求解线性规划的不可行点方法

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摘要

线性规划问题是约束优化中最简单的一类问题，它是在一组线性的等式或者不等式的约束下求解一个线性函数的最大值或最小值。线性规划问题广泛应用于工农业、军事、交通运输、决策管理与规划、科学实验等领域，因此针对它的算法研究一直受到许多学者的关注。

线性规划问题作为一类独立的问题被提出以后，出现了很多求解方法。特别是1984年Karmarkar 提出了第一个多项式时间算法以后，用内点算法求解线性规划问题成为数学规划中的一个热点研究方向。内点算法中的的仿射均衡尺度方法、路径跟踪算法、势函数下降等方法无论从理论上还是具体算法实现上都使得线性规划问题的研究趋向完善。在众多的内点算法中，最有效的一类是不可行的路径跟踪方法，现有的大部分解线性规划的软件包如Cplex, LIPSOL, LOQO, HOPDM, BPMPD等都是基于此类方法。而预测校正方法则是路径跟踪算法中最有效的算法，基于Mehrotra不可行预测校正方法的LIPSOL是目前应用较为广泛的软件包。

本文的目的是在Mehrotra方法的基础上，通过改进校正方向从而进一步提高LIPSOL软件包的效率。在第三章中我们给出一个新的校正方向求解方法，改进后的算法保证迭代点在中心路径的宽邻域内并尽可能的以相近的速度实现可行性与最优性，最后给出了数值实验结果。本文第四章给出了另外一种校正方向的求法：根据前次迭代中得到的点，按照一定的规则构造一个多维的中心因子，再应用多维中心因子求解校正方向，文中给出了理论分析和数值实验结果。在第五章中我们指出了Mehrotra方法的一个缺陷并提出了改进方法。

关键词：线性规划，内点算法，预测校正算法，中心路径。

Linear programming is the simplest problem in constrained optimization. It aims to obtain the maximal(minimal) value of a linear function, subject to a group of linear equations or inequality constraints. Linear programming plays an important role in many fields such as agriculture ,industry, military affairs, transportation, managements, scientific experiments and so on, and it has received much attention in optimization world.

Since the birth of linear programming, many algorithms for solving it were proposed, including the Simplex method by Dantzig. In 1984 Karmarkar surprised the world by announcing a fast polynomial-time algorithm. After that interior-point methods became one of the most popular research area in mathematical programming. We can solve linear programming with affine-scaling methods, path-following methods, potential-reduction algorithms and other interior-point methods. Among many interior-point methods, the infeasible-primal-dual path-following algorithms are the most effective. Now most codes for solving linear programming are based on these algorithms, such as Cplex, LIPSOL, LOQO, HOPDM, BPMPD and so on. The predictor-corrector methods are the most effective among infeasible-primal-dual path-following algorithms. The matlab codes 'LIPSOL' are based on Mehrotra infeasible predictor-corrector method.

Our main purpose is to increase the efficiency of LIPSOL by improving the corrector direction in the Mehrotra predictor-corrector method. In Chapter 3, we present a new method to compute corrector direction. The point during each iteration is limited in the wide neighborhood of the central path, and the rates to achieve the feasibility and optimality are almost equivalent. In the end, we present the numerical results. In Chapter 4, we propose another method to obtain corrector direction: based on the previous point in the algorithm we construct a vector of centrality parameter according to some rules. Applying with the new parameter, we can get a corrector direction by solving linear equations. Analysis of the improved algorithm and numerical results are presented at the end of Chapter 4. In the last chapter, a deficiency of Mehrotra method is pointed out. We provide a technique to overcome it.

Keywords: linear programming, interior-point method, predictor-corrector method, central path.