

Summary: This worksheet corresponds to sections 4.2, 4.3 in the textbook.

How to find the fundamental pair for 2nd-order ODE with constant coefficients:

1. If two simple **real** roots: $r = r_1, r = r_2$ where $r_1 \neq r_2$, then

fundamental pair is $\{e^{r_1 t}, e^{r_2 t}\}$.

2. If one multiple **real** root: $r = r_1$ with multiplicity 2, then

fundamental pair is $\{e^{r_1 t}, te^{r_1 t}\}$.

3. If a pair of **complex** roots: $r = \alpha \pm \beta i$, then

fundamental pair is $\{e^{\alpha t} \cos(\beta t), e^{\alpha t} \sin(\beta t)\}$.

Examples

1. Write down the characteristic equations and solve the roots for each of the following:

(a) $y'' - y' - 20y = 0$.

(b) $2y'' + 8y' + 8y = 0$.

(c) $y'' - 2y' + 5y = 0$. (Use quadratic formula)

2. Find a fundamental pair of solutions for each of the following, then write down the general solution.

(a) $2y'' + 5y' - 12 = 0$.

(b) $y'' + 6y' + 13 = 0$.

(c) $4y'' - 4y' + 1 = 0$.

3. Solve the initial value problem

$$y'' - 2y' + y = 0 \text{ with } y(0) = -1, y'(0) = 2.$$