内点信赖域算法及其应用

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Abstract

界约束的非线性规划问题是一类特殊的约束非线性规划问题，一方面由于这类问题和无约束的优化问题 非常的相似，仅有非常简单的界约束，另一方面，这类问题又是最简单的约束优化问题。内点 信赖域方法是求解这类问题的一个重要方法。本文对求解界约束非线性规划问题的内点信赖域算法 进行了研究，提出了一种新的内点信赖域算法求解此类问题。并将内点信赖域算法应用到求解 大规模图象恢复问题。对于这种大规模的问题，我们还提出了一种子空间的内点信赖域算法，大大提高了 求解图象恢复问题的效率。

内点算法是起源于线性规划的一类重要的优化算法，现在已被广泛应用于非线性规划，组合优化，互补问题等 优化问题。而信赖域方法最早是用来求解无约束优化问题的。信赖域算法的特点在于不是用线搜索的方式 确定步长，而是通过求解信赖域子问题来得到试探步，然后计算目标函数的真实下降量和模型的预估下降量的 比值来决定接受或者拒绝该试探步。对于界约束的问题，内点算法和信赖域算法可以相结合来设计出新 的适合于该类问题的算法。

对于界约束的非线性规划问题，现在已经有了很多的算法。这些算法主要可以分成三大类，即 积极集法，投影梯度法和内点法。我们将在第二章对这些方法做一个简单的介绍。

对于求解界约束非线性规划问题的内点信赖域算法，Coleman和Li等很多学者已经做了许多深入的研究。 在本文的第三章，我们提出了一个新的内点信赖域算法，其中应用到了投影技巧和曲线搜索技巧。 我们从理论上证明了我们提出的这个新的算法具有全局收敛性。我们对一组标准的测试问题进行了 数值实验，结果表明我们的算法优于Coleman和Li在1996年所提出的算法。因此我们 的算法应该有着非常强的实用价值。

在本文的第四章，我们将内点信赖域算法应用到数字图象处理领域来求解非负图象恢复问题。 图象恢复问题目前在遥感科学，大气成像分析等领域受到广泛的关注。由于图象恢复问题本身 的物理意义，对问题添加一个非负约束是非常必要的，从而这个问题可以归结为一个非负约束的 非线性规划问题。对于这类问题，以前的方法一般都是通过引入其他参数或者变换来处理非负约束， 从而将问题转化为一个无约束的问题，而且一般都是在全空间来求解这个问题。我们提出了一个 子空间的内点信赖域算法来求解这个问题。首先，这个方法不需要引入参数或者变换；而且，子空间技术的 应用使得我们的方法非常适合于求解大规模问题。针对我们的算法中的主要运算是矩阵向量乘法这一 特点，我们设计了一个新的算法来计算稀疏矩阵和向量的乘法运算，极大的减少了运算量。数值结果也 表明我们的算法对于大规模的图象恢复问题是非常有效的。

最后，我们在第五章，对我们的工作进行了总结，并对以后的工作以及遗留的问题进行了简单的讨论。

关键词: 内点算法，信赖域方法，非线性规划，界约束，非负约束，子空间，共轭梯度法， 图象恢复

Interior Point Trust Region Method and Its Applications

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The box constrained nonlinear programming problems are special problems in constrained nonlinear programming. They are very similar to unconstrained problems except simple box constraints, and they are the simplest constrained problems. The interior point trust region method is an important method for such problems. In this thesis, we give some investigations in applying the interior point trust region method to solve the box constrained nonlinear programming problems. We also give an application of the interior point trust region method for solving the image restoration problems, where the subspace technique is used due to the large scale of the problems.

The interior point method orients from Linear Programming and now has been widely applied to Nonlinear Programming, Combinatorial Optimization, Complementarity Problem, and so forth. The trust region method, originally designed for unconstrained optimization problems, is different from the line search method in deciding the stepsize. Actually, the trust region method does not carry out line searches, instead it either accepts or rejects the step based on the ratio of the actual reduction of the object function and predicted reduction of the quadratic model. For the box constrained problems, it can be solved by the algorithm combining the interior point and trust region methods.

There are many existing methods for box constrained nonlinear programming problems. The frequently used methods are active set method, projected gradient method, and interior point method. In Chapter 2, we give a survey on these methods for box constrained problems.

The interior-point trust-region approaches for simple bound constrained problems have been investigated by Coleman and Li and many other authors. In Chapter 3, we present a novel algorithm to compute the trial step in the interior-point trust-region framework. The projection and curvilinear search techniques are implemented. We proved the global convergence property of our method. Our new algorithm is tested for a set of standard test problems, and the numerical results show that our new algorithm outperforms the existing methods of the similar type. This indicates that our algorithm has practical potential.

In Chapter 4, we give an application of the interior point trust region method to image restoration problems. Image restoration problems appear frequently in remote sensing and astronomical image analysis. For image restoration problems, it is reasonable to add nonnegative constraints because of the physical meaning of the image. Then the problem can be expressed as a quadratic programming with nonnegative constraints. In the previous research, the parameterization technique is needed to reduce the problem to an unconstrained optimization problem and the problem is solved in a full space. We propose a subspace interior point trust region method to solve this type of problems, and our method does not need parameterization. We use the subspace technique because the dimension of image restoration problems is always very large, and the subspace technique can greatly reduce the computational costs so that large scale problems can be solved in reasonable time. Because the main cost of our algorithm is the operation of matrix-vector multiplication, we also present a novel approach for reducing the cost of sparse matrix-vector multiplication. The numerical experiments show that this method is suitable for large scale image restoration problems.

Finally, we give some conclusions and expectations in the Chapter 5.

Keywords: interior point method, trust region method, nonlinear programming, bound constrained, nonnegatively constrained, subspace, conjugate gradient method, image restoration