# MATH 241. FINAL EXAM

**Name:**____________________________________________________

**Instructions: This is a closed-book exam with two 3" x 5" index cards allowed during the exam.** Do not simplify unless indicated. No calculators are permitted. Show all your work, especially the work related to the methods taught in this course.

| Problem | Grade |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| **Total** | |

**There is a total of 200 points.**

**1.** Parts (a) and (b) are independent.

**(a) (10 Points)** Find parametric equations and symmetric equations for the line $L$ containing points $(-2, 0, 1)$ and $(1, 2, -1)$.

**(b) (10 Points)** Find an equation for the plane containing points $P(1, -3, 1)$, $Q(1, -6, 2)$ and $R(5, -1, 0)$.

**2.** The position of a particle at time $t$ is given by $\boldsymbol{r}(t) = \sin(t)\ \boldsymbol{i} + \cos(t)\ \boldsymbol{j} + t^2\ \boldsymbol{k}$. Let curve $C$ be the curve traced out by the particle in the time interval $0 \le t \le 2$, i.e with parametrization

$$\boldsymbol{r}(t) = \sin(t)\ \boldsymbol{i} + \cos(t)\ \boldsymbol{j} + t^2\ \boldsymbol{k}, \qquad 0 \le t \le 2.$$

**(a) (10 Points)** Compute $a_{\boldsymbol{T}}$, i.e. the tangential component of acceleration. Notice that $a_{\boldsymbol{T}}$ is a function of time $t$.

**Hint:** You may want to use the formula

$$a_{\boldsymbol{T}}(t) = \frac{\boldsymbol{r}'(t) \cdot \boldsymbol{r}''(t)}{|\boldsymbol{r}'(t)|}.$$

**(b) (15 Points)** Evaluate

$$\int_C 2x\ dx + 2y\ dy + 3\ dz.$$

**3. (25 Points)** Consider the region

$$D = \{(x, y) : \ x \geq 0, \ y \geq 0, \ x + y \leq 1\}.$$

Find the absolute maximum and minimum of the function $f(x, y) = (x + y)^2 + y^2 - 2y$ on the region $D$.

**4. (10 Points)** Consider the problem of finding the extreme values of $f(x, y) = 2x - 2y$ subject to the constraint $x^2 + 2y^2 = 6$. Set up the Lagrange system (three equations when using the method of Lagrange multipliers). Please only set up the equations. No need to solve them.

**5. (15 Points)** Find the linear approximation of the function $f(x, y) = x^2y + y^3$ at $(2, 1)$ and use it to approximate $f(2.01, 0.98)$.

**6. (20 Points)** Let $D$ be region bounded by $x = y^2$ and $x = y + 2$. Let $\rho(x, y) = x^2 + y^2$ be the density function. Set up the iterated integral for the mass of $D$. Do not evaluate.

**7.** Parts (a) and (b) are independent.

**(a) (15 Points)** Let $E$ be the solid region inside the half-cylinder $x^2 + y^2 = 4$ with $y \geq 0$ and between the planes $z = 0$ and $z = y+3$. Use cylindrical coordinates to set up an iterated integral for

$$\iiint_E y \, dV.$$

Do not evaluate.

**(b) (15 Points)** Let $E$ be the solid region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 9$ in the first octant. Set up an iterated integral for

$$\iiint_E x \, dV.$$

Do not evaluate

**8. (15 Points)** Let $S$ be the part of the plane $2x + y + z = 12$ in the first octant. Set up the iterated integral corresponding to the surface integral

$$\iint_S xz \, dS.$$

Do not evaluate.

**9. (20 Points)** Use the Divergence Theorem to evaluate the following integral:

$$\iint_S \left(2xy\, \boldsymbol{i} + \left(z^2 - y^2\right) \boldsymbol{j} + \left(e^x + z\right) \boldsymbol{k}\right) \cdot \boldsymbol{n}\, dS,$$

where $S$ is the part of the cylinder $x^2 + y^2 = 4$ between $z = 2$ and $z = 5$ with the caps (disks which seal it off) at both ends, oriented inwards.

**10. (20 Points)** Let $C$ be the intersection of the cylinder $x^2+y^2=9$ with the plane $y+z=6$ with counterclockwise orientation when viewed from above. Apply Stokes' Theorem to the integral

$$\int_C y\ dx + z\ dy + xz\ dz$$

and proceed until you have an iterated double integral. Do not evaluate.