Math 241 Exam 3

Instructions: Do not simplify unless indicated. No calculators are permitted. Show all your work, especially the work related to the methods taught in this course.

1. (20 Points) Use the fundamental theorem of line integrals to compute

$$\int_C (e^x y\, \boldsymbol{i} + (e^x - 4y)\, \boldsymbol{j}) \cdot d\boldsymbol{r}$$

where C is the curve with parametrization:

$$\boldsymbol{r}(t) = t\sin(\pi t)\, \boldsymbol{i} + t^3\, \boldsymbol{j} \quad 0 \leq t \leq 1.$$

2. (a) (10 Points) Evaluate the following iterated integral

$$\int_0^2 \int_x^2 y\, dy\, dx$$

(b) (15 Points) Let E be the region bounded by the planes $y = x, y = 2x, x = 1, z = 0$ and $z = 6$. If the density at a point is given by $f(x, y, z) = x$, set up the iterated integral for the mass of E. Do not evaluate.

3. (20 Points) Let E be the solid under the cone $z = \sqrt{x^2 + y^2}$, inside the sphere $x^2 + y^2 + z^2 = 9$ and above the xy-plane. Use spherical coordinates to rewrite the integral

$$\iiint_E z\, dV$$

as an iterated integral. Do not evaluate.

4. (20 Points) Let R be the region bounded by the lines $y = x, y = 3x, y = \frac{4}{x}$ and $y = \frac{1}{x}$ in the first quadrant. Use a change of variables to rewrite

$$\iint_R xy\, dA$$

as an iterated integral in the uv-plane. Do not evaluate.

5. (15 Points) Use Green's Theorem to compute

$$\int_C \left(y - \sqrt{1 + x^4}\right)\, dx + \left(\cos(1 + y^4) - x\right)\, dy$$

where C is the boundary of the circle $x^2 + y^2 \leq 1$ oriented counterclockwise.