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**Chinese Mathematical Society** 

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# **Conference Program**

8:15~8:30	Opening	Ceremony	Ro	om 1
8:30~9:30	Grimshaw (Keynote)		Room 1	
9:30~10:00	Coffee break			
	MS1 (I)	MS3 (I)	MS5 (I)	MS4 (I)
Organizer	Degasperis	Mei/Liu	Brand/Carr	Chen/Morandotti
Location	Room 2	Room 3	Room 1	Room 4
10:00~10:30	Ablowitz	Arikawa	Davis	Arie
10:30~11:00	Degasperis	Jensen	Bradley	Desyatnikov
11:00~11:30	Z. Feng	Mei	Adams	Harvey
11:30~12:00	Ghofraniha		Gou	Trillo
12:00~12:30	Grava			Z. Li
12:30~14:00	lunch			
	MS1 (II)	MS3 (II)	MS5 (II)	MS10
Organizer	Degasperis	Mei/Liu	Brand/Carr	B. Feng/Matsuo
Location	Room 2	Room 3	Room 1	Room 4
14:00~14:30	Kamvissis	Liao	Sommer	Matsuo
14:30~15:00	Miller	H.Liu	Brand	Miyatake
15:00~15:30	Qiao	P.Liu	Dyke	Furihata
15:30~16:00		Coffee	break	
16:00~16:30	Santini	Madsen	Ohberg	Sheu
16:30 <b>~</b> 17:00	Situ	Kharif	Cai	Sun
17:00~17:30	Trillo	Pedersen	Haddad	B. Feng

June 12

8:00~9:00	Mikhailov (I	Keynote)	Roo	m 1
9:00~9:30	Coffee break			
	MS2 (I)	MS6 (I)	MS5 (III)	MS9 (I)
Organizer	Matveev	Pelinovsky	Brand/Carr	Clarkson
Location	Room 2	Room 3	Room 1	Room 4
9:30~10:00	Matveev	Malomed	Fromhold	Buckingham
10:00~10:30	Dubard	Nixon	Piazza	Grava
10:30~11:00	Degasperis	Zezyulin	Astrakharchik	Kecker
11:00~11:30	Gaillard	D'Ambroise	Carr	Marchant
11:30~12:00	Gelash	Alfimov	Su	Miller
12:00~12:30	Chabchoub	Shi	Flach	Virtanen
12:30~14:00		lun	ich	
14:00~18:00	Tour to the Fragrant Mountain			
18:30~20:30	Conference Reception			

June 13

	MS2 (II)	MS6 (II)	MS7 (I)	MS5 (IV)
Organizer	Matveev	Pelinovsky	Menyuk/Dudley	Brand/Carr
Location	Room 3	Room 4	Room 5	Room 2
8:00~8:30	J. He		Menyuk	Zhai
8:30~9:00	Kibler	Pelinovsky	Dudley	Zuelicke
9:00~9:30	Klein	Zhu	Gat	Julia-Diaz
9:30~10:00	Ling	Hang	Hölzer	B. Wu
10:00~10:30		Coffee	break	
10:30~11:00	Clarkson	Mostafazadeh	Malomed	Duan
11:00~11:30	Smirnov	Konotop	Eggleton	W.Liu
11:30~12:00		Graefe	Harvey	
12:00~13:30		lunch		
	MS3 (III)	MS6 (III)	MS9 (II)	MS11 (I)
Organizer	Mei/Liu	Pelinovsky	Clarkson	Liu/Popowicz
Location	Room 2	Room 3	Room 4	Room 5
13:30~14:00	Lynett	Cartarius	Mikhailov	Popowicz
14:00~14:30	Sergeeva	Park	Olver	Lechtenfeld
14:30~15:00	Brocchini	Joglekar	Clarkson	B. Feng
15:00~15:30	Coffee break			
	MS2 (III)	MS8	MS7 (II)	C3 (I)
Organizer	Matveev	Rothos	Menyuk/Dudley	Tian
Location	Room 2	Room 3	Room 5	Room 4
15:30 <b>~</b> 16:00	Yan	Alfimov	Lushnikov	Brody
16:00 <b>~</b> 16:30	Yang	Trichtchenko	Chembo	Clark Duran
16:30 <b>~</b> 17:00	Wabnitz	Zezyulin	Wai	Friedland
17:00~17:30	Slunyaev	Curtis	de Sterke	Hattam
47.00 40.00	0			Khawaja
17:30~18:00	Sergeeva		liday	
19:00~20:30		dinn	ier	

June 14

	MS4 (I)	MS11 (II)	C1 (I)
Organizer	Chen/Morandotti	Liu/Popowicz	Antar
Location	Room 2	Room 3	Room 4
8:00~8:30	DelRe	Krutov	Antar
8:30~9:00	Wabnitz	Zuo	Bagci
0.00 0.20	LL Zana		Driben
9:00~9:30	H. Zeng	E. Fan	Driben
9:30~10:00	Y. Hu	Levi	Gat
10:00~10:30		Coffee	break
	MS4 (II)	MS11 (III)	C3 (II)
Organizer	Chen/Morandotti	Liu/Popowicz	Clark
Location	Room 2	Room 3	Room 4
10:30~11:00	De Angelis	Lou	Rashid
11:00~11:30	Efremidis	Qu	C. Yu
11:30~12:00	F. Ye	R. Lin	
12:00 <b>~</b> 12:30	G. Huang	D. Wu	
12:30~14:00		lun	ch
		MS11 (IV)	C1 (II)
Organizer		Liu/Popowicz	Bakirtas
Location		Room 2	Room 3
14:00~14:30		Z. Zhu	Gazeau
44.00 45.00		V. Ohar	Goksel
14:30~15:00		Y. Chen	Jiang
15:00 <b>~</b> 15:30		Zullo	Louis
15:30~16:00		K. Tian	Yulin
16:00~16:30		Coffee	break
		C2	
Presider		Qu	
Location		Room 2	
16:30 <b>~</b> 16:50		Chang	
16:50 <b>~</b> 17:10		Fujishima	
17:10 <b>~</b> 17:30		Grahovski	
17:30~17:50		Hay	
17:50 <b>~</b> 18:10		Yamane	
18:10 <b>~</b> 18:30		Zhao	
19:00~20:30		dinn	er
Note	0	ne posters will n June 13 and 14 (f	rom 8:00 to 19:00)

