

Singular Solitons and Fourier Transform on Riemann Surfaces

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Abstract:

A lot of singular solutions to the soliton equations like KdV are known. They do not have any direct physical meaning in the Theory of Nonlinear Waves. Corresponding Schrodinger operators do not have any direct meaning in the Quantum Mechanics as well as in the spectral theory of operators in the standard Hilbert Spaces. Indeed, a lot of mathematical works were dedicated to them in the literature. It was found in the late 1980s that singular soliton-type constructions can be used for the right definition of Fourier series on the Riemann Surfaces needed for the operator quantization of strings. In the recent works joint with P.Grinevich, we constructed and investigated the analog of Fourier Transform on Riemann Surfaces. We found that indefinite analogs of Hilbert Spaces are needed. In particular, famous singular Schrodinger operators (like Lamé' operator on the whole line) with singular soliton-type potential are in fact self-adjoint in the indefinite Hilbert Space.