An Efficient and Effective Nonlinear Solver of Large Scale Petroleum Reservoir Simulation

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Milestones in recent years

>>

TWO months

Seq. Commercial Code

MIPS R10K (1998)

0.94 hrs

Our Parallel Code,

LSSC-II 64CPU (2002)

Xeon 2G with Myrinet

Computation Capacity (64 CPUs case)

1998 → → → → 2002

CPU frequency improves : 5 times

Coarsely parallelization : 8 times

Parallel strategy tuning : 8 times

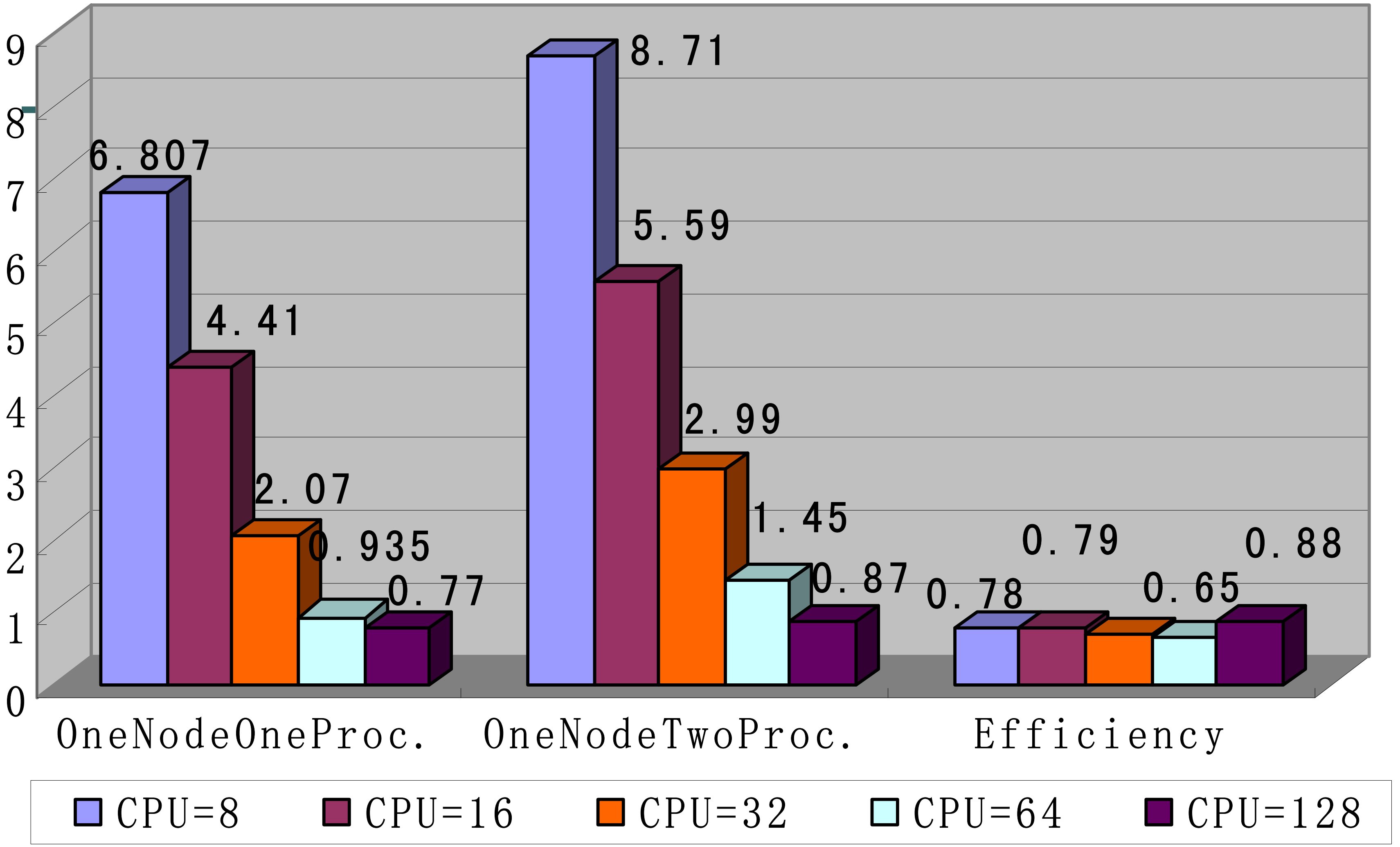
Algorithm acceleration : 5 times

TOTAL CAPACITY : 5x8x8x5=1600

Why this para code is efficient ?

- **Newton-Krylov-Schwartz solution strategy**
  - Robust solver is choosed
  - Avoid frequent global message passing hand and foot
- **MPI-based parallelism strategy**
  - Communication frequency minimization
  - Thoroughly data-based parallelization

- **Dealing with wells & faults using parallel strategy**
  - Wells are divided properly and conquered by its owned processor
  - Faults are localized hand and foot, and treated by a group of processors which own this fault
  - Group communication instead of global communication
- **File I/O process is parallel in order to avoid I/O bottleneck**



9
8
7
6
5
4
3
2
1
0
6.807
4.41
2.07
0.935
0.77
8.71
5.59
2.99
1.45
0.87
0.78
0.79
0.65
0.88
OneNodeOneProc.
OneNodeTwoProc.
Efficiency
CPU=8
CPU=16
CPU=32
CPU=64
CPU=128

Why this para code is effective?

- An elaborate nonlinear solver