June 15

# **List of Talks**

# **KEYNOTE TALKS**

This conference will have two keynote speakers:

**1. Roger Grimshaw** (Loughborough University, U.K.) "The effect of rotation on internal solitary waves"

**2. Alexander Mikhailov** (University of Leeds, U.K.) "Integrable PDEs, D $\Delta$ Es and P $\Delta$ Es"

# **MINISYMPOSIA**

## **MS1.** Dispersive Nonlinear Vector Waves: Solitons and Shocks

#### **Organizer:**

Antonio Degasperis (University of Rome, Italy)

Several wave phenomena are modeled by vector fields, namely by vector valued functions of coordinates and time. Describing vector wave propagation, if affected by nonlinear coupling, requires numerical computations as well as analytical methods of investigation. Integrable models, when applicable, even if only approximate, play an important role because of our ability of constructing explicit solutions and of using spectral techniques. The target of this MiniSymposium is to present recent contributions to this subject, in both experimental contexts and in mathematical schemes of computation. Special focus is on nonlinear models of relevance in physics which possess exact or approximate explicit solution and/or shock-wave solutions.

#### List of Speakers:

- 1) Mark J. Ablowitz (University of Colorado, U. S. A.) "Solitons and interactions of dispersive shock waves"
- 2) Antonio Degasperis (University of Rome, Italy) "Vector semi-rational soliton solutions"
- 3) Zhaosheng Feng (University of Texas-Pan American, U. S. A.) "Solitary wave solution to the KdV-Burgers-type equation"
- 4) Neda Ghofraniha (Sapienza University of Rome, Italy) "Experiments on optical spatial shocks in random media"
- 5) Tamara Grava (SISSA, Italy) "Hamiltonian PDEs and dispersive shock waves"
- 6) Spirydon Kamvissis (University of Crete, Greece) "Perturbation of the periodic Toda lattice"
- 7) Peter Miller (University of Michigan, U. S. A.) "Internal waves with small dispersion"
- 8) Zhijun Qiao (University of Texas, U. S. A.)"Integrable system with peakon, weak kink, and kink-peakon interactional solutions"

9) Paolo Santini (Sapienza University of Rome, Italy) "Nonlinear wave propagations in multidimensions and wave breaking in Nature"

10) Guohai Situ (Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, China)

"Observation of the Berezinskii-Kosterlitz-Thouless transition in a photonic lattice"

11) Stefano Trillo (University of Ferrara, Italy)"Optical dispersive shock waves in parametric wave mixing"

# MS2. Rogue Waves 2013: Theory and Experiment

#### **Organizer:**

V. B. Matveev (Université de Bourgogne, France)

The aim of this mini-symposium is to join together the active researchers in the field and to expose the most recent results of the last 2 years both concerning the theoretical and experimental results concerning the studies of the rogue waves. Nowadays a phenomenon of the appearance of rogue waves was observed in quite different areas ranging from the ocean rogues waves, rogue waves in plasma and optical fibers,- to the rogue waves in Bose-condensates, finances and atmosphere. The most impressive theoretical results are connected with studies of the related modulation instabilities for the nonlinear models although the linear approach rests still important. Special attention will be paid to the recently discovered phenomena of multiple rogue waves generation in various nonlinear integrable models.

#### List of Speakers:

- 1) Amin Chabchoub (Technishe Universität; Hamburg, Germany) "Experiments on NLS solutions in the focusing and defocusing regime"
- 2) Peter Clarkson (University of Kenterberry, U. K.) "Rational solutions of the Boussinesq equation"
- 3) Antonio Degasperis (Sapienza University of Rome, Italy) "Resonant interactions and rogue waves"
- 4) Philippe Dubard (Université de Bourgogne, France)"New results on NLS-KP-I correspondence and multiple rogue waves events"
- 5) Pierre Gaillard (Université de Bourgogne, France) "Deformations of higher rogue Peregrine breathers and monstrous polynomials"
- 6) Andrei Gelash (Novosibirsk State University, Russia )"On the nonlinear stage of Modulation Instability"
- 7) Jingsong He (Ningbo University, China) "New patterns of higher order rogue waves"
- 8) Bertrand Kibler (Université de Bourgogne, France)"Rogue waves: rational solutions and wave turbulence theory"
- 9) Christian Klein (Université de Bourgogne, France)
   "Algebro-geometric solutions, solitons and breathers of Davey-Stewartson and Nonlinear Schrodinger equations "

10) Liming Ling (Institute of Applied Physics and Computational Mathematics, China)"High order rogue waves solutions of NLS and DNLS equations"

11) Vladimir B. Matveev (Université de Bourgogne, France)
 "Large parametric asymptotic of the multi-rogue waves solutions of the NLS equation and extreme rogue wave solutions of the KP-I equation"

- 12) A.O. Smirnov (St.Petersbourg State University of Aerospace Instrumentation, Russia) "Multi-phase freak waves"
- 13) Zhenya Yan (Institute of Systems Science, Chinese Academy of Sciences, China) "Nonautonmous rogue waves and interactions in some nonlinear physical models"
- 14) Jianke Yang (University of Vermont, U. S. A)"Dynamics of rogue waves in the Davey-Stewartson equations"
- 15) Stefan Wabnitz (Università di Brescia, Brescia, Italy) "Optical tsunami"
- 16) Alexey Slunyaev (Russia Nizhny Novgorod State Technical University, Russia)"Coherent wave groups causing rogue waves"
- 17) Anna Sergeeva (Russia Nizhny Novgorod State Technical University, Russia)"Extreme events in numerical simulations of irregular surface water waves"

# **MS3.** Tsunami and Coastal Dynamics

#### **Organizers:**

Chiang C. Mei (Massachusetts Institute of Technology, U. S. A.) Philip Liu (Cornell University, U. S. A.)

