

# Titles and Abstracts

## 1. James Adler

- **Title:** Numerical Approximation of Asymptotically Disappearing Solutions of Maxwell's Equations

- **Abstract:** This work is on the numerical approximation of incoming solutions to Maxwell's equations, whose energy decays exponentially with time (asymptotically disappearing), meaning that the leading term of the back-scattering matrix becomes negligible. For the exterior of a sphere, such solutions are obtained by Colombini, Petkov and Rauch by specifying a maximal dissipative boundary condition on the sphere and setting appropriate initial conditions.

We consider a mixed finite element approximation of Maxwell's equations in the exterior of a polyhedron whose boundary approximates the sphere. We use the standard Nedelec-Raviart-Thomas elements and a Crank-Nicholson scheme to approximate the electric and magnetic fields. We set discrete initial conditions with standard interpolation, modified so that these initial conditions are divergence-free. We prove that with such initial conditions, the fully discrete approximation to the electric field is weakly divergence-free for all time. We show numerically that the finite-element solution approximates well the asymptotically disappearing solutions constructed analytically when the mesh size becomes small. Future work includes developing multilevel solvers for the resulting linear systems.

## 2. Yanlai Chen

- **Title:** Superconvergence Properties of Variable-degree HDG Methods on non-conforming meshes

- **Abstract:** The discontinuous Galerkin method is known to handle nonconforming meshes easily. In the simple setting of steady-state, pure diffusion, the presence of hanging nodes does degrade the order of accuracy of the classic discontinuous Galerkin methods for a wide class of meshes. This can also be shown to hold for the so-called hybridizable discontinuous Galerkin methods. However, since these methods are more accurate than all the classic discontinuous Galerkin methods, one wonders if the presence of the hanging nodes has an

adverse effect on their remarkable superconvergence properties on unstructured meshes. We provide the first, rigorous a priori error analysis that shows that the superconvergence properties of the hybridizable discontinuous Galerkin methods remain unaffected by the presence of hanging nodes for the so-called semimatching nonconforming meshes. We also present quantification of the degradation when general nonconforming meshes are used. Finally we discuss an extension of these results to case in which the degree of the local polynomial approximation changes from element to element.

### 3. Bo Dong

- **Title:** A New Multiscale Discontinuous Galerkin Method for the One-dimensional Stationary Schrödinger Equation
- **Abstract:** We develop and analyze a new multiscale discontinuous Galerkin (DG) method for one-dimensional stationary Schrödinger equations with open boundary conditions which have highly oscillating solutions. Our method uses a smaller finite element space than the WKB local DG method proposed by Wang and Shu (J. Comput. Phys. 218:295–323, 2006) while achieving the same order of accuracy with no resonance errors. We prove that the DG approximation converges optimally with respect to the mesh size  $h$  in  $L^2$  norm without the typical constraint that  $h$  has to be smaller than the wave length. Numerical experiments were carried out to verify the second order optimal convergence rate of the method and to demonstrate its ability to capture oscillating solutions on coarse meshes in the applications to Schrödinger equations.

### 4. Xiaobing Feng

- **Title:** A Finite Element Method for Linear Elliptic PDEs in Non-divergence Form
- **Abstract:** In this talk I shall present a newly developed finite element method for approximating strong solutions of a class of linear elliptic PDEs in non-divergence form whose leading coefficients are only continuous. Such PDEs are building blocks of fully nonlinear Hamilton-Jacobi-Bellman equations arising from stochastic optimal control and financial mathematics. The proposed method can be viewed as a  $C^0$  DG method, it is very simple to implement and that can be done using any standard finite code. On the other hand, the convergence analysis of the method is quite involved and very technical, it requires to establish a (finite element) discrete Calderon-Zygmund theory, which will be the focus of this talk. Numerical experiments will be presented to demonstrate the effectiveness of the proposed finite element method. If time permits, extensions to IP-DG methods will also be discussed. This is a joint work with Michael Neilan of University of Pittsburgh.

