14.2 Limits and Continuity (We do not use this much)

1. Definition of Limits

Notation $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) = L$

The limit of $f(x,y)$ as (x,y) approaches (a,b) is L.

What does it mean when we say (x,y) approaches (a,b)?

The distance between Point (x,y) and Point (a,b) $\to 0$.

Example: Find $\displaystyle\lim_{(x,y)\to(1,2)} \sin(x)\, y$.

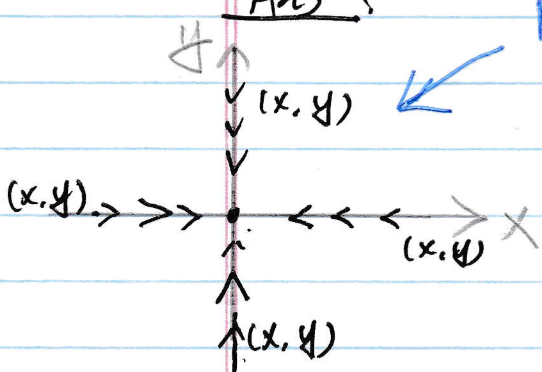
Ans: Just plug in: $\displaystyle\lim_{(x,y)\to(1,2)} \sin(x)\, y = (\sin(1))(2)$

$= 2\sin(1)$.

Sometimes not possible to plug in.

Text-Ex 1: Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exist.

Ans: Different ways of Point $(x,y) \to (0,0)$.



If we plug $(0,0)$ into the expression, we get $\frac{0}{0}$.

This cannot tell us anything. The limit may or may not exist.

Now assume $(x, y) \to (0,0)$ in the way:

$$y = 0, \quad x \to 0. \quad (x \neq 0).$$

Then $\dfrac{x^2 - y^2}{x^2 + y^2} = \dfrac{x^2}{x^2} = 1$

However if $(x, y) \to (0,0)$ in the following way

$$x = 0, \quad y \to 0 \quad (y \neq 0)$$

Then $\dfrac{x^2 - y^2}{x^2 + y^2} = \dfrac{-y^2}{y^2} = -1$

Two different numbers

$\Rightarrow$ limit does not exist.

<u>Remark</u>: For $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^2+y^2}$, although we get $\frac{0}{0}$ when we plug in, one can still prove the limit exists and is equal to 0 (not required).

2. <u>Continuity</u>

Function f is continuous at (a, b) if

a point in 2D

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

The functions we deal with in this class are continuous at most points. An example of discontinuity:

$$f(x,y) = \begin{cases} \dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is not continuous at $(0,0)$.