Long the source of inspiration for artists, poets and writers, ocean waves have fascinated and challenged scientists and engineers for centuries for their intricate dynamics requiring powerful tools for their analysis and prediction, and ingeneous technology for their control and utilization. Scientific advances in the past few decades on nonlinear-wave dynamics in general owe a good deal to the developments in ocean surface waves. Whether due to wind, earthquakes or human activities, their occurrence, propagation and impact on shorelines and global climate are of immense importance to the environment and to human livelihood.

Research activities among scientists and engineers have been catapulted by recent tsunami in Indian ocean and Japan and by frequent occurence of freak waves. On the other hand the mutual influence of sea waves on coast lines have always been a challenging topic requiring the contributions from separate disciplines. In this minisymposium we have invited active researchers from all continents to discuss several fascinating aspects of ocean wave physics, with particular emphasis on the dynamical behavior in shallow seas and impact on the coast. In addition to mainly theoretical analyses, there will be papers dealing with experimental investigations and environmental implications. We hope this minisymposium will not only highlight the accomplishments and needs in water-wave studies, but also stimulate the exchange of ideas with other branches of wave physics.

#### List of Speakers:

1) Taro Arikawa (Port and Airport Research Institute, Japan)

"Development of solid-gas-liquid 3-phase calculation system coupling CADMAS-SURF /3D and DEM"

2) Maurizio Brocchini (Polytechnic University of Marche, Italy)"The influence of frictional Chezy contributions on the shoreline motion"

3) Atle Jensen (University of Oslo, Norway) "Kinematics of solitary wave runup"

4) Christian Kharif (Ecole Centrale Marseille, France)
 "Vorticity effect on modulational instability: Application to rogue waves"

5) Shijun Liao (Shanghai Jiao Tong University, China) "On the multiple steady-state fully resonant water waves"

- 6) Hua Liu (Shanghai Jiao Tong University, China) "Impacts of local tsunamis on China coasts"
- 7) Philip L-F Liu (Cornell University, U. S. A.)"Evolution of boundary layer flows under a transient long wave"
- 8) Per A. Madsen (Technical University of Denmark, Denmark) "Asymptotic solutions to the Cauchy-Poisson problem in 2D and 3D"
- 9) Chiang C. Mei (Massachusetts Institute of Technology, U. S. A. ) "Nonlinear long waves over a muddy beachh"
- 10) Geir Pedersen (University of Oslo, Norway)"Waves due to sub-aerial slides in narrow waterways. Experiments and computational challenges"
- 11) Patrick Lynett (University of Southern California, U. S. A.) "Dynamic, tsunami-induced nearshore currents"

12) Anna Sergeeva (Institute of Applied Physics, Nizhny Novgorod State Technical University, Russia)

"KdV turbulence in shallow water"

# **MS4.** Nonlinear Optical Waves and Novel Phenomena

#### **Organizers:**

Zhigang Chen(San Francisco State University, U. S. A.)Roberto Morandotti(Université du Québec, Canada)

The field of nonlinear photonics has been hot and dynamically changing. This mini-symposium aims to bring together experts from the field of nonlinear photonics and related areas, including but not limited to novel optical wave phenomena such as wave dynamics in plasmonic waveguide arrays and optical microcavities, nonlinear frequency conversion and wave mixing, spatial solitons and vortices, nonlinear beam shaping and self-accelerating beams, and optical switching in nonlinear photonic crystals. The scope of this symposium covers both theoretical and experimental study on linear and nonlinear wave phenomena in a variety of optical structures and material systems.

#### List of Speakers:

- Yi Hu (Université du Québec, Canada; Nankai University, China) "Self-accelerating beams through spectrum-engineering"
- 2) Stefano Trillo (University of Ferrara, Italy) "Nonlinear waves emitting resonant radiation: new scenarios"

3) Nikos Efremidis (University of Crete, Greece), D. N. Christodoulides (CREOL, University of Central Florida, U. S. A.), Z. Chen (San Francisco State University, U. S. A.)
 "Navigating accelerating beams along arbitrary trajectories"

4) Ady Arie (Tel-Aviv University, Tel-Aviv, Israel) "Shaping and twisting light beams using nonlinear holograms"

5) A.S. Desyatnikov (The Australian National University, Australia), D. Buccoliero (The Australian National University, Australia), M. R. Dennis (University of Bristol, U. K.), and Yu. S. Kivshar (The Australian National University, Australia)
"Vortex knots in self-trapped laser beams"

6) Stefan Wabnitz, T. Hansson and D. Modotto (University of Brescia, Brescia, Italy)
 "Nonlinear dynamics of multiwave mixing in optical microcavities"

7) Eugenio DelRe, Fabrizio di Mei, Claudio Conti, Aharon J.Agranat, and Jacopo Parravicini (University of Rome, Italy; Hebrew University of Jerusalem, Israel)

