Mathematical and Statistical Methods for Imaging

Habib Ammari* Josselin Garnier†

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Abstract

The aim of the summer school is to introduce fundamental mathematical and statistical tools, probabilistic approaches, and inversion and optimal design methods to address emerging modalities in medical imaging, nondestructive testing, and environmental inverse problems. Many mathematical and computational challenging problems arise in emerging imaging techniques and they often lead to the investigation of fundamental problems in various branches of mathematics. The summer school describes state of-the-art in asymptotic imaging, stochastic modelling, and analysis of wave propagation phenomena. It throws a bridge across these different aspects of mathematical imaging. It provides deep understanding of the different scales in the physical problem and an accurate modelling of the uncertainty and noise sources in order to derive the best imaging functional in the sense that it achieves the optimal trade-off between signal-to-noise ratio and resolution. The summer school also leads the participants to appreciate the practical implementations and performance evaluations of the described imaging methods.

Plan

Lecture 1: Applications of wave imaging, Introduction to active array imaging, passive array imaging, time reversal experiments, and cross correlation imaging techniques.
Illustrations and comparisons in homogeneous media and in random media.
Introduction to probabilistic and statistical tools.

Lecture 2: Introduction to integral equations techniques; Some useful identities: reciprocity, Green's identities, Helmholtz-Kirchhoff identity.
A quick introduction to geometric optics.

*Department of Mathematics and Applications, Ecole Normale Supérieure, 45 Rue d'Ulm, 75005 Paris, France (habib.ammari@ens.fr).

†Laboratoire de Probabilités et Modèles Aléatoires & Laboratoire Jacques-Louis Lions, Université Paris VII, 75205 Paris Cedex 13, France (garnier@math.jussieu.fr).

Lecture 3: Introduction to Random Matrix Theory.

Structure of the response matrix in the presence of electronic noise, in the presence of cluttered noise due to a random medium in the single-scattering approximation, in the presence of cluttered noise due to a random medium in the multiple-scattering regime.

Statistical hypothesis testing. Detection test and localization of a point target.

Optimal (weighted subspace) migration techniques for a point target and for an extended target.

Lecture 4: Born approximations, shape-from-moments problem, Radon transform, small-volume asymptotic expansions.

Lecture 5: Least-square imaging, reverse-time imaging, Kirchhoff imaging, weighted subspace imaging, topological derivative based imaging.

Basic resolution theory in homogeneous media.

Lecture 6: Properties of the polarization tensors.

Lecture 7: Wave propagation in random media: The one-dimensional case.

One-dimensional random processes, a little bit of theory and simulation techniques.

Limit theorems for differential equations with random coefficients: homogenization and diffusion approximation.

Applications to imaging and time-reversal in random media. Resolution and statistical stability analysis.

Lecture 8: Resolution enhancement, optimal control algorithms, imaging extended targets.

Lecture 9: Wave propagation in random media: The paraxial regime.

Multi-dimensional random processes, a little bit of theory and simulation techniques.

The white-noise paraxial regime. Theoretical (statistical) aspects and simulation. Importance of the Wigner distribution.

Applications to imaging and time-reversal in random media.

Lecture 10: Imaging extended targets, resolution and stability analysis, shape derivative.

Lecture 11: Passive sensor imaging using ambient noise.

The ideal case when the noise sources are uniformly distributed.

The problem when the noise sources are spatially localized. Use of geometric optics approximation.

Role of scattering due to a random medium: trade-off between enhanced resolution due to directional diversity enhancement and signal-to-noise ratio reduction.

Lecture 12: Electromagnetic invisibility, cloaking enhancement, metamaterials.

Basic references

H. Ammari and H. Kang, *Polarization and Moment Tensors with Applications to Inverse Problems and Effective Medium Theory*, Applied Mathematical Sciences, Vol. 162, Springer, New York, 2007.

J.-P. Fouque, J. Garnier, G. Papanicolaou, and K. Sølna, *Wave Propagation and Time Reversal in Randomly Layered Media*, Springer, New York, 2007.

Notes are provided during the summer school.