

On Friedrichs–Poincaré-type inequalities [☆]

Weiying Zheng

^a *Institute of Computational Mathematics, Academy of Mathematics and System Sciences,
Chinese Academy of Sciences, PO Box 2719, Beijing 100080, PR China*

^b *Bureau of Basic Research, Chinese Academy of Sciences, Beijing 100864, PR China*

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Abstract

Friedrichs- and Poincaré-type inequalities are important and widely used in the area of partial differential equations and numerical analysis. Most of their proofs appearing in references are the argument of reduction to absurdity. In this paper, we give direct proofs of Friedrichs-type inequalities in $H^1(\Omega)$ and Poincaré-type inequalities in some subspaces of $W^{1,p}(\Omega)$. The dependencies of the inequality coefficients on the domain Ω and some sub-domains are illustrated explicitly.

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1. Introduction

Friedrichs-type inequalities and Poincaré-type inequalities are very important tools and widely used in the area of partial differential equations and numerical analysis. They are frequently used in proving the existence of the solution of partial differential equation and in finite element error estimates. These inequalities ensure that the solution is in a

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E-mail addresses: zwy@lsec.cc.ac.cn (W. Zheng).