"Observation of diffraction-free submicrometer visible laser beam propagation through nanodisordered ferroelectrics"

- 8) Costantino De Angelis (Università degli Studi di Brescia, Italy)
   "Binary plasmonic waveguide arrays: energy localizarion, modulational instability and gap solitons"
- 9) Guoxiang Huang (East China Normal University, China) "Slow and fast light bullets and vortices in coherent atomic systems"
- 10) John Harvey (Physics Department, University of Auckland, New Zealand) "Parametric amplification and cascaded FWM in optical fibers"
- 11) Fangwei Ye (Shanghai Jiao Tong University, China) "Sub-wavelength plasmonic lattice solitons"
- 12) Zhiyuan Li (Institute of Physics, CAS, China)"Nonlinear photonic crystal for optical switching and logic functionality"
- 13) Heping Zeng (East China Normal University, China)"High-power fiber optical frequency combs"

# MS5. Quantum Degenerate Systems

### **Organizers:**

Joachim Brand	(Massey University, New Zealand)
Lincoln Carr	(Colorado School of Mines, U. S. A.)

Nonlinear waves are ubiquitous phenomena for ultra-cold gases in the regime of quantum degeneracy, in particular Bose-Einstein condensates, where internal friction is suppressed or absent. Recent years have seen progress in the experimental study of nonlinear wave processes but have also diversified the field of quantum degenerate gases to include novel symmetries, trapping geometries, and artificial gauge potentials. The physical structure of the underlying quantum system becomes important for strongly correlated gases, e.g. in low dimensions, where mean-field theory fails. Interesting connections to nonequilibrium phase transitions are realised where nonlinear wave dynamics is relevant for defect formation and decay. This minisymposium will provide a broad perspective on quantum degenerate systems and highlight recent developments of the field.

#### List of Speakers:

- 1) Allan Adams (MIT, U. S. A.) "Holography and turbulence"
- 2) Grigory Astrakharchik (Universitat Politecnica de Catalunya, Spain) "Lieb's soliton-like excitations in harmonic trap"
- 3) Ashton Bradley (University of Otago, New Zealand) "Inverse energy cascade in two-dimensional quantum vortex turbulence"
- 4) Joachim Brand (Massey University, New Zealand)"Dark solitons in superfluid Fermi gases: Dispersion relation and snaking instability"
- 5) Qing-Yu Cai (CAS Wuhan Institute of Physics and Mathematics, China) "Towards experimentally testing the paradox of black hole information loss"
- 6) Lincoln Carr (Colorado School of Mines, U. S. A.)"Macroscopic quantum tunneling in Bose-Einstein condensates"
- 7) Matthew Davis (University of Queensland, Australia)"Nonequilibrium superfluidity and internal convection in finite temperature Bose gases"
- 8) Hao Duan (Tsinghua University, China)"Ultracold collision in the presence of synthetic spin-orbit coupling"

- 9) Paul Dyke (Rice University, U. S. A.)"Interactions of bright matter-wave solitons with a potential defect"
- 10) Sergej Flach (Massey University, New Zealand)"Correlated metallic few-particle bound states in quasiperiodic potentials"
- 11) Mark Fromhold (University of Nottingham, U. K.)"Nonlinear wave dynamics of cold quantum matter near atom chips"
- 12) Shih-Chuan Gou (National Changhua University of Education, Taiwan, China)
   "Kibble-Zurek scaling and its breakdown for spontaneous generation of Josephson vortices in
   Bose-Einstein condensates"
- 13) Laith Haddad (Colorado School of Mines, U. S. A.)"Solitons in armchair and zigzag geometries in the nonlinear Dirac equation"
- 14) Bruno Juliá-Díaz (University of Barcelona, Spain)"Fractional quantum Hall phases of two-component ultracold bosonic gases"
- 15) Wu-Ming Liu (CAS Institute of Physics, China)"Controlling phase separation of a two-component Bose-Einstein condensate by confinement"
- 16) Patrick Öhberg (Heriot-Watt University, U. K.)"The Bose gas as a simulator of interacting gauge theories"
- 17) Francesco Piazza (Technical University of Munich, Germany)"Critical velocity and current-phase relation of dilute ultracold bosonic atoms"
- 18) Ariel Sommer (MIT and University of Chicago, U. S. A.)"Heavy solitons in a fermionic superfluid"
- 19) Yi Su (CAS Institute of Theoretical Physics, China)"Macroscopically superposed ground states of a dipolar Bose gas in a double-well potential"
- 20) Biao Wu (Peking University, China) "Bright solitons in spin-orbit coupled Bose-Einstein condensates"
- 21) Hui Zhai (Tsinghua University, China)"Dynamics of spin-orbit coupled quantum gases"
- 22) Uli Zuelicke (Victoria University of Wellington, New Zealand) "Low-dimensional spin-orbit-coupled BECs: Inspirations from semiconductor spintronics"

# **MS6.** Nonlinear Waves in PT-symmetric Potentials

## Organizer

Dmitry Pelinovsky (McMaster University, Canada)

This minisymposium will cover the recent advances in analysis and modelling of nonlinear waves in continuous and discrete nonlinear Schrödinger equation with PT-symmetric potentials. The potentials of the nonlinear Schrödinger equations are said to be PT-symmetric if they are symmetric with respect to a composition of the parity and time reversal transformations. A typical example of such potentials is a local space-dependent potential with an even real part and an odd imaginary part.

Properties of both linear and nonlinear models with such potentials have been widely studied in the past five years. The minisymposium will feature talks of both applied mathematicians and physicists reporting the recent works on the subject. The main emphasis of this minisymposium will be on questions of global existence of time-dependent solutions and existence of stable localized solutions in such models.

