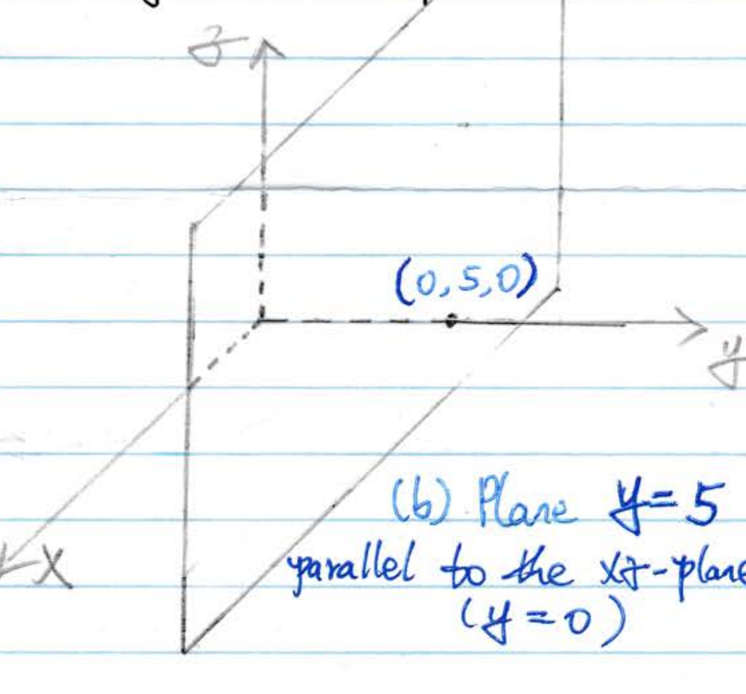
<u>Example</u>: determine whether the following points are in the first octant. $(1, 2, -3)$, $(-1, -1, 2)$, $(3, 0.1, 2)$.
No $\qquad$ No $\qquad$ Yes

5. Examples of surfaces in 3D

- Text-Ex 1: (a) Which points satisfy $z = 3$? What is the surface given by the collection of all such points?
(b) What about $y = 5$?



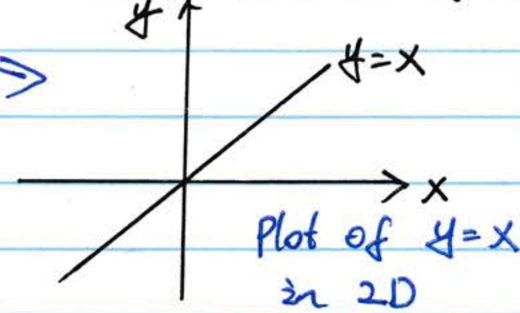
(a) Plane $z = 3$
parallel to the xy-plane ($z = 0$)



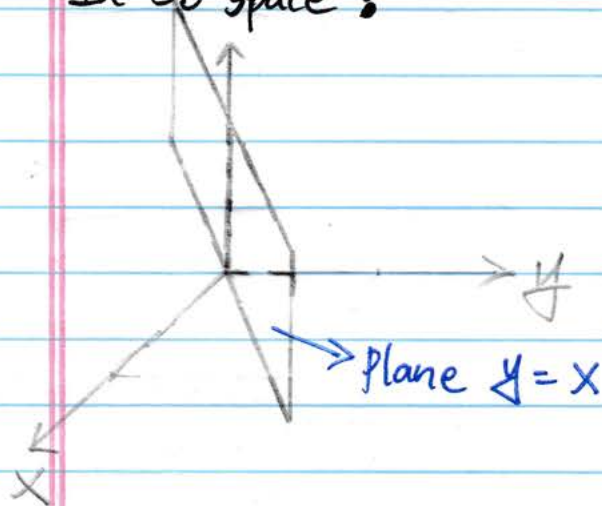
(b) Plane $y = 5$
parallel to the xz-plane ($y = 0$)

• Text-Ex 3 : Sketch the surface in $\mathbb{R}^3$ represented by $y = x$.

