Some first order algorithms and their iteration complexities

A contraction method with implementable proximal regularization for linearly constrained convex programming

Abstract. The proximal point algorithm (PPA) is classical, and it is implicit in the sense that the resulting proximal subproblems may be as difficult as the original problem. In this talk, we show that with appropriate choices of proximal parameters, the application of PPA to the linearly constrained convex programming

$$\min\{\theta(x)\,|\,Ax=b,\ x\in\mathcal{X}\},$$

can result in easy proximal subproblems. In particular, under some practical assumptions on the objective function, these proximal subproblems become implementable in the sense that they all have closed-form solutions or can be efficiently solved up to a high precision. We thus present a contraction method with implementable proximal regularization for linearly constrained convex programming, and its global convergence is proved easily under the analytic framework of contraction type methods.

A relaxed customized proximal point algorithm for separable convex programming

For solving the separable linearly constrained convex optimization problem

$$\min\{\theta_1(x)+\theta_2(y)\,|\,Ax+By=b,\ x\in\mathcal{X},\ y\in\mathcal{Y}\},$$

the alternating direction method (ADM) has attracted wide attention due to its significant efficiency and easy implementation. In ADM, λ is the Lagrangian multipliers of the linear constraints $Ax+By-b=0$ and the variable x plays only an intermediate role. Thus, the iteration is from $v^k=(y^k,\lambda^k)$ to $v^{k+1}=(y^{k+1},\lambda^{k+1})$. In this talk, motivated by proximal point algorithm (PPA), we propose a relaxed customized proximal point algorithm. The prediction step is similar as in the popular ADM but changes the order of y and λ. However, the update correction allows to use a relaxation factor $\gamma\in(0,2]$. The convergence is proved in the framework of PPA and the complexity result is presented as well. The computational load of the proposed method is the same as the classical one, preliminary numerical results indicates that our method is much faster than the popular ADM.

Alternating direction methods with substitutions for general separable convex programming

Abstract. It remains a theoretical challenge to prove the convergence when the alternating direction method (ADM) is extended to solve the linearly constrained convex programming whose objective function is separable to more than two blocks:

$$\min\Big\{\sum_{i=1}^{m}\theta_i(x_i) | \sum_{i=1}^{m} A_i x_i = b,\ x_i \in \mathcal{X}_i,\ i = 1,\cdots,m\Big\}$$

In this talk, our purpose is to show the convergence of this extension when the output of the ADM is corrected by an easy (forward or backward) substitution step at each iteration. We show that the sequence generated by the blend of ADM and substitution is contractive to the solution set of the problem under consideration, and so its global convergence can be established easily under the framework of contraction type methods. In addition, we prove the $\boldsymbol{O(1/t)}$ iteration complexity for the ADM with substitution by using the tool of variational inequality.

On the $O(1/t)$ convergence rate of the projection and contraction methods for variational inequalities with Lipschitz continuous monotone operators

Abstract. Recently, Nemirovski's analysis indicates that the extragradient method has the $O(1/t)$ convergence rate for variational inequalities with Lipschitz continuous monotone operators. For the same problems, in the last decades, we have developed a class of Fejér monotone projection and contraction methods. Until now, only convergence results are available to these projection and contraction methods, though the numerical experiments indicate that they always outperform the extragradient method. The reason is that the former benefits from the 'optimal' step size in the contraction sense. In this paper, we prove the convergence rate under a unified conceptual framework, which includes the projection and contraction methods as special cases and thus perfects the theory of the existing projection and contraction methods. Preliminary numerical results demonstrate that the projection and contraction methods converge twice faster than the extragradient method.