#### List of Speakers:

- 1) G.L. Alfimov (National Research University of Electronic Technology, Moscow, Russia) "Nonlinear modes of the nonlinear Schrodinger equation with complex periodic potential"
- 2) Holger Cartarius (University of Stuttgart, Germany) "Bose-Einstein condensates in PT-symmetric double-well and multi-well setups"
- 3) Jennie D'Ambroise (Bard College, U. S. A.) "Eigenstates of lattices with PT-symmetric defects"
- 4) Eva-Maria Graefe (Imperial College, U. K.) "Mean-field approximation for PT-symmetric many-boson systems"
- 5) Yogesh Joglekar and Derek Scott (IUPUI, U. S. A.) "Competing PT-symmetric potentials and re-entrant PT-symmetric phase"
- 6) Vladimir Konotop (University of Lisbon, Portugal)"Nonlinearities supporting localized modes in PT-symmetric lattices"
- 7) Ali Mostafazadeh (Koc University, Istambul, Turkey)"Spectral singularities, unidirectional invisibility and PT-symmetry"
- 8) Sean Nixon (University of Vermont, U. S. A.)"Nonlinear dynamics of wave packets in PT-symmetric optical systems"

- 9) Namkyoo Park (Seoul National University, South Korea) "PT-symmetric wave dynamics in photonic molecular systems"
- 10) Dmitry Pelinovsky (McMaster University, Canada) "Nonlinear stationary states in PT-symmetric lattices"
- 11) Zhiwei Shi (Guangdong University of Technology, China)"Spatial solitons in defocusing Kerr media with PT-symmetric potentials"
- 12) Chao Hang (East China Normal University, China) "PT symmetry with a system of three-level atoms"
- 13) Yi Zhu (Tsinghua University, China)"Nonlinear wave packets in deformed honeycomb lattices"
- 14) Dmitry Zezyulin (Universidade de Lisboa, Portugal) "Solitons in PT-symmetric nonlinear lattices"
- 15) Boris Malomed (University of Tel Aviv, Israel)"Stability of symmetric and antisymmetric solitons in PT-symmetric couplers"

# **MS7.** Frontiers of Nonlinear Waves in Optics

#### **Organizers:**

C.R. Menyuk (University of Maryland at Baltimore County, U. S. A.) J.M. Dudley (University of Franche-Comte, France)

Since the invention of the laser, progress in nonlinear optics and the mathematics of nonlinear waves have gone together. Mathematical concepts like solitons, similaritons, and accordions have enriched our understanding of optical communication systems, modelocked lasers, and light propagation and scattering through a wide variety of media. The experimental study of supercontinuum generation, optical filamentation, optical combs, and photonic crystal fibers have motivated the development of new mathematical and computational techniques. The goal of this minisymposium is to bring together both experimental and mathematical experts in nonlinear optics to discuss recent progress and exchange ideas.

#### List of Speakers:

- Curtis Menyuk and Shaokang Wang (University of Maryland Baltimore County, U. S. A.)
   "Finding the stability of modelocked laser pulses over a broad parameter range"
- 2) John Dudley (University of Franche-Comté, France) "From rogue waves to randomness in nonlinear supercontinuum generation"
- 3) Omri Gat and David Kielpinski (OG = Hebrew University, Israel; DK = Griffith University, Australia)

"Frequency comb injection locking of mode locked lasers"

- 4) Philipp Hölzer and Philip Russell (Max-Planck Institute for Light, Germany) "Nonlinear fiber optics with ultraviolet light and high fields"
- 5) Boris Malomed (Tel-Aviv University, Israel) "Bright optical solitons from defocusing nonlinearities"
- 6) Benjamin Eggleton (University of Sydney, Australia)"Nonlinear optical phononics: Harnessing stimulated Brillouin scattering in nanoscale circuits"
- 7) John Harvey (The University of Auckland, New Zealand) "Similaritons and chirpedfibre lasers"

8) Pavel Lushnikov (University of New Mexico, U. S. A.)
 "Logarithmic scaling in the catastrophic self-focusing (collapse) of a laser beam in Kerr media"

9) Yanne Chembo and Curtis Menyuk (YC=University of Franche-Comté, France; CM=University of Maryland Baltimore County, U. S. A.)

"Lugiato-Lefever model for optical frequency comb generation based on monolithic whispering gallery mode resonators"

10) Li Feng and Ping-Kong Alex Wai (Hong Kong Polytechnic University, Hong Kong, China)

"Modeling of frequency domain mode-locked fiber lasers"

- 11) Martijn de Sterke (University of Sydney, Australia)"Linear and nonlinear frozen light in waveguides"
- 12) F. Ömer Ilday (Bilkent University, Turkey)"Nonlinearity engineering in photonics: from mode-locked lasers to nonlinear laser lithography"

# **MS8. Localized Excitations in Nonlocal PDEs**

## **Organizer:**

Vassilis M Rothos (Aristotle University of Thessaloniki, Greece)

The evolution equations can be used to illustrate many striking features of nonlinear waves, each of which has been understood by a combination of methods from scientific computations and from the theory of PDEs and geometric dynamical systems. The minisymposium will bring together specialists in the nonlinear nonlocal evolution equations. Recent results were obtained in the context of localized modes of nonlocal lattice equations, stability of vortices in nonlocal Nonlinear Schrödinger (NLS) equations with an external potential, embeded solitons in nonlocal PDEs and nonlocal water waves equations. Speakers of the minisymposium will bring the latest development in these and other problems.