  - Inexact Newton iteration is used
  - When IN don't converge satisfactorily, the iteration will be damped or backtracking
  - If IN is difficult to converge or even diverge, initial approximation will rebuild by using BFGS
  - Cutting time step by half is the third choice

- **A well-tuned linear solver**
  - Parallel Preconditioned FGMRES
  - FGMRES(10+), maximum iteration limitation :88
  - Adopt an iterative technique as its preconditioner :
    - ILU+GMRES(10) as a preconditioner : max. # restart: 3
  - Tolerance criteria is related to :
    - Nolinear iteration history evaluation
    - Residual of linear iteration vector,Problem scale

- **A distinct preconditioner**
  - It is based on a multipurpose oblique projection correction method
  - There are three types of oblique projection:
    - A → A_P   global matrix to pressure submatrix
    - A → A_Omega   DDM as a projection
    - A → A_Coarse   AMG as a projection
  - There are eight preconditioning components:
    - Relaxed ILU decomposition

- Additive Schwarz used for DDM
- AMG for initial guess & two level schwarz method
- Watts correction for anisotropy of permeability
- Constraint Residual Precond for BLK model
- CRPextended precond for compositional model
- Shur complement & Multi step Method for coupled system
- Decoupling operator used as left precond

It is adaptive, there are 10 difficult level, higher level, more powerful, and more overhead.