<u>Ans</u> : Think of 2D first



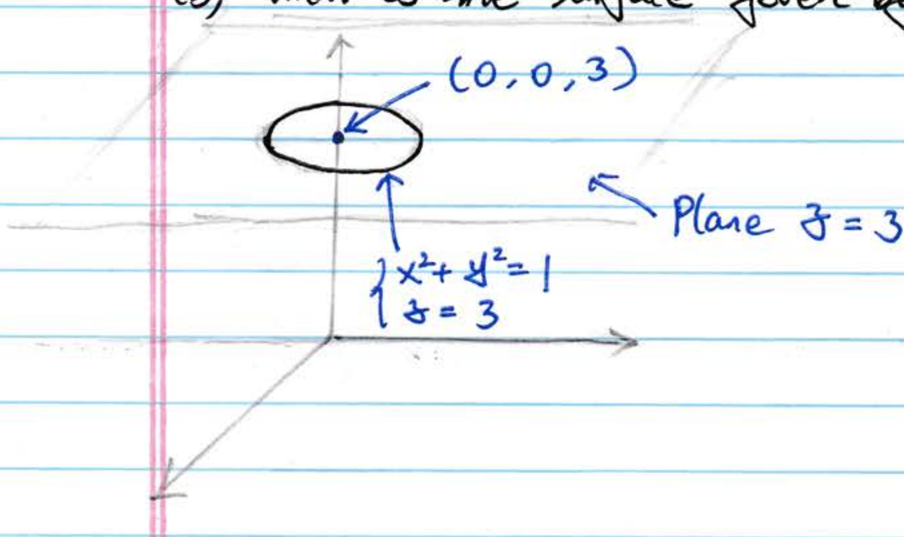
In 3D space :

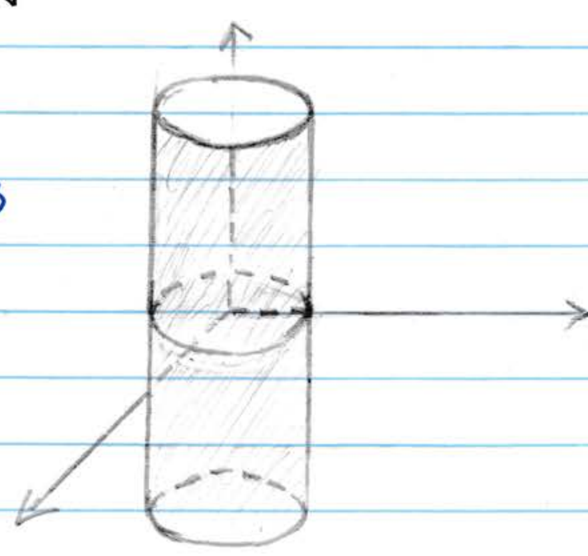


• Text-Ex 2 : (a) Which points (x, y, z) satisfy

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 3$$

(b) What is the surface given by $x^2 + y^2 = 1$ in $\mathbb{R}^3$?





<u>6.</u> In 3D space, we denote distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ by $|P_1 P_2|$, and

$$|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

- Example: Find the lengths of the sides of the triangle PQR, where $P = (6,-1,-1)$, $Q = (8,0,1)$, $R = (9,-2,-1)$.

Ans:
$$|PQ| = \sqrt{(8-6)^2 + (0-(-1))^2 + (1-(-1))^2}$$
$$= \sqrt{4+1+4} = \sqrt{9} = 3$$
$$|QR| = \sqrt{(9-8)^2 + (-2-0)^2 + (-1-1)^2}$$
$$= \sqrt{1+4+4} = \sqrt{9} = 3$$
$$|PR| = \sqrt{(9-6)^2 + (-2-(-1))^2 + (-1-(-1))^2}$$
$$= \sqrt{9+1+0} = \sqrt{10}$$

- Is this triangle isosceles? Yes b/c $|PQ| = |QR|$
- Is it a right triangle? No. Check using Pythagorean Thm
$$3^2 + 3^2 \neq (\sqrt{10})^2$$

7. Equation of a sphere with center $C(x_0, y_0, z_0)$ and radius r:
$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2.$$

Similarly, for closed (open) ball
$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \leq (<)\ r^2$$

- Example: Consider the sphere with equation
$$(x+2)^2 + (y-3)^2 + z^2 = 16,$$
what is the center of the sphere? Radius?

Ans: Rewrite the equation as

$$(x-(-2))^2 + (y-3)^2 + (z-0)^2 = 4^2 .$$

so the center is $(-2, 3, 0)$ and the radius is 4.

- Text-Ex 6: Show that $x^2+y^2+z^2+4x-6y+2z+6=0$ is the equation of a sphere, and find its center and radius.

Ans: Complete squares.

$$x^2+y^2+z^2+4x-6y+2z+6=0$$

group terms of x, y, z respectively

$$\underbrace{x^2+4x}_{=(x+2)^2-4} + \underbrace{y^2-6y}_{=(y-3)^2-9} + \underbrace{z^2+2z}_{(z+1)^2-1} + 6 = 0$$

$$(x+2)^2-4 + (y-3)^2-9 + (z+1)^2-1 + 6 = 0$$

Move the constants to the right and compute

$$(x+2)^2+(y-3)^2+(z+1)^2 = 4+9+1-6 = 8 .$$

Proceed as the last example

$$(x-(-2))^2+(y-3)^2+(z-(-1))^2 = (2\sqrt{2})^2$$

$\Rightarrow$ Center $(-2, 3, -1)$, Radius: $2\sqrt{2}$.