

Algebraic Multigrid Methods

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This lecture series is based on an invited review article entitled “Algebraic Multigrid Methods” which is being completed by the lecturer J. Xu (in joint with L. Zikatanov) and will appear in Acta Numerica 2017. It is a self-contained short course for an overview of AMG methods for solving large scale systems of equations such as those from the discretization of partial differential equations. AMG is often understood as the acronym of “Algebraic Multi-Grid”, but it can also be understood as “Abstract Muti-Grid”. Indeed, as it will demonstrate in this course, how and why an algebraic multigrid method can be better understood in a more abstract level. In the literature, there are a variety of different algebraic multigrid methods that have been developed from different perspectives. In this course, we try to develop a unified framework and theory that can be used to derive and analyze different algebraic multigrid methods in a coherent manner. Given a smoother R for a matrix A , such as Gauss-Seidel or Jacobi, we prove that the optimal coarse space of dimension n_c is the span of the eigen-vectors corresponding to the first n_c eigen-vectors $\bar{R}A$ (with $\bar{R}R = R + R^T - R^TAR$). We also prove that this optimal coarse space can be obtained by a constrained trace-minimization problem for a matrix associated with $\bar{R}A$ and demonstrate that coarse spaces of most of existing AMG methods can be viewed some approximate solution of this trace-minimization problem. Furthermore, we provide a general approach to the construction of a quasi-optimal coarse space and we prove that under appropriate assumptions the resulting two-level AMG method for the underlying linear system converges uniformly with respect to the size of the problem, the coefficient variation, and the anisotropy. Our theory applies to most existing multigrid methods, including the standard geometric multigrid method, the classic AMG, energy-minimization AMG, unsmoothed and smoothed aggregation AMG, and spectral AMGe.

Tentative Schedule:

Day 1

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Lecture 1.1 Introduction, Linear vector spaces and duals

Lecture 1.2 Model problems and discretization

Lecture 1.3 Graphs and sparse matrices

Day 2

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Lecture 2.1 Basic iterative methods

Lecture 2.2 Abstract multigrid methods: Part I

Lecture 2.3 Abstract multigrid methods: Part II

Day 3

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Lecture 3.1 Strength of connections

Lecture 3.2 Coarsening algorithms: Part I

Lecture 3.3 Coarsening algorithms: Part II

Day 4

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Lecture 4.1 Trace-min, energy-AMG

Lecture 4.2 Aggregation AMG

Lecture 4.3 Classic AMG

Day 5

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Lecture 5.1 Adaptive AMG

Lecture 5.2 Examples: Part I

Lecture 5.3 Examples: Part II

The prerequisite of the short course is a basic knowledge of numerical linear algebra and numerical partial differential equations. Some homework problems will be assigned and some research topics will be suggested. This short course is part of the course named “Multilevel Iterative Methods”. More information about the course can be found online at

<http://lsec.cc.ac.cn/~zhangcs/include/teaching.html>

Lecture Time: Oct 31 (Monday) — Nov 4 (Friday), 8:30am—11:30am

Location: S Hall, AMSS (中科院数学与系统科学研究院思源楼一楼大报告厅)