$$T_C := T_{CoarseGrid} := T_{Watts} \bigcup T_{AMG}$$

$$T_1 := T_{AddSchwarz} \bigcup T_{\mathrm{Re}\ laxILU}$$

$$T_2 = P_1 (P_1^T A P_1)^{-1} P_1^T$$

$$T_3 = P_2 (P_2^T A P_2)^{-1} P_2^T$$

$$T_D A T_{Right} (T_{Right}^{-1} x) = T_D b$$

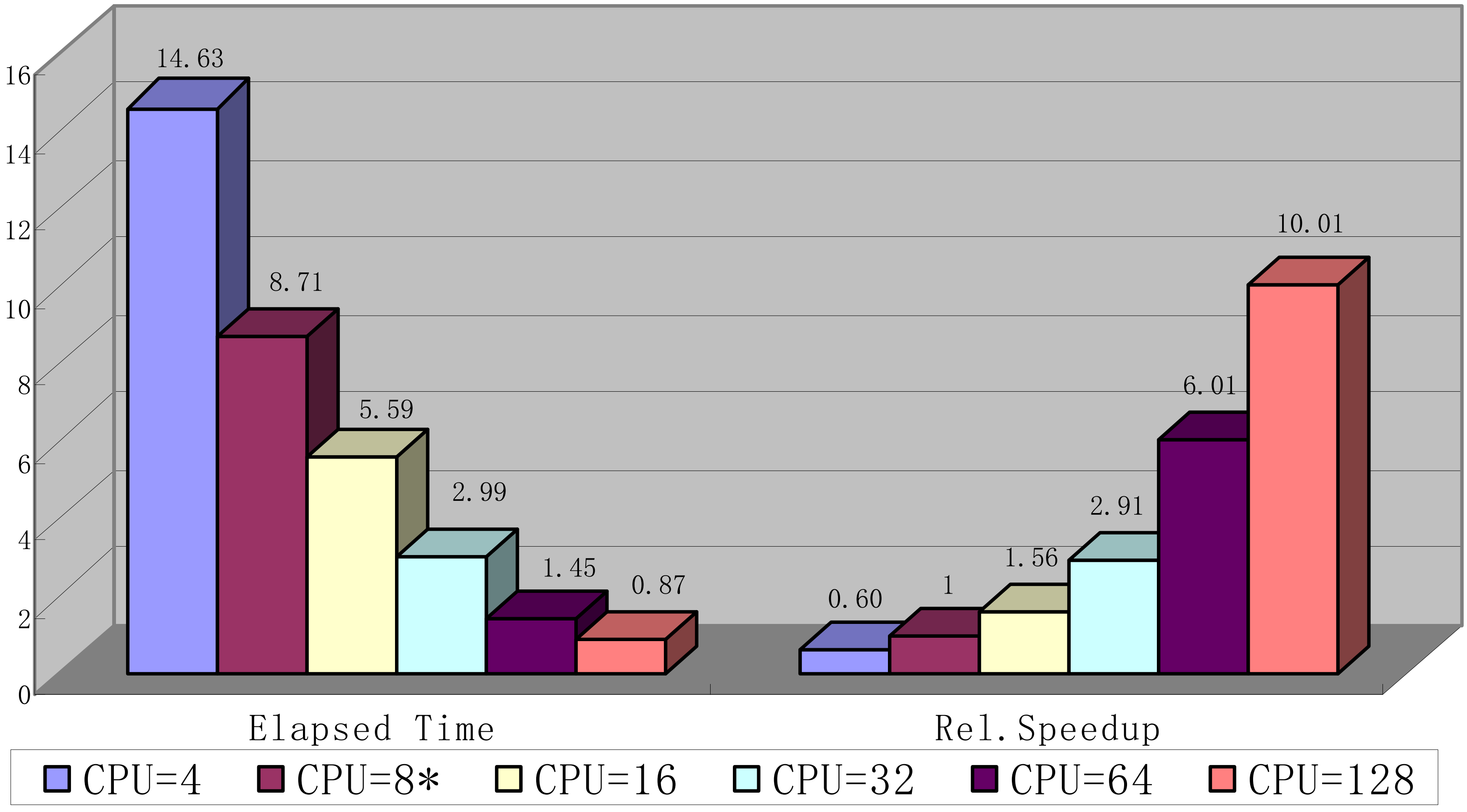
$$B = T_D \bigcup T_{Right}$$

$$I - A T_{Right} = (I - T_3)(I - T_C)(I - T_2)(I - T_1)$$

Practical test cases and results

- **1159K scale BLK data from DaQing**
  - 291 wells, 31.5 years history matching
- **1382K scale BLK data from ShengLi**
  - 326 wells, 14 years history matching
- **5529K scale BLK data from ShengLi**
  - 326 wells, 14 years history matching
- **430K scale BLK data from CNOOC**
  - 29 wells, 22 years history matching

1159K scale BLK data from DaQing



4 x 6 x 8 = 192
4 x 6 x 8 x 12 = 2304

- One time step needs 4 Newton iteration steps
- One Newton step needs 6 FGMRES Steps
- One FGMRES step needs 8 linear iterations
- One linear iteration needs 12 precond. iters.

