

Abstract

During the last half century computer simulation has become the leading tool for studying the behavior of man-made and natural physical systems, and increasingly for biological and social systems as well. Such simulations begin with a mathematical model which is then discretized numerically so it can be solved or investigated on a computer. The validity of these studies depends on the accuracy of the numerical discretization, that is, on the convergence of the numerical solution to the exact solution of the mathematical model as the discretization is refined. The accuracy of numerical discretization, in turn, depends on the consistency and stability of the discretization method used. This theme, that consistency and stability imply convergence, recurs throughout numerical analysis, and is especially important in the numerical solution of partial differential equations. But the concept of numerical stability can be subtle and elusive. Even simple examples can yield unexpected, even catastrophic, results, and the development of stable numerical algorithms remains elusive for important classes of problems. We will survey these ideas through a variety of examples, and describe some modern tools from geometry and topology which are taking their place along side the more classical analytic tools for designing and understanding stable numerical discretizations.