## 5. Xiaoming He

- **Title:** Dual-porosity-Stokes Model and Finite Element Method for Coupling Dual-porosity Flow and Free Flow
- **Abstract:** We propose and numerically solve a new model considering transient flow in dual porosity media coupled with free flow in embedded conduits. Such situation arises, for example, for fluid flows in tight/shale reservoirs with macro-fractures or horizontal wellbores. The flow in dual-porosity media, which consists of both matrix and micro fractures, is described by a dual-porosity model. And the flow in the conduits is governed by the Stokes equation. The two models are coupled through four physically valid interface conditions on the interface between dual-porosity media and conduits, which play a key role in a physically faithful simulation with high accuracy. All the four interface conditions are constructed based on fundamental properties of the traditional dual-porosity model and Stokes-Darcy model. Due to the Stokes equation and the interface conditions which can provide more detail features of the flow around the conduits, the original dual-porosity model should be adjusted for the term of conduits/wells correspondingly. The weak formulation is derived for the proposed model and the existence and uniqueness of the solution are proved for the wellposedness of the model. A finite element method is proposed based on the weak formulation and the convergence is analyzed. Three numerical experiments are presented to validate the proposed model and illustrate the features of both the model and numerical method, such as the optimal convergence rate of the numerical solution, the detail flow information around conduits, and the applicability to the real world problems.

## 6. Wenrui Hao

- **Title:** A Reduced Basis Homotopy Continuation Method For Computing Multiple Solutions of Nonlinear Differential Equations
- **Abstract:** This talk will be based on a novel reduced basis homotopy continuation method for computing multiple solutions of nonlinear differential equations. Motivating from the series representation of solutions to differential equations, we construct an approximation space using the idea of reduced basis method for the differential equation parameterized by the index of each series term on the right-hand side. The resulting approximation space is of very low dimension which is well suited when applying numerical homotopy continuation method to compute multiple solutions to nonlinear differential equations. Various numerical examples will be illustrated the efficiency of the method.

## 7. Xuehai Huang

- **Title:** Quasi-optimal Convergence Rate for an Adaptive Hybridizable  $C^0$  Discontinuous Galerkin Method for Kirchhoff Plates
- **Abstract:** In this work, we present an adaptive hybridizable  $C^0$  discontinuous Galerkin method (AHCDGM) for Kirchhoff plates. Some reliable and efficient a posteriori error estimators are produced for this HCDG method. Quasi-orthogonality and discrete reliability are established with the help of a post-processing moment and discrete Helmholtz decomposition. Based on these, the contraction property between two consecutive loops and complexity of AHCDGM are studied thoroughly.

## 8. Hengguang Li

- **Title:** Finite Element Approximations of Singular Solutions in the  $W_p^1$  Norm
- **Abstract:** Consider elliptic equations on 2D polygonal domains with singular solutions due to the non-smoothness of the boundary. We first establish the stability of the finite element solution in the  $W_p^1$  norm on a class of graded meshes. Then, we give regularity requirements in weighed Sobolev/Holder spaces on the given data and specific parameter-selection criteria for graded meshes, such that the resulting numerical approximation achieves the optimal convergence rate in  $W_p^1$ . Numerical results are provided.

## 9. Jie Liu

- **Title:** A Second-order Stable Explicit Interface Advancing Scheme for FSI with Both Rigid and Elastic Structures and Its Application to Fish Swimming Simulations
- **Abstract:** In this paper, we present a temporally second-order numerical scheme for fluid-structure interaction (FSI) problems in which the structure may be rigid or elastic. The explicit treatment of the interface motion and the semi-implicit treatment of all the other terms make the scheme very efficient. We prove an energy inequality of the scheme which shows that the explicit treatment of the interface motion does not damage the stability. An exact solution for FSI is derived. We use it to numerically check that our scheme converges at rate  $O(\Delta t^2 + h^{m+1})$  when we use  $P_m/P_{m-1}/P_m$  finite elements for fluid velocity, fluid pressure and structure velocity. We also test its performance on the benchmark problem of a laminar incompressible channel flow around a compressible elastic structure. As our fluid-structure system can model both active motion and passive deformation of structures, we apply our scheme to study the locomotion of articulated structures in viscous fluids. We propose an elastic-rigid fish model which obeys all the local balance laws at the deforming interfaces. It can faithfully capture the vorticity generation and drag/thrust generation at these