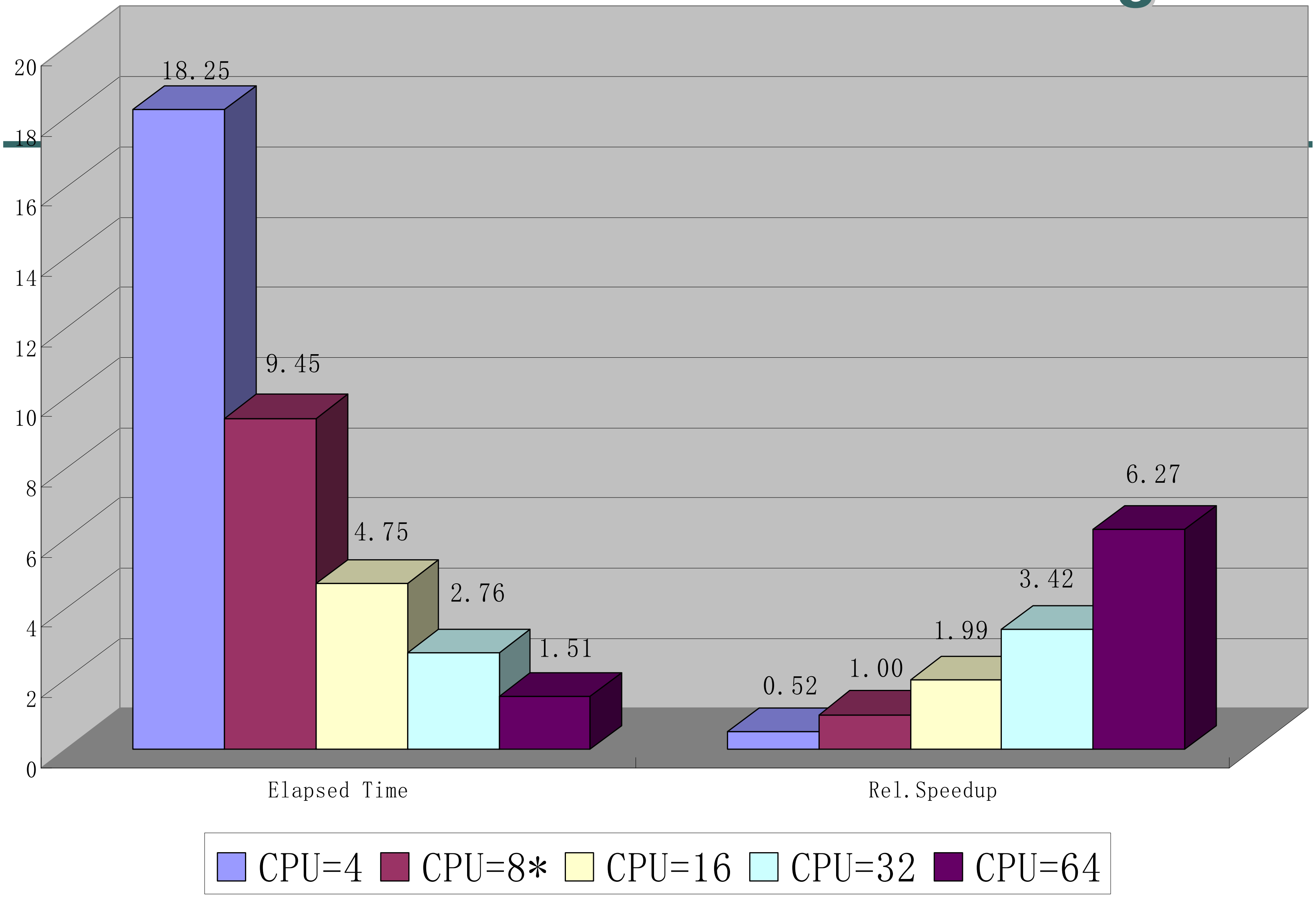
- One Newton step needs 3000 scale reductions
- One FGMRES step needs 500 scale reductions
- One linear iteration needs 65 scale reductions
- One precond.iteration needs 6 scale reductions

