Non-linear dynamics of a thin soliton

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ABSTRACT:

The aim of this talk is to investigate the dynamics of a soliton in a general form, without direct reference to an equation to the order parameter, in terms of the energy $E_s(V, \mu)$ of a soliton in an uniform medium where V is the velocity of the soliton and μ is the chemical potential. The number of atoms in the soliton can be calculated as $N_s \equiv \int_{-\infty}^{\infty} (n - \bar{n})\, dx = -\partial E_s/\partial\mu$. If the medium is placed in an external potential $U(x)$, the motion of a "thin" soliton, whose thickness is small in comparison to characteristic length scale of the potential, is defined by the equation of conservation of energy $E_s(dX/dt, \mu - U(X)) = E_0$ or

$$m_I d^2X/dt^2 = -N_s \partial U/\partial X, \tag{1}$$

where the 'inertial mass" $m_I = 2\partial E_s/\partial V^2$. In typical examples both N_s and m_I are negative. For example, a soliton in BEC, described by the GP-equation, moves as a particle of mass $2m$.

Important and nontrivial problems arise in connection with the phase imprinting experiment, where an optical perturbation creates a soliton with given jump $\Delta\varphi$ of the phase of the order parameter. Assuming that the system possesses Galilean invariance and the equation for order parameter can be obtained by an energy minimization, one can calculate the relation $V(\Delta\varphi)$ from the equation

$$\int_0^V \frac{\partial E_s}{\partial V}\frac{dV}{V} = mN_sV + \frac{m}{m_B}\hbar\bar{n}\left[\Delta\varphi - \pi\right], \tag{2}$$

where $m_B = m$ for the bosonic superfluid and $m_B = 2m$ for the fermionic one.

This equation gives a relation between the "canonical" momentum of the soliton $p_c = \int_0^V \frac{\partial E_s}{\partial V}\frac{dV}{V}$ and the "physical" one mN_sV. The physical meaning of this relation is discussed. It follows that the maximum value of $p_c$is $\pi(m/m_B)\hbar\bar{n}$. For a soliton in a harmonic trap with frequency ω_x, equation (2) gives the equation for the frequency of small oscillations of a soliton

$$\left(1 - \frac{\omega^2}{\omega_x^2}\right) m_I = \frac{\hbar m\bar{n}}{m_B}\left[\frac{d}{dV}\Delta\varphi\right]_{V\to 0}. \tag{3}$$

Confirmation of this equation in real or numerical experiments would be an important check of the theory. Introducing E_s is quite useful for 2D and 3D problems, where it plays role of the surface tension of the soliton. For example, an element of the soliton surface with proncipal radii of curvature R_1 and R_2 moves along its normal with the acceleration

$$d^2X/dt^2 = \frac{E_s}{m_I}\left(R_1^{-1} + R_2^{-1}\right). \tag{4}$$

This relation can be used to investigate solitons stability and solve problems of dynamic of soliton "bubbles", which can be interesting from experimental point of view.