Superregular solitonic solutions in Nonlinear Schrödinger Equation

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Abstract:

It is known since 1971 that the Nonlinear Schrödinger (NLSE) is a system completely integrable by the Inverse Scattering Method (ISTM). The NLSE has a simple solution, the monochromatic wave with frequency depended on amplitude - the condensate. The condensate is unstable with respect to modulation instability. The first solitonic solution of NLSE in the presense of condensate was fount in 1977 by E.A. Kuznetsov. This is a standing soliton which oscillate in time. Since this moment different types of solitonic solutions were found such as Akhmediev breather and Peregrine soliton. Now general N-solitonc solution is known.

In terms of ISTM solitonic solution is characterized by location of poles in the plane of spectral parameter. In the case of condensate the plane has a cut from $-A$ to A, where A is the condensate amplitude. This fact essentially complicate properties of solitonic solutions thus not all soliton's "species" are studied still. There are also important question: what is a nonlinear stage of modulation instability? In spatial dimension $D = 2, 3$, the answer is known - modulation instability leads to formation of finite time singularities - collapses. In dimension $D = 1$ collapses are forbidden. However in this case development of modulation instability leads to formation of "extreme" (rogue, freak) waves where energy density exceeds the mean level by order of magnitude.

We study solitonic solutions of the focusing NLSE in the presence of a condensate by using the dressing method. We find a general N-solitonic solution and separate a special designated class of "regular solitonic solutions" that do not disturb phases of the condensate at infinity by coordinate. All regular solitonic solutions can be treated as localized perturbations of the condensate. If we assume that the modulation instability develops from localized perturbation, only regular solution can be used as model for its nonlinear behavior.

The central result of our work is following. We find an important class of "superregular solitonic solutions" which are small perturbations at certain a moment of time. Then they develop into pairs of different solitons. This describes the nonlinear stage of the modulation instability of the condensate and can be treated as a sort of "integrable turbulence" appearing as a result of nonlinear development of the modulation instability.