- Scale / Broadcast = 91.06%
- Vector / Broadcast = 8.93%

- Neighbor Exchanges / Broadcast = 24.52%

- Average timestep : 90 days
- For CASE1, Delta(t) is larger, so the needed linear iterations & precond. iterations is larger than CASE2.

1382K scale BLK data from ShengLi



5 x 7 x 5 = 175

5 x 7 x 5 x 6 = 1050

- One time step needs 5 Newton iteration steps
- One Newton step needs 7 FGMRES Steps
- One FGMRES step needs 5 linear iterations
- One linear iteration needs 6 precond. iters.

- One Newton step needs 1500 scale reductions
- One FGMRES step needs 220 scale reductions
- One linear iteration needs 44 scale reductions
- One precond.iteration needs 9 scale reductions

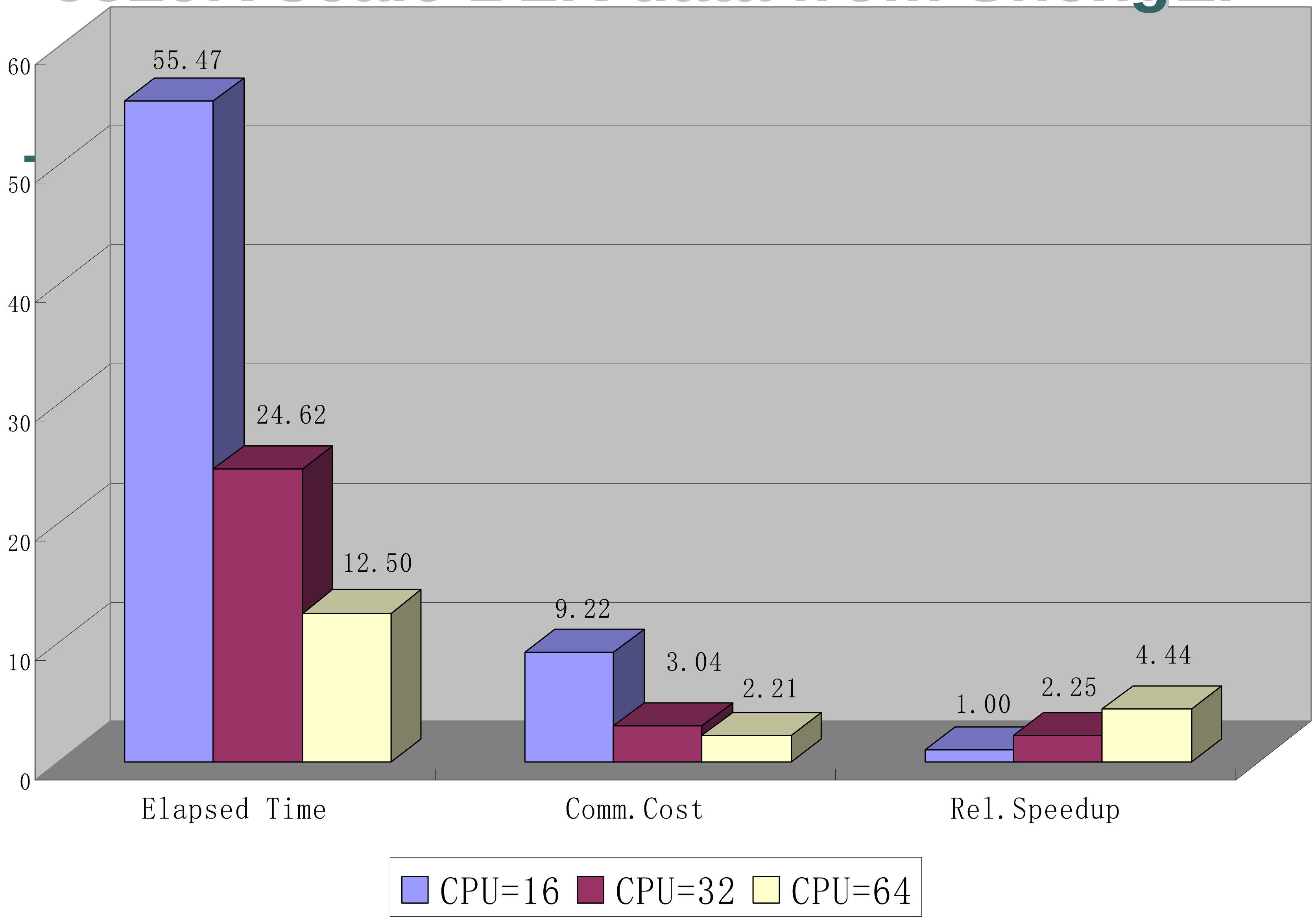
- Scale / Broadcast = 84.54%
- Vector / Broadcast = 15.45%

