Knots in a conformal nonlinear sigma model

Chang-Guang Shi and Minoru hirayama
Department of Mathematics and Physics, Shanghai University of Electric Power
Pingliang road 2010, Shanghai, China
Tel: 021-65430410
email: shichangguang@shiep.edu.cn;
hirayama@jodo.sci.u-toyama.ac.jp

Abstract:

Recently, electromagnetic knots have attracted much attention.They are the solutions of Maxwell equations possessing knot structures. They might be important in plasma physics and fluid dynamics. On the other hand, Ferreira succeeded in obtaining the $3+1$ dimensional exact solutions of a model, which we refer to as the conformal nonlinear σ model (CNLSM), for a complex scalar field. It is expected that the CNLSM has connections to the low energy limit of the Yang-Mills theory and the Skyrme-Faddeev model.

The Lagrangian density of CNLSM is given by $LF = -\frac{1}{4}H\mu\nu H^{\mu\nu}$, $H\mu\nu = \frac{1}{2}n \cdot (\partial\mu n \times \partial\nu n)$ Here n denotes a point on S^2, $n = (n^1, n^2, n^3), \sum a = 1^3 n^a n^a = 1$, which is related to the complex field u by the stereographic projection $u = \frac{n^1+in^2}{1-n^3}$. The field equations are given by

$$\partial\mu(H^{\mu\nu}\partial\nu u) = 0, \tag{1}$$

$$\partial\mu(H^{\mu\nu}\partial\nu u^*) = 0. \tag{2}$$

Because of the conformal symmetry of electromagnetism and CNLSM, solutions of both theories cannot be energetically-stable. The solutions of both theories, however, can be topologically-stable in the sense that conserved topological numbers called Hopf indices can be defined for them.

The present authors have recently pointed out that Ferreira's solution of CNLSM supplies us with a class of exact magnetic knot configurations in some electric charge and current distributions. We have also shown that electromagnetic knot solutions of Maxwell equations can yield solutions of CNLSM other than Ferreira's solutions.

In this talk, we discuss the properties of the magnetic knots induced by Ferreira's Hopf soliton solutions of CNLSM. The electric charge and current densities associated with these magnetic knots should satisfy some constraints. Their behavior for some cases of Hopf indices are investigated analytically and numerically.

Acknowledgements This research was partially supported by the National Natural Science Foundation of China (Grant No.10601031) and the Innovation Program of Shanghai Municipal Education Commission(Grant No. 09ZZ183).

References:

1. Ranada1A.F.Rañada, Lett. Math. Phys.**18**, 97 (1989).
2. A.F.Rañada and J.L.Trueba, Phys. Lett. **A** 202 (1995).
3. L.Ferreira, J. High En. Phys. 0603:075 (2006). arXiv:hep-th/060123v2.
4. L.Faddeev and A.J.Niemi, Phys. Rev. Lett. **82**, 1624(1999).
5. C-G Shi and M.Hirayama, J. High En. Phys.0903:089(2009).