deforming interfaces. Our computation shows that a planar three-link fish can propel itself in a viscous fluid by periodically change its shape variables. Mesh refinement study is also performed.

#### 10. Xingfeng Liu

- **Title:** Compact Implicit Integration Factor Method for High Order Differential Equations
- **Abstract:** In this talk, we will present an efficient integration factor method for solving a family of semilinear fourth-order parabolic equations, in which the bi-Laplace operator is explicitly handled and the computational cost and storage remain the same as to the classic integration factor methods for second-order problems. In particular, the proposed method can deal with not only stiff nonlinear reaction terms but also various types of homogeneous or inhomogeneous boundary conditions.

#### 11. Zhonghua Qiao

- **Title:** Error Analysis of a Mixed Finite Element Method for The Molecular Beam Epitaxy Model
- **Abstract:** We investigate the error analysis of a mixed finite element method with CrankNicolson time-stepping for simulating the molecular beam epitaxy (MBE) model. The fourth-order differential equation of the MBE model is replaced by a system of equations consisting of one nonlinear parabolic equation and an elliptic equation. Then a mixed finite element method requiring only continuous elements is proposed to approximate the resulting system. It is proved that the semi-discrete and fully discrete versions of the numerical schemes satisfy the nonlinearity energy stability property, which is important in the numerical implementation. Moreover, detailed analysis is provided to obtain the convergence rate. Numerical experiments are carried out to validate the theoretical results.

#### 12. Abner J. Salgado

- **Title:** Piecewise Polynomial Interpolation in Weighted Sobolev Spaces and Applications to Optimal Control Problems
- **Abstract:** We develop a piecewise polynomial approximation theory in weighted Sobolev spaces with Muckenhoupt weights. We construct a quasi-interpolation operator and derive optimal error estimates on simplicial shape regular meshes and anisotropic rectangular meshes. The interpolation theory extends to cases when the error and function regularity require different weights. Concerning the applications to the optimal control theory, we consider: a linear quadratic

optimal control problem involving a nonuniformly elliptic equation; a problem with a pointwise tracking objective and one where the control is the amplitude of secondary forces, modeled as point masses.

### 13. Andreas Vogel

- **Title:** The Finite Element / Finite Volume Software Framework UG4
- **Abstract:** We give an overview on the infrastructure of the finite element / finite volume software framework UG4 (unstructured grids) that uses grid-based methods to solve a broad variety of partial differential equations. In particular, we highlight the implementation aspects of higher order finite volume methods for several kind of physical applications and comment on parallelization and scalability for matrix solvers on high performance computer cluster with hundred thousands of computing cores.

### 14. Feng Wang

- **Title:** On a Parallel Robin-Robin Iterative Method for the Mixed Finite Elements
- **Abstract:** In this talk, we shall discuss a parallel Robin-Robin domain decomposition method for the mixed finite element approximation of the second order elliptic problems. By choosing suitable parameters, we obtain a optimal (with respect to the mesh size) algorithm. Our theory is based on the spectral analysis of the Dirichlet to Neumann operator for the mixed finite element method. Some numerical experiments are presented to confirm our results.

### 15. Huayi Wei

- **Title:** Introduction to Two New Triangle Mesh Optimization Methods
- **Abstract:** We proposed two new triangle mesh optimization methods which are simple, robust, fast and local size invariable. First, motivated by the relationship of stiffness matrix and triangle angles, we construct an idea stiffness matrix and then get the optimal positions of the mesh nodes by solving two symmetric positive definite linear systems. Second, based on the triangle radius ratio, we construct a block Jacobi optimization scheme. In every iterate step, we need to solve two symmetric positive definite linear systems. For the linear system appeared in above methods, AMG can be used to solve them quickly.