It is higher than case1 because of more slant wells increase the frequency of vector comm.

- Neighbor Exchanges / Broadcast = 40.89%

- Average time step : 30 days
- For CASE2, Delta(t) is smaller, so the needed linear iteration & precond. iteration are less than that of CASE1

5529K scale BLK data from ShengLi



8 x 8 x 6 = 384

8 x 8 x 6 x 6 = 2304

- One time step needs 8 Newton iteration steps
- One Newton step needs 8 FGMRES Steps
- One FGMRES step needs 6 linear iterations
- One linear iteration needs 6 precond. iters.

- One Newton step needs 2200 scale reductions
- One FGMRES step needs 290 scale reductions
- One linear iteration needs 47 scale reductions
- One precond.iteration needs 9 scale reductions

- Scale / Broadcast = 89.18%
- Vector / Broadcast = 10.81%

- Neighbor Exchanges / Broadcast = 38.44%

- Average time step : 30 days

430K scale BLK data from CNOOC

- Grids : 101x91x47 = 431977
- Faults : 36
- Wells : 7+22
- Elapsed time :
  - 2p : 7382s 4p : 3222s
  - 8p : 2157s 16p : 1204s
  - 32p : 1376s

- Average time step : 320 days
- For CASE4, Delta(t) is largest, so the needed linear iterations & precond. iterations is largest among these CASEs

- Scale / Broadcast = 76.80%
- Vector / Broadcast = 23.19%
- Neighbor Exchanges / Broadcast = 15.63%

It is higher than others because 36 faults increase the frequency of vector comm.

Behind these results ……

- **If the number of grids improves 4 times, then the elapsed time will improve about 9 times, where**
  - **the frequency of global reduction improves about 3 times**
  - **the number of Newton step improves about 2 times**
  - **the computation complexity of linear iteration improves about 4 times**

- For one million grid scale problem, every 100 time step of discrete parabolic equations will need
  - 300-500 newton steps,
  - 2400-3500 FGMRES steps,
  - 14K-20K linear iterations,
  - 0.5M-1M global communications,
- and it can be simulated in **one hour** on Beowulf clusters **(128 Nodes).**

- For five million grid scale problem, every 100 time step of discrete parabolic equations will need
  - 600-1000 newton steps,
  - 5000-8000 FGMRES steps,
  - 30K-40K linear iterations,
  - 1M-2M global communications,
- and it can be simulated in **three hours** on Beowulf clusters **(256 Nodes).**

- **Global Communication**
  - The ratio of scale reduction exceeds 80%
  - The ratio of vector reduction is less than 20%
  - The frequency of neighbor's exchanging messages is less than 50% of global communication
  - So, the communication process of scale reduction is a bottleneck of simulation
  - Algorithm & parallelism strategy should be adopted in order to satisfy minimization of scale reduction