Math 231 Work Sheet 6 NAME:

Brief Table of Laplace Transforms

Function	Example
$\mathcal{L}[0] = 0$	N/A
$\mathcal{L}[c] = \frac{c}{s}$	$\mathcal{L}[231] = \frac{231}{s}$
$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$	$\mathcal{L}[t^3] = \frac{3!}{s^4}$
$\mathcal{L}[e^{at}] = \frac{1}{s-a}$	$\mathcal{L}[e^{5t}] = \frac{1}{s-5}$
$\mathcal{L}[e^{at}t^n] = \frac{n!}{(s-a)^{n+1}}$	$\mathcal{L}[e^{2t}t^4] = \frac{4!}{(s-2)^5}$
$\mathcal{L}[\cos(bt)] = \frac{s}{s^2+b^2}$	$\mathcal{L}[\cos(7t)] = \frac{s}{s^2+49}$
$\mathcal{L}[\sin(bt)] = \frac{b}{s^2+b^2}$	$\mathcal{L}[\sin(7t)] = \frac{7}{s^2+49}$
$\mathcal{L}[e^{at}\cos(bt)] = \frac{s-a}{(s-a)^2+b^2}$	$\mathcal{L}[e^{5t}\cos(3t)] = \frac{s-5}{(s-5)^2+9}$
$\mathcal{L}[e^{at}\sin(bt)] = \frac{b}{(s-a)^2+b^2}$	$\mathcal{L}[e^{5t}\sin(3t)] = \frac{3}{(s-5)^2+9}$
$\mathcal{L}[af(t)+bg(t)] = a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$	$\mathcal{L}[2+5t] = \mathcal{L}[2] + 5\mathcal{L}[t] = \frac{2}{s} + 5\left(\frac{1}{s^2}\right)$

Laplace Transforms and Step Functions

$$\mathcal{L}[u(t-a)f(t-a)] = e^{-as}\mathcal{L}[f(t)]$$

1. (a) Find $\mathcal{L}[u(t-2)e^{4(t-2)}(t-2)^2]$. Hint: pull out $u(t-a)$ which changes to e^{-as} and change all $t-a$ to t inside [ ] then continue.

(b) Find $\mathcal{L}[u(t-1)(t+1)]$. Hint: Write $t+1$ as a function of $t-1$, then proceed as (a).

(c) Find $\mathcal{L}^{-1}[e^{-3s}\frac{5!}{s^6}]$. Hint: For $\mathcal{L}^{-1}[e^{-as}J(s)]$, first find $j(t)$ with $\mathcal{L}[j(t)] = J(s)$, then replace the t by $t-a$ and put a $u(t-a)$ in front.

2. Use Laplace transform to solve the IVP:

$$y'' - 2y' + 2y = 0 \text{ with } y(0) = 2, y'(0) = 0.$$

3. Solve the initial value problem:

$$y' - 2y = f(t) \quad \text{with} \quad y(0) = 0,$$

where

$$f(t) = \begin{cases} 0, & t < 3, \\ 1, & 3 < t < 4, \\ 0, & t > 4. \end{cases}$$