Math 231 Work Sheet 5 NAME:

Summary: This worksheet corresponds to sections 7.2, 7.3, and 7.4 in the textbook.

Table of Laplace transform

Function	Example
$\mathcal{L}[0] = 0$	n/a
$\mathcal{L}[c] = \frac{c}{s}$	$\mathcal{L}[42] = \frac{42}{s}$
$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$	$\mathcal{L}\left[t^3\right] = \frac{3!}{s^4}$
$\mathcal{L}[e^{at}] = \frac{1}{s-a}$	$\mathcal{L}\left[e^{5t}\right] = \frac{1}{s-5}$
$\mathcal{L}[\cos(bt)] = \frac{s}{s^2+b^2}$	$\mathcal{L}[\cos(7t)] = \frac{s}{s^2+49}$
$\mathcal{L}[\sin(bt)] = \frac{b}{s^2+b^2}$	$\mathcal{L}[\sin(7t)] = \frac{7}{s^2+49}$
$\mathcal{L}[e^{at}t^n] = \frac{n!}{(s-a)^{n+1}}$	$\mathcal{L}\left[e^{2t}t^4\right] = \frac{4!}{(s-2)^5}$
$\mathcal{L}[e^{at}\cos(bt)] = \frac{s-a}{(s-a)^2+b^2}$	$\mathcal{L}\left[e^{5t}\cos(3t)\right] = \frac{s-5}{(s-5)^2+9}$
$\mathcal{L}[e^{at}\sin(bt)] = \frac{b}{(s-a)^2+b^2}$	$\mathcal{L}\left[e^{5t}\sin(3t)\right] = \frac{3}{(s-5)^2+9}$
$\mathcal{L}[af(t) + bg(t)] = a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$	$\mathcal{L}[2+5t] = \mathcal{L}[2] + 5\mathcal{L}[t] = \frac{2}{s} + 5\left(\frac{1}{s^2}\right)$

1. Find Laplace transforms by using the table above.

(a) $\mathcal{L}[3t]$

(b) $\mathcal{L}[3 + t^2 + e^{-4t}\cos(t)]$

(c) $\mathcal{L}[(1 + e^{-t})^2]$

(d) $\mathcal{L}[e^{2t}t^3 - \sin(t)]$

2. Find the inverse Laplace transforms by reversing the procedure. Sometimes we need to manipulate the function first.

(a) Find $\mathcal{L}^{-1}[F(s)]$ for $F(s) = \dfrac{3}{s-7}$, i.e. find y with $\mathcal{L}[y] = \frac{3}{s-7}$.

(b) Find $\mathcal{L}^{-1}[F(s)]$ for $F(s) = \dfrac{9}{(s+1)^3}$, i.e. find y with $\mathcal{L}[y] = \frac{9}{(s+1)^3}$.

(c) Find $\mathcal{L}^{-1}[F(s)]$ for $F(s) = \dfrac{3s-1}{s^2-2s-3}$.

(d) Find $\mathcal{L}^{-1}[F(s)]$ for $F(s) = \dfrac{3s-1}{s^2-2s+5}$.