# Recent Progress on Dynamic Stability and Global Regularity of 3D Incompressible Euler and Navier-Stokes Equations

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**Abstract:**

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the seven Millennium Open Problems posted by the Clay Mathematical Institute. We review some recent theoretical and computational studies of the 3D Euler equations which show that there is a subtle dynamic depletion of nonlinear vortex stretching due to local geometric regularity of vortex filaments. The local geometric regularity of vortex filaments can lead to tremendous cancellation of nonlinear vortex stretching, thus preventing a finite time singularity. Our studies also reveal a surprising stabilizing effect of convection for the 3D incompressible Euler and Navier-Stokesequations. Finally, we present a new class of solutions for the 3D Euler and Navier-Stokes equations, which exhibit very interesting dynamic growth property. By exploiting the special structure of the solution and the cancellation between the convection term and the vortex stretching term, we prove nonlinear stability and the global regularity of this class of solutions.