随机循环矩阵的有限等距性质研究

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摘　要

压缩感知是近年来国际上热门的研究方向，其广泛应用于数字相机、医学成像、遥感成像、地震勘探等领域. 其主要思想为：利用信号的稀疏性，通过尽量少的观测信息恢复信号. 对压缩感知的研究主要分为两方面，一方面是对观测矩阵的研究；另一方面是对重建信号算法的研究. 本文主要是研究了压缩感知中的观测矩阵，探讨了部分循环矩阵、Hankel 矩阵、Teoplitz 矩阵的有限等距性质，证明了对于 k 稀疏的向量，从 $n\times n$ 的随机矩阵中随机选取 q 行的部分随机矩阵 P 大概率满足 k-阶有限等距性质，其中对于部分 Bernuolli 随机循环矩阵，q 的量级优化至 $q=\mathcal{O}(k\log^2 k\log n)$. 2010年，Rauhut等人的研究结果为 $q=\mathcal{O}(k\log n)^{3/2}$. 2014年，Krahmer等人的研究结果为 $q=\mathcal{O}(k\log^2 k\log^2 n)$；对于部分 Gauss 随机矩阵，$q$ 的量级为 $q=\mathcal{O}(k\log^2 n\log^2(k\log n))$，与的研究结果近似，我们给出了新的证明方法. 另外我们将上述结果推广至 Hankel 矩阵上.

首先，本文证明了部分 Bernuolli 随机矩阵在 $q=\mathcal{O}(k\log^2 k\log n)$ 的条件下满足 k-阶有限等距性质. 主要证明过程是：先证明了 Bernuolli 随机矩阵大概率近似具有酉矩阵的性质，进而与已有研究结论相结合可以得到部分 Bernuolli 随机矩阵满足有限等距性质的条件. 并且有效的将随机抽取行的行数 q 的量级做了优化 $q=\mathcal{O}(k\log^2 k\log n)$.

其次，鉴于上述结果主要是针对稀疏度较大的向量 $k\leq\mathcal{O}\left(\sqrt{n/\log(n/k)}\right)$，我们将结果推广至更一般的情况，放宽了对于向量稀疏程度 k-的限制. 在保证大概率的情况下，由服从 Bernuolli 和 Gauss 分布的随机向量生成的循环矩阵和 Hankel 矩阵中随机从中抽取 q 行作为观测矩阵仍然可以满足 k-阶有限等距性质. 对于部分 Bernuolli 随机循环矩阵，$q=\mathcal{O}(k\log^2 k\log n)$，与之前结果相比量级有所提高；对于部分 Gauss 随机循环矩阵，$q=\mathcal{O}(k\log^2 n\log^2(k\log n))$，我们给出了新的证明.

关键词：压缩感知，随机循环矩阵，有限等距性质，Bernuolli 分布，Gauss 分布

Abstract

Compressed sensing is a recent popular research direction internationally, which widely used in digital camera, medical imaging, remote sensing imaging, seismic exploration and other fields. Its main idea is: Under the condition that the signal is sparse, restore the signal with as little observational information as possible. The study of compressed sensing is mainly divided into two aspects, one to study the observation matrix; The other to study the reconstruction signal algorithm. This paper mainly focuses on the study of observation matrix in compressed sensing, and do studies on the restricted isometry property of partial circulant matrices, partial Hankel matrices, and partial Teoplitz matrices. For the k sparse vectors, randomly select q rows from an observation matrix of $n\times n$, to make its high probability satisfy the restricted isometry property of order k. For partial Bernuolli circulant matrix, the level of q is better, $q=\mathcal{O}(k\log^2 k\log n)$. Results of previous research mainly focused on partial circulant matrix. The result of Krahmer in 2010 is $q=\mathcal{O}(k\log n)^{3/2}$. The result of Krahmer in 2014 is $q=\mathcal{O}(k\log^2 k\log^2 n)$. For partial Gauss circulant matrix, the level of $q=\mathcal{O}(k\log^2 k\log^2 n)$ equal to the result of Krahmer. We have developed new proof methods. Moreover, we promote the results to the Hankel matrix.

Firstly, this paper proves that partial Bernuolli stochastic matrices satisfy the restricted isometry property of order k under the condition of $q = \mathcal{O}(k \log^2 k \log n)$. The main proofs are as follows: Firstly, we prove that Bernuolli stochastic matrices has the property of Unitary matrices with high probability, then combine with existing research conclusions and find out the condition when partial Bernuolli stochastic matrices satisfy restricted isometry property. We also effectively optimize the row level of randomly drawn rows, $q = \mathcal{O}(k\ log^2 k\ log^2 n)$.

Moreover, the above results are mainly for vectors with large sparseness, $k \leq \mathcal{O}\left(\sqrt{n/\log(n/k)}\right)$. We promote the results to a more general case and relax the restriction on vector sparsity k. In the case of ensuring high probability, we randomly drawn row q from the circulant matrices and Hankel matrices generated from the Bernuolli and Gauss distributions as the observation matrix, and find that it can still satisfy the restricted isometry property of order k . In other words, choosing these matrices as the observation matrix can still recover the vector x with high probability. At the same time, for a matrix generated from random vectors obeying Bernoulli' s distribution, $q = \mathcal{O}(k \log^2 k \log n)$, the level of q remains the same as before; For a matrix generated from a random vector obeying the Gauss distribution, $q = \mathcal{O}(k \log^2 n \log^2(k \log n))$.

Keywords:Compressed Sensing, Subsampled Random Convolutions, Restricted Isometry Property, Bernuolli distribution, Gauss distribution