Math 241 Exam 1 9:15 AM - 10:05 AM

1. (a) (10 Points) Find the center and radius of the sphere with equation

$$x^2 + 2x + y^2 + z^2 - 2z = 0.$$

(b) (10 Points) Find the angle between vectors $\boldsymbol{a} = \langle 1, -2, 2\rangle$ and $\boldsymbol{b} = \langle 1, -1, 0\rangle$.

2. (15 Points) Compute the distance from point $P(3, 0, -4)$ to the plane $2x-2y+z-5 = 0$.

3. (20 Points) Consider line L_1 with equation $\boldsymbol{r}(t) = (-2 + t)\boldsymbol{i} + (3 - t)\boldsymbol{j} + 4\boldsymbol{k}$ and line L_2 with equation $\boldsymbol{r}(t) = (-2 - t)\boldsymbol{i} + (3 + 3t)\boldsymbol{j} + (4 - t)\boldsymbol{k}$. Find the equation of the plane containing both line L_1 and line L_2. Write this into the form $ax + by + cz = d$.

4. (20 Points) Find the symmetric equations for the tangent line to the curve $\boldsymbol{r}(t) = 2t^2\boldsymbol{i} - t\boldsymbol{j} + t^3\boldsymbol{k}$ at the point when $t = 1$.

5. Consider the curve $\boldsymbol{r}(t) = \langle \sin(t),\ 2e^t,\ 1 - 2t\rangle$.
(a) (10 Points) Find the unit tangent vector $\boldsymbol{T}$ at $t = 0$.
(b) (15 Points) Compute the curvature at $t = 0$.

Some Formulas from the Textbook

$$\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|}\,, \quad \boldsymbol{N}(t) = \frac{\boldsymbol{T}'(t)}{|\boldsymbol{T}'(t)|}\,, \quad \boldsymbol{B}(t) = \boldsymbol{T} \times \boldsymbol{N}\,, \quad \kappa(t) = \frac{|\boldsymbol{T}'(t)|}{|\boldsymbol{r}'(t)|} = \frac{|\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)|}{|\boldsymbol{r}'(t)|^3}$$

$$a_{\boldsymbol{T}}(t) = \frac{\boldsymbol{r}'(t) \cdot \boldsymbol{r}''(t)}{|\boldsymbol{r}'(t)|}\,, \quad a_{\boldsymbol{N}}(t) = \frac{|\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)|}{|\boldsymbol{r}'(t)|}$$