

**Math 241 Exam 2** 9:15 AM - 10:05 AM

**Instructions:** Do not simplify unless indicated. No calculators are permitted. Show all your work, especially the work related to the methods taught in this course.

1. **(15 Points)** Find the directional derivative of  $f(x, y, z) = xy + yz$  in the direction of  $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$  at the point  $(1, 2, 3)$ .

2. **(20 Points)** We have the following formula for electrical power  $P$  (in watts) in terms of voltage  $V$  (in volts) and resistance  $R$  (in ohms)

$$P = \frac{V^2}{R}.$$

If the voltage  $V$  is **decreasing** at 2 volts/second while the resistance  $R$  is **increasing** at 3 ohms/second, then at what rate is the electrical power  $P$  changing when  $V = 50$  and  $R = 10$ ?

3. **(25 Points)** Let  $f(x, y) = xy^2 - y^2 + x^3 - 12x$ . Find all critical points of  $f$  and categorize each critical point as a local maximum, local minimum or saddle point.

Note: There are four critical points.

4. **(15 Points)** Let  $R$  be the region inside both the circle  $r = 2\sin(\theta)$  and the first quadrant. Set up the iterated double integral in polar coordinates for  $\iint_R x\sqrt{x^2 + y^2} dA$ . **Do not evaluate.**

5. **(25 Points)** Use Lagrange Multipliers to find the maximum and minimum values of the function  $f(x, y) = x - 2y + 1$  subject to the constraint  $x^2 + 4y^2 = 8$ .