Soliton reflection in defocusing NLS with linearizable boundary conditions

Gino Biondini and Anh Minh Bui Boi
Department of Mathematics, State University of New York, Buffalo, NY 14260, U.S.A.
Tel: 716-645-6284, email: biondini@buffalo.edu, ambuiboi@buffalo.edu

Abstract:

Boundary value problems (BVPs) for integrable nonlinear evolution equations have received renewed interest over the past decade. In particular, a general methodology was proposed in [5], based on an extension of the inverse scattering transform. In all of the approaches to the BVP, however, the problem only truly becomes linear for certain choices of boundary conditions (BCs), which are therefore called linearizable. For the nonlinear Schrödinger (NLS) equation $iq_t + q_{xx} - 2\nu|q|^2 q = 0$ the linearizable BCs for the BVP on the half line $x > 0$ are $q_x(0,t) - \alpha q(0,t) = 0$, with α an arbitrary real constant.
For the focusing NLS ($\nu = -1$) with linearizable BCs, it was shown in [3] that discrete eigenvalues appear in quartets (as opposed to pairs in the initial value problem), and explicit relations exist for the corresponding norming constants. Here we discuss BVPs for the defocusing NLS ($\nu = 1$) with non-zero BCs at infinity and linearizable BCs at $x = 0$. We show that, in the BVP, discrete eigenvalues appear in symmetric pairs in the interval $(-q_0, q_0)$, and a corresponding symmetry exists for the norming constant. As in the focusing case, the apparent reflection of each soliton at the boudary of the spatial domain (see figures) is then due to the presence of a "mirror" soliton, with equal amplitude and opposite velocity, located beyond the boundary.

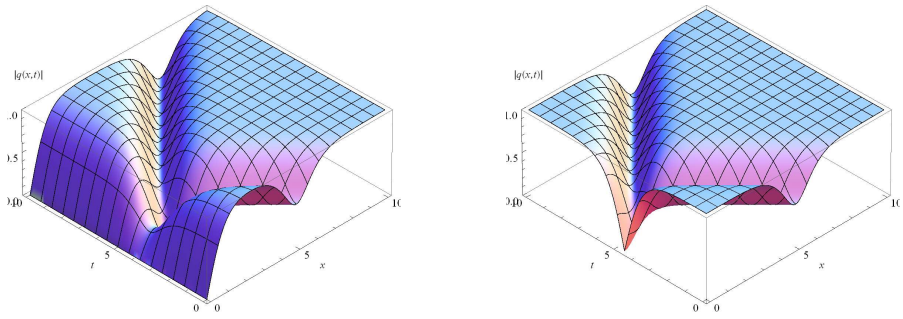

Figure 1: Soliton reflection with Dirichlet BC (left) and Neumann BC (right).

References:

1. M.J. Ablowitz and H. Segur, J. Math. Phys. **16**, 1054-6 (1975)
2. R.F. Bikbaev and V.O. Tarasov, J. Phys. A **24**, 2507-16 (1991)
3. G. Biondini and G. Hwang, J. Phys. A **42**, 205207 (2009)
4. A.S. Fokas, Phys. D **35**, 167-85 (1989)
5. A.S. Fokas, J. Math. Phys. **41**, 4188-237 (2000)
6. V.O. Tarasov, Inv. Prob. **7**, 435 (1991)
7. V.E. Zakharov and A.B. Shabat, Sov. Phys. JETP **37**, 823-828 (1973)