## List of Speakers:

- 1) G.L.Alfimov (National Research University of Electronic Technology, Moscow, Russia) "Discrete spectrum of nonlinear modes in weakly nonlocal problems: a mechanism to emerge"
- 2) Chris Curtis (University of Colorado Boulder, U. S. A.) "Stability of solutions to a non-local Gross-Pitaevskii equation"
- 3) Olga Trichtchenko (University of Washington, U. S. A.) "Stability of water waves in the presence of surface tension"

4) D. A. Zezyulin (University of Lisbon, Portugal)

"Localized modes in nonlocal nonlinear Schrödinger equation with PT-symmetric parabolic potential"

## MS9. Universality of Nonlinear Behaviour

## **Organizer:**

Peter Clarkson (University of Kent, U. K.)

In this minisymposium, the focus will be the study of nonlinear equations, in particular integrable systems. The modern theory of integrable systems and the modern theory of nonlinear waves are intimately related topics, with the Korteweg-de Vries, nonlinear Schrödinger and Boussinesq equations, being the prototypical examples.

The Painlevé equations, discovered about a hundred years ago, are special amongst nonlinear ordinary differential equations in that they are integrable?due to their representation as Riemann-Hilbert problems. The Painlevé equations also arise as symmetry reductions of nonlinear waves equations such as the Korteweg-de Vries, nonlinear Schrödinger and Boussinesq equations. Further they are nonlinear analogues of the classical special functions with a plethora of remarkable properties.

The minisymposium will be focus on some of the recent developments in the research on integrable models which are deeply connected to the theory of nonlinear waves.

### List of Speakers:

- Robert Buckingham (University of Cincinnati, U. S. A.)
   "Applications of Painlevé functions to nonlinear wave equations"
- 2) Peter Clarkson (University of Kent, U.K.)"Painlevé equations nonlinear special functions"

3) Tamara Grava (SISSA, Trieste, Italy)

"Solution of the generalized nonlinear Schrödinger equation in the semiclassical limit and Painlevé equations"

- 4) Thomas Kecker (University College London, U. K.) "Solutions of ODEs with movable algebraic singularities"
- 5) Timothy Marchant (University of Wollongong, Australia) "Solitary waves and dispersive shock waves in colloidal media"
- 6) Alexander Mikhailov (University of Leeds, U. K.)"Wave fronts and soliton webs--exact solutions of the 2D Volterra system"
- 7) Peter Miller (University of Michigan, U. S. A.)"Asymptotics of rational solutions of the inhomogeneous Painlevé-II equations"
- 8) Sheehan Olver (University of Sydney, Australia)"Numerical calculation of finite random matrix statistics, and the onset of universality"

9) Jani Virtanen (University of Reading, U. K.) "Riemann-Hilbert problems in Hardy spaces"

# **MS10.** Numerical Methods for Hamiltonian PDEs

**Organizers:** 

Bao-Feng Feng( The University of Texas-Pan American, U. S. A. )Takayasu Matsuo( The University of Tokyo, Japan )

Many important PDEs in nonlinear wave problems are Hamiltonian systems, among which some possess bi-Hamiltonian structure, thus, become integrable and admit infinite number of conservation laws. In the past two decades, there have been major advances in the study of numerical methods for Hamiltonian PDEs. Structure-preserving methods such as symplectic and multi-symplectic methods have been developed. On the other hand, the integrable method seems a new and promising one for integrable Hamiltonian PDEs.

The purpose of this organized minisymposium is to bring together researchers from both integrable system and numerical PDEs to discuss recent advances on numerical aspects of Hamiltonian PDEs.

## List of Speakers:

1) Takayasu Matsuo ( The University of Tokyo, Japan )

"Discretization of a nonlocal nonlinear wave equation preserving its variational structure"

2) Yuto Miyatake (The University of Tokyo, Japan)"Structure-preserving discretizations for Ostrovsky-type nonlocal nonlinear wave equations"

3) Daisuke Furihata (Osaka University, Japan)

"An attempt to design a fast and structure preserving scheme for Feng equation"

4) Tony Sheu (National Taiwan University, Taiwan, China)

"Development of a Hamiltonian-conserving combined compact difference scheme for simulating CH equation at different initial conditions and investigating head-on collision of solitons "

- 5) Yajuan Sun (AMSS, Chinese Academy of Sciences, China) "Structure-preserving numerical integrators for peakon b-family equations"
- 6) Bao-Feng Feng (The University of Texas-Pan American, U. S. A.) "Self-adaptive moving mesh methods for a class of nonlinear wave equations"

# **MS11.** Integrable and Super Integrable Systems

## **Organizers:**

Q. P. Liu (China University of Mining and Technology, China) Ziemowit Popowicz (University of Wroclaw, Poland)

The theory of modern integrable systems originated from the work on the celebrated Korteweg-de Vries equation, a prototype water wave model. The aim of this minisymposia is to bring together the experts in the theory of integrable systems, including classical integrable systems (both continuous and discrete) and supersymmetric integrable systems and will be focused on the recent achievements of this field.

#### List of Speakers:

1) Ziemowit Popowicz (University of Wroclaw, Poland) "The generalizations of the peakon's systems"

2) Olaf Lechtenfeld (Universität Hannover, Germany) "Calogero-type models with maximally extended superconformal symmetry"

3) Federico Zullo (University of Kent, U. K.)

"Hamiltonian flows from Bäcklund transformations: an application to the Ablowitz-Ladik hierarchy."

