单相和两相薄层颗粒流模型的数值方法与GPU计算研究

摘要

2016 年 7 月 1 日

薄层颗粒流模型是浅水方程在颗粒崩塌流中的推广，可用于研究滑坡碎屑流、泥石流、火山灰流等地质灾害流动问题。发展准确高效的求解薄层颗粒流模型的数值方法及程序，有助于这些地质灾害的评估和防治研究。本论文中，我们针对两个比较著名的薄层颗粒流模型，即单相流的 Savage-Hutter(SH) 模型和两相流的 Pitman-Le 模型，研究相应的数值方法，并实施 GPU 并行计算。

SH 模型是将无数个干颗粒组成的堆积物处理成连续的库仑材料，从满足库仑本构关系的不可压 Navier-Stokes 方程出发用深度平均方法导出的偏微分方程组。库仑介质区别于一般流体的一个特征是它能抵抗一定大小的剪切应力而保持静止。我们应用 Davis 的二阶预估-校正的 Godunov 型有限体积法求解 SH 模型。该方法对守恒变量使用 MUSCL 重构，对数值通量使用 HLLC 格式。我们在求解过程中加入了颗粒流的静力学条件和停止准则。数值算例验证了修改后的 Davis 方法能准确地模拟颗粒激波和颗粒在缓坡上停止的现象并能稳健地处理干湿前沿情形。用该方法的 GPU 并行程序模拟了两个实际问题，一个是液滴型颗粒射流实验，计算中采用了更符合实际的变摩擦角摩擦律；另一个是圆柱颗粒绕流实验，计算中采用贴体的极坐标和大网格数，捕捉到了前人文献中未曾捕捉到的真空区域后的小楔形激波。

Pitman-Le 两相薄层颗粒流模型包括液体和固体的质量方程和动量方程，并且耦合守恒和非守恒的动量交换项。该模型对于那些需要 Riemann 解算器的数值方法带来很大的麻烦，不仅因为没有解析的特征系统，而且因为在某些特殊的流动条件下方程系统会出现复特征值的情形。无振荡中心 (Non-Oscillatory Central，NOC) 格式由于无需求解 Riemann 问题而显得更有吸引力。我们将时空交错的 NOC 格式推广于 Pitman-Le 两相流模型的数值求解，对非守恒项和河床斜率源项使用了一种简单的离散格式，配合额外计算的自由面高程变量对由 NOC 格式计算出的深度变量进行修正，保证了格式的完全平衡性 (well-balancedness)。对于在某些流动条件中出现复特征值的情形，我们采用数值的相间拽力项确保系统的双曲性。通过几个典型的两相薄层流算例验证了所发展的 NOC 格式可以处理极端的流动条件包括形成真空区和出现复特征值等情形，表明格式有较强的健壮性。

在 GPU 计算方面，我们对求解 SH 模型的 Davis 方法实施基于 CUDA 的 GPU 计算，给出了一种 GPU 上分配二级指针的技巧，方便了串行程序向 GPU 程序的移植。对求解 Pitman-Le 两相流模型的 NOC 格式进一步实施了单节点多 GPU 计算，分别采用 OpenMP-CUDA 和 Multistream-CUDA 这两种并行策略。计算测试发现在小网格数时 Multistream-CUDA 比 OpenMP-CUDA 稍慢，但在大网格数时稍快，这表明采用不同多 GPU 策略的并行效率会产生较小的差异。然而多 GPU 的计算效率更多地受到单 GPU 加速比性能的影响。

关键字：Savage-Hutter 模型，停止准则，两相，浅水流，HLLC 格式，NOC 格式, 完全平衡，GPU 计算

Abstract

Shallow granular flow models are extensions of traditional shallow water equations to granular avalanche flows. They are used to study landslide debris flows, mud flows, and pyroclastic currents. Development of accurate and efficient numerical methods and codes for shallow granular flow models are helpful to evaluation and protection of the geological disasters. In this thesis, we study numerical methods for the Savage-Hutter (SH) equations and for the Pitman-Le two-phase shallow granular flow equations, which are two popular models in the study of shallow granular flows, and implement GPU computing to accelerate numerical computation.

Treating a finite mass of countless dry granular piles as continuum Coulomb materials, the SH model is a system of partial differential equations derived from the incompressible Navier-Stokes equations obeying Coulomb-type constitutive relations by using depth-averaged approach. Distinguished from general fluids, Coulomb medium can resist certain magnitude of shear stress to remain static. We apply Davis' second-order predictor-corrector Godunov type method to solve SH equations. The method uses MUSCL reconstruction for conservative variables and HLLC scheme for numerical fluxes. Static resistance conditions and stopping criteria are incorporated into the solution algorithm. Numerical examples verify that the modified Davis' method can simulate the granular flows with shocks and slope deposits accurately and deal with dry-wet fronts robustly. The implemented GPU parallel computing code is used to simulate two application examples, one is the granular jet experiment with teardrop-shaped shocks described by variable friction coefficients, another is the granular avalanches around a circular cylinder. By adopting body-fitted coordinate and large grid number, we capture the small wedge shock behind the vacuum region, which is not reproduced in literature.

The two-phase shallow granular flow model due to Pitman and Le consists of mass and momentum equations for both the solid and fluid phases, coupled together by conservative and non-conservative momentum exchange terms. Numerical methods based on Riemann problem solutions for such models may face difficulties because complex eigenvalue may occur in some special flow conditions. As a Riemann problem-free method, Non-oscillatory central (NOC) differencing schemes become attractive. We extend a space-time staggered NOC scheme to numerically solve the Pitman-Le two-phase granular flow model. Simple discretization schemes for the non-conservative and bed slope source terms and a correction procedure for the updating of the depth variables are proposed to ensure the well-balanced property. For some flow conditions associated with complex eigenvalues, the interphase drag force term is numerically adopted to ensure the hyperbolicity of the equations. Several typical numerical

tests in two-phase shallow granular flow problems show the NOC scheme can overcome the difficulty associated with complex eigenvalues occurring in some flow conditions and model wet/dry fronts and vacuum appearance robustly.

In the aspect of GPU computing, Davis' method for solving SH equations has been implemented on single GPU based on CUDA. A skill for allocating memory of second-rank pointer on GPU is given and it makes porting serial programme to GPU programme convenient. Multi-GPU computing in a single node has been further carried out with NOC schemes for the Pitman-Le two-phase granular flow model. Two parallel strategies, OpenMP-CUDA and Multistream-CUDA, have been adopted. Comparison of parallel efficiencies between the two strategies shows that the Multistream-CUDA strategy can be slightly slower than the OpenMP-CUDA strategy with small grid number, but faster with large grid number, which indicate different Multi-GPU strategy will not affect the performance of GPU parallel computing significantly. However, the efficiency of Multi-GPU is greatly affected by the capability of single GPU speedup.

keywords: Savage-Hutter equation, stopping criterion, two-phase, shallow flow, HLLC scheme, NOC, well-balanced, GPU computing