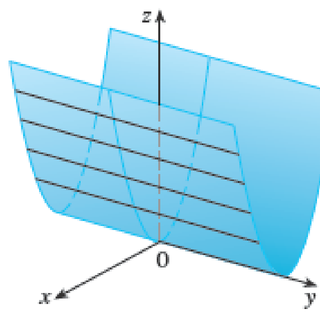
Math 241 Section 12.6

1. Sketching $3D$ surfaces with an equation not involving one of the x, y, z. Plot a $2D$ curve in one coordinate plane (xy-plane, xz-plane or yz-plane), and then move the curve along one direction.

Text-Example 1. Sketch the graph of the surface $z = x^2$.

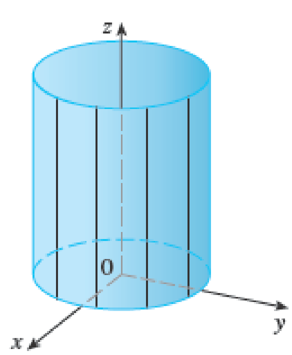
Solution: 1. Equation doesn't involve $y \Longrightarrow$ The surface contains point (x, y, z) for any y if $z = x^2$.
2. Draw the intersection of the surface with the xz-plane (also called traces within the xz-plane).
3. Move the graph obtained in the xz-plane along the y-axis to form the graph in $3D$. See the figure below from the textbook.



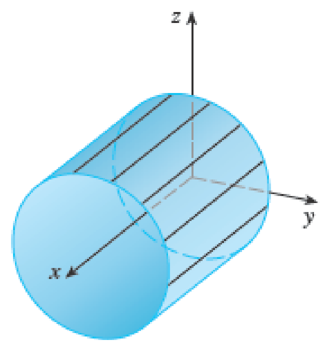
Related concepts: This is a **cylinder**, i.e. a surface that consists of all lines (called **rulings**). In this example, the rulings are parallel to the y-axis.

Text-Example 2. Sketch the graph of the surfaces
(a) $x^2 + y^2 = 1$ (b) $y^2 + z^2 = 1$.

Solution: (a) 1. Equation doesn't involve $z \Longrightarrow$ The surface contains point (x, y, z) for any z if $x^2 + y^2 = 1$.
2. Draw the intersection of the surface with the xy-plane (also called traces within the xy-plane), which is the unit circle.
3. Move the graph obtained in the xy-plane along the z-axis to form the graph in $3D$. See the figure below from the textbook. The rulings are parallel to the z-axis.

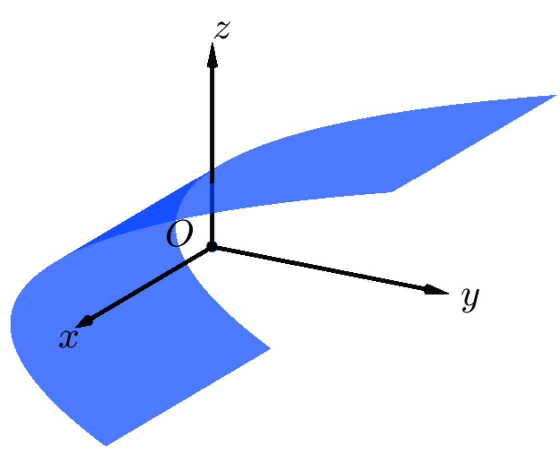


(b) 1. Equation doesn't involve $x \Longrightarrow$ The surface contains point (x, y, z) for any z if $y^2 + z^2 = 1$.
2. Draw the intersection of the surface with the yz-plane (also called traces within the yz-plane), which is the unit circle.
3. Move the graph obtained in the xy-plane along the x-axis to form the graph in $3D$. See the figure below from the textbook. The rulings are parallel to the x-axis.



Example. Sketch the graph of the $3D$ surface $y = z^2 + z - 1$.

Solution: 1. Equation doesn't involve x but in $3D \Longrightarrow$ The surface contains point (x, y, z) for any x if $y = z^2 + z - 1$.
2. Draw the intersection of the surface with the yz-plane, which is a graph of a quadratic function.
3. Move the graph obtained in the xy-plan along the x-axis to form the graph in $3D$. See the figure below drawn by MATLAB.