4) Andrey Krutov (Ivanovo State Power University, Russia)

"On the (non)removability of spectral parameters in  $\mathbb{Z}_2$ -graded zero-curvature representations and its applications"

5) D. F. Zuo (University of Science and Technology of China, China) "Euler equations related to the generalized Neveu-Schwarz algebra"

6) Engui Fan (Fudan University, China)"Super extension of Bell polynomials with applications to supersymmetric equations"

7) D. Levi (Universita' degli Studi di Roma Tre, Italy)"Partial difference schemes and commuting operators on the lattice"

8) S. Y. Lou (Ningbo University, China)

"Nonlocal symmetries, conservation laws and related reduction solutions of integrable systems"

9) Changzheng Qu (Ningbo University, China)"Peakons and stability of the modified μ-Camassa-Holm equation"

- 10) Runliang Lin (Tsinghua University, China) "Bilinear identity for an extended KP hierarchy"
- 11) Derchyi Wu (Academia Sinica, Taipei, China) "Integrable twisted hierarchies with  $D_2$  symmetries"
- 12) Z. N. Zhu (Shanghai Jiaotong University, China)"Solitons and dynamic properties of the coupled semidiscrete Hirota equation"
- 13) Yong Chen (East China Normal University, China)"A maple package for generating one-dimensional optimal systems of finite dimensional Lie algebra"
- 14) Bao-Feng Feng (The University of Texas-Pan American, U. S. A.) "Integrable semi-discretizations for the short wave limit of the Degasperis-Procesi equation"
- 15) Kai Tian (China University of Mining and Technology, China)"Behavior of Hamiltonian structures under supersymmetric reciprocal transformations"

# **CONTRIBUTED TALKS**

# C1. Nonlinear Optics:

- 1) Nalan Antar (Istanbul Technical University, Turkey) "Lattice solitons in cubic-saturable media with external potentials"
- 2) Mahmut Bagci, Ilkay Bakirtas, Nalan Antar (Istanbul Technical University, Turkey) "Vortex and dipole solitons in defect lattices"
- 3) Ilkay Bakirtas (Istanbul Technical University, Turkey)
   "Solitons and wave collapse in nonlocal nonlinear Schrödinger type equations"
- 4) Rodislav Driben (Tel Aviv University, Israel; Paderborn University, Germany)
   "Super-focusing and mirroring of Airy pulses propagating in fibers with third order dispersion"

5) Rodislav Driben (Tel Aviv University, Israel; Paderborn University, Germany)
 "Newton cradle in optics. Understanding the fission of higher-order solitons under the action of the higher-order dispersion"

- 6) Omri Gat (Hebrew University of Jerusalem, Israel)"Finite size effects in wave packet scattering off a dispersion band gap"
- 7) Maxime Gazeau (Université Lille 1 Sciences et Technologies, France)"Nonlinear pulse propagation in optical fibers with randomly varying birefringence"
- 8) Izzet Göksel, Nalan Antar, Ilkay Bakirtas (Istanbul Technical University, Turkey) "Solitons of cubic-quintic nonlinear Schrödinger equation with various external potentials"
- 9) Hui Jiang (University of Nottingham, U. K.) "Wave propagations in periodically curved waveguide arrays"
- 10) Simon Louis (The University of Wollongong, Australia)"Optical solitary waves in thermal media with non-symmetric boundary conditions"
- 11) Alexey Yulin (Universidade de Lisboa, Portugal)"Domain walls and dissipative solitons in polariton systems"
- 12) Alexey Yulin (Universidade de Lisboa, Portugal)"Inter-soliton interactions mediated by dispersive waves in optical fibers"

# C2. Integrable Systems:

- 1) Chueh-Hsin Chang (National Taiwan University, Taiwan, China) "Study of the scattering problem of Camassa-Holm equation"
- 2) Hironobu Fujishima (Optics R&D Center, CANON INC., Japan)"The apprpximate method and its application for the AKNS--type linear scattering problem by

descritizing the initial wave packet"

- 3) Georgi G. Grahovski (University of Leeds, U. K.)
   "Integrable discretisations of the nonlinear Schrödinger equation on Grassmann algebras"
- 4) Mike Hay (Università degli Studi di Roma Tre, Italy) "Lattice modified KdV hierarchy from a Lax pair expansion"
- 5) Hideshi Yamane (Kwansei Gakuin University, Japan) "Long-time asymptotics for the defocusing integrable discrete nonlinear Schrödinger equation"
- 6) Li-Chen Zhao (Institute of Applied Physics and Computational Mathematics, China) "Rogue wave solutions of a three-component coupled nonlinear Schrödinger equation"

# C3. General aspects of nonlinear waves:

- 1) Dorje C. Brody (Brunel University, U. K.) "Nonlinearity and constrained quantum motion"
- 2) Simon Clarke (Monash University, Australia)"Nonlinear effects on the focussing of tsunami due to underwater lenses"
- 3) Angel Duran (University of Valladolid, Spain)"Numerical generation of traveling wave profiles with the Petviashvili method"
- 4) Lazar Friedland (Hebrew University of Jerusalem, Israel) "Autoresonant wave dynamics in Vlasov-Poisson systems"
- 5) Laura Hattam (Monash University, Austrilia) "Modulation theory for the steady fKdVB equation- constructing periodic solutions"
- 6) Usama Al Khawaja (United Arab Emirates University, United Arab Emirates) "Directional flow of solitons with asymmetric potential wells: Soliton diode"

7) Abdur Rashid (Gomal University, Pakistan)

"Legendre pseudospectral method for solving three-dimensional non-linear hyperbolic partial differential equations"

8) Ching-Hao Yu (National Taiwan University, Taiwan, China)

"A dispersively accurate compact finite difference method for the Degasperis-Procesi equation"

## POSTER

Fatma Aydogmus (Istanbul University, Turkey)
 "Two-dimensional vector field visualization of Gursey instantons"

