Math 231. Quiz 6 (23 minutes) **Name:**______________

Brief Table of Laplace Transforms

Function	Example
$\mathcal{L}[0] = 0$	N/A
$\mathcal{L}[c] = \frac{c}{s}$	$\mathcal{L}[231] = \frac{231}{s}$
$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$	$\mathcal{L}[t^3] = \frac{3!}{s^4}$
$\mathcal{L}[e^{at}] = \frac{1}{s-a}$	$\mathcal{L}[e^{5t}] = \frac{1}{s-5}$
$\mathcal{L}[e^{at}t^n] = \frac{n!}{(s-a)^{n+1}}$	$\mathcal{L}[e^{2t}t^4] = \frac{4!}{(s-2)^5}$
$\mathcal{L}[\cos(bt)] = \frac{s}{s^2+b^2}$	$\mathcal{L}[\cos(7t)] = \frac{s}{s^2+49}$
$\mathcal{L}[\sin(bt)] = \frac{b}{s^2+b^2}$	$\mathcal{L}[\sin(7t)] = \frac{7}{s^2+49}$
$\mathcal{L}[e^{at}\cos(bt)] = \frac{s-a}{(s-a)^2+b^2}$	$\mathcal{L}[e^{5t}\cos(3t)] = \frac{s-5}{(s-5)^2+9}$
$\mathcal{L}[e^{at}\sin(bt)] = \frac{b}{(s-a)^2+b^2}$	$\mathcal{L}[e^{5t}\sin(3t)] = \frac{3}{(s-5)^2+9}$
$\mathcal{L}[af(t) + bg(t)] = a\mathcal{L}[f(t)] + b\mathcal{L}[g(t)]$	$\mathcal{L}[2 + 5t] = \mathcal{L}[2] + 5\mathcal{L}[t] = \frac{2}{s} + 5\left(\frac{1}{s^2}\right)$

Other Properties of Laplace Transforms

$$\mathcal{L}[u(t-a)f(t-a)] = e^{-as}\mathcal{L}[f(t)], \quad \mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$$

$$\mathcal{L}^{-1}[e^{-as}F(s)] = f(t-a)u(t-a), \text{ where } f(t) = \mathcal{L}^{-1}[F(s)]$$

$$\mathcal{L}[y'](s) = s\mathcal{L}[y](s) - y(0), \quad \mathcal{L}[y''](s) = s^2\mathcal{L}[y](s) - y'(0) - sy(0).$$

1. (13 points) Solve the initial value problem:

$$y' + y = f(t) \text{ with } y(0) = 0,$$

where

$$f(t) = \begin{cases} 0, & 0 < t < 1, \\ 2, & 1 < t < 2, \\ 0, & t > 2. \end{cases}$$

2. (7 points) Let $y(x) = \sum_{n=0}^{+\infty} a_n x^n$, find the power series coefficients of $2y' - y$, i.e. for

$$\sum_{n=0}^{+\infty} c_n x^n = 2y' - y,$$

express c_n in terms of coefficients $a_0, a_1, a_2, \cdots$ for all n.