### 16. Yongke Wu

- **Title:** Superconvergence and Recovery Type a Posteriori Error Estimation for Hybrid Stress Finite Element Method

- **Abstract:** Superconvergence and a posteriori error estimators of recovery type are analyzed for the 4-node hybrid stress quadrilateral finite element method proposed by Pian and Sumihara (Int. J. Numer. Meth. Engrg., 1984, 20: 1685-1695) for linear elasticity problems. Uniform superconvergence of order  $O(h^{1+\min\{\alpha,1\}})$  with respect to the Lamé constant  $\lambda$  is established for both the recovered gradients of the displacement vector and the stress tensor under a mesh assumption, where  $\alpha > 0$  is a parameter characterizing the distortion of meshes from parallelograms to quadrilaterals. A posteriori error estimators based on the recovered quantities are shown to be asymptotically exact. Numerical experiments confirm the theoretical results.

## 17. Xiaoping Xie

- **Title:** Analysis of a Two-level Algorithm for HDG Methods for Diffusion Problems
- **Abstract:** We analyze an abstract two-level algorithm for hybridizable discontinuous Galerkin (HDG) methods in a unified fashion. We use an extended version of the Xu-Zikatanov (X-Z) identity to derive a sharp estimate of the convergence rate of the algorithm, and show that the theoretical results also apply to weak Galerkin (WG) methods. The main features of our analysis are twofold: one is that we only need the minimal regularity of the model problem; the other is that we do not require the triangulations to be quasi-uniform. Numerical experiments are provided to confirm the theoretical results. This work is joint with Binjie Li and Shiquan Zhang.

## 18. Yifeng Xu

- **Title:** An Adaptive Finite Element Method for Reconstruction of the Robin Coefficient
- **Abstract:** In this talk, I introduce an adaptive finite element method to recover the Robin coefficient involved in a diffusion system from some boundary measurement. Unlike the standard approach for direct problems, the relevant a posteriori error estimator is derived from convergence analysis of the adaptive algorithm. It is proved that the adaptive algorithm guarantees a convergent subsequence of discrete solutions in an energy norm to some exact triplet (the Robin coefficient, state and costate variables) satisfying the optimality system of the least-square formulation with Tikhonov regularization for the concerned inverse problem. Finally, numerical examples are reported to illustrate the performance of the algorithm. This is a joint work with Jun Zou at The Chinese University of Hong Kong.

## 19. Wenjun Ying

- **Title:** A Space-time Adaptive Algorithm for Cardiac Electrical Dynamics
- **Abstract:** The electrical wave propagation in the heart may be modeled by a set of singularly perturbed reaction-diffusion equations. Due to the presence of multiple temporal and spatial scales in the system, the cardiac electrical dynamics typically involves sharp wave fronts. This talk will introduce a space-time adaptive algorithm for dynamically, locally and adaptively capturing the propagating electrical waves. The algorithm places fine grids and small timesteps in the region where the electrical wave has large gradient in time or space while it works with coarse grids and large timesteps in other regions where the electrical wave changes slowly. The subdomains covered by the fine and coarse grids naturally form a non-overlapping domain decomposition of the global problem domain. The grids of the algorithm are locally structured and consist of rectangular cells. They are non-matching on the subdomain interface with hanging nodes. The algorithm works with the Dirichlet-Neumann decomposition. It imposes the Dirichlet boundary condition on the fine grid side of the subdomain interface and the Neumann boundary condition on the coarse grid side. The space discretization of the PDE in this work is made with the piecewise bilinear or trilinear finite elements on quadrilateral and hexahedral grids, respectively. This talk will also describe a composite grid iteration method for the resulting system on the multilevel grids. Numerical simulations will be presented.

