

Principles of Systematic Upscaling

Combining Multigrid techniques with Renormalization Group ideas from theoretical physics has given rise to the Systematic Upscaling (SU) paradigm. For complex multiscale physical systems, where first-principle equations (or statistical rules) are known at some microscopic scale, SU is a general and rigorous computational methodology for deriving, scale after scale, suitable variables and operational rules for processing the system at increasingly larger (eventually macroscopic) scales. No scale separation is assumed; as in multigrid, small scale ratio between successive levels is in fact important to ensure slowdown-free simulations at all scales. Unlike multigrid, however, at each scale computations need to be performed only in relatively few representative "windows" (subdomains, each containing relatively few variables). Examples will be given in terms of local-interaction systems at equilibrium, and extensions will briefly be discussed to long-range interactions, dynamic systems, low temperatures, and other types of problems.