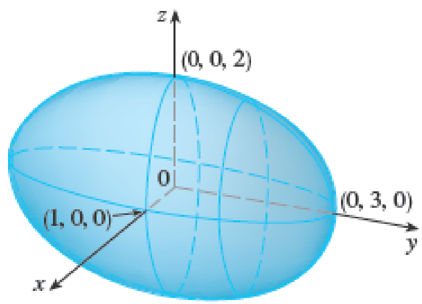


2. Quadric surfaces (not required, brief introduction).

Text-Example 3. Sketch of an ellipsoid given by the equation

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1.$$

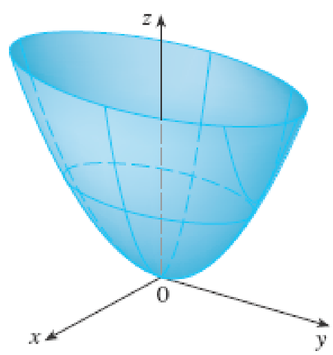
See the figure below from the textbook. Notice that the trace in any coordinate plane (xy-plane, xz-plane, yz-plane) is an ellipse. The surface intersects the x,y,z-axes at points $(1, 0, 0)$, $(-1, 0, 0)$, $(0, 3, 0)$, $(0, -3, 0)$, $(0, 0, 2)$ and $(0, 0, -2)$.



Text-Example 4. Sketch of an elliptic paraboloid given by the equation

$$z = 4x^2 + y^2.$$

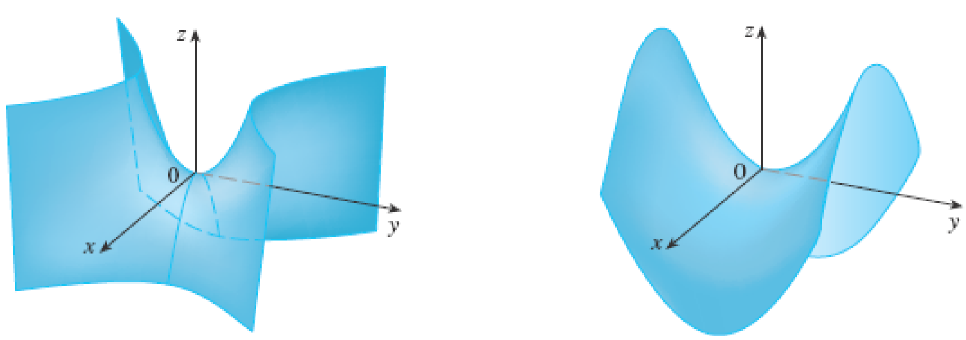
See the figure below from the textbook.



Text-Example 5. Sketch of a hyperboloid paraboloid given by the equation

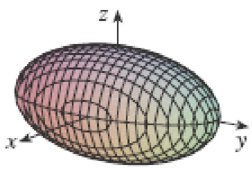
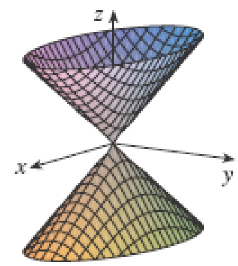
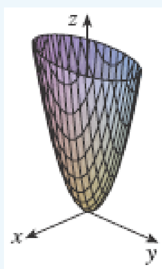
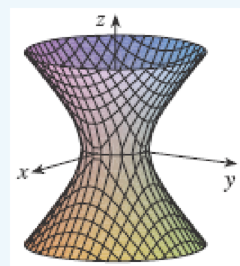
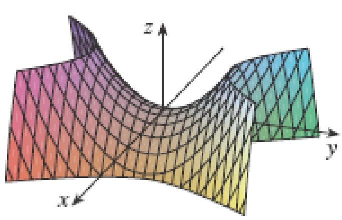
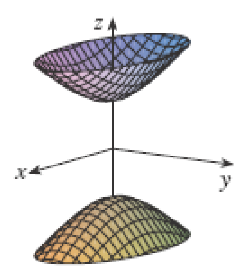
$$z = y^2 - x^2.$$

See the figure below from the textbook.



Text-Table 1. Summary of graph of quadric surfaces.

Graphs of Quadric Surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.