Convergent evolving finite element methods for geometric flows

Abstract

Geometric flows of closed surfaces are important in a variety of applications, ranging from the diffusion-driven motion of the surface of a crystal to models for biomembranes and tumor growth. Basic geometric flows are mean curvature flow (described by a spatially second-order evolution equation) and Willmore flow and the closely related surface diffusion flow (described by spatially fourth-order evolution equations).

Devising provably convergent surface finite element algorithms for such geometric flows has long remained an open problem, going back to pioneering work by Dziuk in 1988. Recently, a solution to this problem for various geometric flows including those mentioned above was found and explored in joint work with Balάzs Kovάcs and Buyang Li.

The proposed algorithms discretize evolution equations for geometric quantities along the flow, in our cases the normal vector and mean curvature, and use these evolving geometric quantities in the velocity law interpolated to the finite element space. This numerical approach admits a convergence analysis, which yields optimal-order H^1-norm error bounds.