## 20. Xiu Ye

- **Title:** New Class of Finite Element Methods: Weak Galerkin Methods
- **Abstract:** Newly developed weak Galerkin finite element methods will be introduced for solving partial differential equations. Weak Galerkin methods have the flexibility of employing discontinuous elements and share the simple formulations of continuous finite element methods at the same time. The Weak Galerkin method is an extension of the standard Galerkin finite element method where classical derivatives were substituted by weakly defined derivatives on functions with discontinuity. Recent development of weak Galerkin methods will be discussed in the presentation.

## 21. Shuo Zhang

- **Title:** Some Results on Structure Preserving Finite Elements
- **Abstract:** The feature of preservation of structure has been finding more and more significance in designing discretisation schemes. In this talk, we will present some structure preserving finite elements. Both their construction and also some applications for discretising practical problems will be talked about.



22. **Shun Zhang**

- **Title:** A New Trace Inequality and Its Applications in Error Estimates of Discontinuous Galerkin Methods
- **Abstract:** In this talk, we will present a new trace inequality for functions in  $H(\text{div})$  space with some extra regularity. With this new trace inequality, new a priori and a posteriori error estimates for nonconforming and DG finite elements approximations of elliptic equations with discontinuous coefficients are established. For a priori estimate, these problems are often of very low regularity, we prove new estimate which is both robust and local optimal. For a posteriori error estimate, we prove the error estimator is robust without the assumption of quasi-monotonicity distribution of coefficients.

23. **Wujun Zhang**

- **Title:** Convergence and Rates of Convergence for Monge-Ampere Equation
- **Abstract:** Monge Ampere equation, arising naturally from differential geometry, optimal mass transport, nonlinear elasticity and the other fields of science and engineering, has received considerable attention in recent years. In contrast to the extensive PDE literature, its numerical approximation is still under development.

In this talk, we shall review the weak solutions of Monge-Ampere equations, namely viscosity solution and Alexandroff solution, and their numerical approximations. We show that these numerical methods are monotone and consistent, provided that the mesh grids are translation invariant. We shall also discuss several numerical issues for Monge-Ampere equation, including stability, convergence and rates of convergence of the numerical schemes.

24. **Zhengyu Zhang**

- **Title:** Introduction to Numerical Investigation of Cavitation Bubble
- **Abstract:** Cavitation bubbles often occur in nature and industrial production processes. The numerical investigation of their behavior is of both academic and practical applications values. In this talk, how to use a VOF (volume of fluid) method for an axisymmetric flow to simulate the cavity's behavior in incompressible flow frame will be introduced. Some problems such as how to improve the accuracy in the computation of the curvature of bubble boundary will be discussed.

25. **Liuqiang Zhong**

- **Title:** A-posteriori Error Analysis for a Staggered Discontinuous Galerkin Discretization of the Time-Harmonic Maxwell's Equations

- **Abstract:** In this talk, we present the first a-posteriori error analysis for the staggered discontinuous Galerkin (SDG) method. Specifically, we consider the approximation of the time-harmonic Maxwell's equations by a SDG method, and prove that our residual based a-posteriori error indicator is both reliable and efficient. We validate the performance of the indicator within an adaptive mesh refinement procedure, and show the adaptive meshes and the associated numerical complexity are quasi-optimal for a range of test problems.

## 26. Yunrong Zhu

- **Title:** Auxiliary Space Preconditioner for Linear Elasticity with Weakly Imposed Symmetry
- **Abstract:** In this talk, we present an auxiliary space preconditioner for the mixed finite element approximation of the linear elasticity equations with weakly imposed symmetry. The preconditioner consists of a fast Poisson solver, and  $d$  copies of (vector)  $H(\text{div})$  solvers (such as HX-preconditioner) where  $d$  is the space dimension. We show that the preconditioner is uniform with respect to the mesh size and parameters in the equation. This preconditioner also gives an efficient solver for the pseudo-stress formulation of the Stokes equation.