Numerical Solutions of Delay Differential Equations using Spectral Methods

Ishtiaq Ali

Abstract

Delay differential equations (DDEs), also called the functional differential equations arises in many applications, like biology, medicine, control theory, climate models, population dynamics, electrical networks, and many others, where DDEs are used to constitute basic mathematical models for such real phenomena. Due to the similarities between DDEs and ordinary differential equations (ODEs), less attention was paid to the numerical solutions of DDEs in early stage of research contrary to the fact at that time as solving DDEs numerically is more complicated as compare to the the numerical treatments of ODEs. In DDEs the unknown not only depend on the derivatives of independent variables (time) at present value but also contain an additional derivatives which depend on the solution at some previous time. The principal difficulty in studying DDEs lies in their special transcendental nature. Hence, therefore they are often solved using numerical methods, asymptotic solutions, approximations and graphical approaches.

This thesis presents an efficient numerical approach for delay differential equations with vanishing proportional delays based on spectral methods. In the first part of this thesis we develop a spectral approach for the pantograph type-delay differential equations. A Legendre spectral-collocation method is employed to obtain highly accurate numerical approximations to the exact solution. We also discuss briefly the case of nonlinear and system of pantograph-type delay differential equations. A more straight forward approximation which does not use the quadrature rule is also introduced for the linear pantograph-type delay differential equation. It is proved theoretically and demonstrated numerically that the proposed method converges exponentially provided the data in the given pantograph delay differential equation are smooth. We then, extend our proposed method to more general pantograph delay equation with constant as well as with variable co-efficients. The accuracy of computation of these time-varying coefficients by standard schemes for ODEs is of finite order; in contrast spectral methods are free from this disadvantage and thus optimal in terms of efficiency. The exponential convergence was obtained theoretically which was confirmed by some numerical

examples. We also analyze the convergence properties of the spectral method for delay differential or integral equations with two or more vanishing delays. It is shown that for the pantograph-type functional equations the spectral methods yields the familiar exponential convergence. Various numerical examples are used to illustrate these results.

In the 2nd part of this thesis spectral methods for the integro-delay differential equation is introduced. A Gauss quadrature formula relative to the Legendre weights is used to evaluate the integral term. The method has spectral accuracy, which means that a very accurate solution can be obtained using relatively few data points. Numerical results confirm the theoretical justification of the exponential rate of convergence. Finally, a Jacobi spectral-collocation method is presented for the integro-delay differential equation with weakly singular kernels, followed by some numerical examples.

Keywords: Delay differential equations, Spectral methods, Integro-delay differential equation, Convergence analysis, Numerical examples.