

# Matter-wave solitons and finite-amplitude Bloch waves in optical lattices with a spatially modulated nonlinearity

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We investigate solitons and nonlinear Bloch waves in Bose-Einstein condensates trapped in optical lattices. By introducing specially designed localized profiles of the spatial modulation of the attractive nonlinearity, we construct an infinite number of exact soliton solutions in terms of the Mathieu and elliptic functions, with the chemical potential belonging to the semi-infinite bandgap of the optical-lattice-induced spectrum. Starting from the exact solutions, we employ the relaxation method to construct generic families of soliton solutions in a numerical form. The stability of the solitons is investigated through the computation of the eigenvalues for small perturbations, and also by direct simulations. Finally, we demonstrate a virtually exact (in the numerical sense) composition relation between nonlinear Bloch waves and solitons.

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