MATH 231. EXAM 2

Name:_____

Instructions: This is a closed-book exam, and calculators can only be used to do basic arithmetic operations (not allowed for differentiation and integration). The last page contains a table for integrals and some results from textbook, which might be helpful. Read each problem carefully. You must show your work to receive credit. Partial credit will be given for any work relevant to the problem.

Problem	Grade
1	
2	
3	
4	
5	
Total	

There is a total of 100 points.

Problem 1 (10 points): Determine whether the critical point $(0, 0)$ in the following system is stable or not. Show your work.

$$\begin{cases} \dfrac{dx}{dt} = 5x + 2y \\ \dfrac{dy}{dt} = -5x - y. \end{cases}$$

Problem 2 (15 points): Find the general solution to the equation:

$$y'' - 4y' + 13y = 0.$$

Problem 3 (30 points): Find the general solution to the equation:

$$y'' - 2y' = 4te^{2t} + 5\sin(t).$$

Problem 4 (20 points): Find the general solution to the equation:

$$2y'' + 4y' + 2y = -\frac{e^{-t}}{t^2}.$$

Problem 5 (25 points): Use the elimination method to solve the system with initial values:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} + 2\dfrac{\mathrm{d}y}{\mathrm{d}t} = 2x + 2t + 1, & x(0) = 2, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x - y + t, & y(0) = 2. \end{cases}$$

Variation of Parameters Formula. Consider a 2nd order ODE

$$y'' + a_1(t)y' + a_0(t)y = f(t),$$

and assume its homogeneous version has a fundamental pair $\{Y_1(t), Y_2(t)\}$. Then a particular solution $Y_p(t)$ to the nonhomogeneous ODE is given by:

$$Y_p(t) = Y_1(t) \int \frac{-Y_2(t)f(t)}{W[Y_1, Y_2]}dt + Y_2(t) \int \frac{Y_1(t)f(t)}{W[Y_1, Y_2]}dt.$$

Brief Table for Integrals:

$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x + \sqrt{x^2+a^2}|.$ $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x + \sqrt{x^2-a^2}|, \quad x^2 \geq a^2.$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin(\frac{x}{a}), \quad a^2 \geq x^2.$ $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan(\frac{x}{a}).$

$\int \tan(x)dx = -\ln|\cos x|.$ $\int \cot(x)dx = \ln|\sin x|.$

$\int \sec(x)dx = \ln|\sec x + \tan x|.$ $\int \csc(x)dx = -\ln|\csc x + \cot x|.$

Common trigonometric substitutions:

(1) For integrand involving $\sqrt{a^2 - x^2}$, set $x = a\sin(\theta)$,
(2) For integrand involving $\sqrt{a^2 + x^2}$, set $x = a\tan(\theta)$,
(3) For integrand involving $\sqrt{x^2 - a^2}$, set $x = a\sec(\theta)$,
(4) For $\int \tan^n(x)\sec^{2m}(x)dx$, set $u = \tan(x)$,
(5) For $\int \cot^n(x)\csc^{2m}(x)dx$, set $u = \cot(x)$.