2) Rodislav Driben (Tel Aviv University, Israel; University of Paderborn, Germany)
 "Generation of ultra-compressed solitons with a high tunable wavelength shift in Raman-inactive hollow-core photonic crystal fibers"

3) Ibrahim H. El-Sirafy (Alexandria University, Egypt)
 "On axisymmetrical boundary problem of unsteady motion of micropolar fluid in the half-space"

4) Pierre Gaillard (Université de Bourgogne, France)

"Deformations of higher Peregrine breathers, multi-rogue waves and their analytical expressions"

5) A. A. Gelash (University of Arizona, U. S. A.) "Superregular solitonic solutions in Nonlinear Schrödinger Equation"

6) Yannis Kominis (University of Patras, Greece; National Technical University of Athens, Greece)

"Non-paraxial traveling solitary waves in layered nonlinear media"

7) Chuanzhong Li (Ningbo University, China)"Block algebra in two-component BKP and D type Drinfeld-Sokolov hierarchies"

8) Chuanzhong Li (Ningbo University, China)"Rogue waves of the Hirota and the Maxwell-Bloch equations"

9) Maohua Li (Ningbo University, China)
 "Virasoro type algebraic structure and the analyticity of the constrained discrete KP hierarchy"

10) Yongyao Li (South China Agricultural University, China; Tel Aviv University, Israel)
 "Matter-wave solitons supported by field-induced dipole-dipole repulsion with a spatially modulated strength"

11) Ji Lin (Zhejiang Normal University, China)

"(2+1)- dimensional analytical solutions of the combining cubic-quintic nonlinear Schrödinger equation"

12) Bo Ren (Shaoxing University, China)"The supersymmetric Burgers equation: Bosonization and exact solution"

13) Eren Tosyali (Istanbul University, Turkey)"Search of chaos in Bose-Einstein condansate in tilted bichromatic potential"

14) Tao Xu (China University of Petroleum, China)

"Darboux transformation of the Sasa-Satsuma equation: New solitons and resonant interaction"

15) Jun Yu (Shaoxing University, China)"Bosonization of the supersymmetric Ito equation"
# Abstract

#### Keynote Talks

## The effect of rotation on internal solitary waves

Roger Grimshaw

Department of Mathematical Sciences, Loughborough University, LE11 3TU, U.K. Tel: 44-1509-223480, email: R.H.J.Grimshaw@lboro.ac.uk

## Abstract

In the weakly nonlinear long wave regime, internal oceanic solitary waves are often modeled by the Korteweg-de Vries equation, which is well-known to support an exact solitary wave solution. However, when the effect of background rotation is taken into account, the resulting relevant nonlinear wave equation, the Ostrovsky equation, does not support an exact solitary wave solution. Instead an initial solitary-like disturbance decays into radiating oscillatory waves. In this talk, we will demonstrate through a combination of theoretical analyses, numerical simulations and laboratory experiments that the long-time outcome of this radiation is a nonlinear wave packet, whose carrier wavenumber is determined by an extremum in the group velocity. Whe variable bottom topography is also taken into account, although this process may still take place, some new features emerge such as the formation of secondary undular bores. Keynote Talks

# Integrable PDEs, $D\triangle$ Es and $P\triangle$ Es

## Alexander V. Mikhailov University of Leeds, Leeds, UK

#### Abstract

There are many tight connections between integrable Partial Differential, Differential-Difference and Partial Difference equations (PDEs,  $D\Delta Es$  and  $P\Delta Es$ ). By integrability we understand the existence of infinite hierarchies of conservation laws and/or commuting symmetries.

Having a Lax representation associated with an integrable PDE one can construct an infinite hierarchy of commuting symmetries and a canonical conservation laws. One can apply the Spectral Transform Method to construct exact partial solutions and study a general solution of the PDE.

A Darboux transformation of the Lax structure results in a Bäcklund transformation for the corresponding PDE. The sequence of Bäcklund transformations represents an integrable  $D\Delta E$  whose symmetries and conservation laws are related to symmetries and conservation laws of the PDE and can be derived using the Darboux-Lax representation.

The condition of Bianchi commutativity for Darboux transformations leads to an integrable system of partial difference equations (is a Darboux representation for an integrable  $P \Delta E$ ), whose symmetries are Bäcklund transformations (D  $\Delta E$ s) corresponding to these Darboux transformations. Conservation laws of the P $\Delta E$  are inherited from the conservation laws of the corresponding D $\Delta E$ s.

Although there is not any algorithmic way for construction of Lax, Darboux-Lax or Darboux representation for a given equation (PDEs,  $D\Delta E$  or  $P\Delta E$ ), one can find strong necessary conditions for the existence of symmetries and conservation laws. This approach, known in literature as Symmetry Approach, enable us to solve a number of classification problems for integrable PDEs and  $D\Delta Es$  (i.e. to find all possible integrable equations). There is a promising progress in the theory of integrable  $P\Delta Es$ . The development of this theory requires a reformulation of the whole foundation in the rigorous terms of differential and difference algebra.

# Solitons and interactions of dispersive shock waves

Mark J. Ablowitz

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## Abstract

Historically, solitons first appeared in the context of shallow water waves. Interestingly, two dimensional interacting solitons with "X, Y" as well as ones with more complex structure can frequently be seen on at beaches. These solutions are related to the unidirectional Kadomtsev-Petviashvili and two-directional Benney-Luke equations. Also, in many applications, including water waves and nonlinear optics, dispersive shock waves (DSWs) appear. One of the key models of DSWs is the Korteweg-de Vries (KdV) equation. The long time behavior of the KdV equation for general step-like behavior is investigated. While multi-step data evolve with multiphase dynamics at intermediate times, these interacting DSWs eventually merge to form a single-phase DSW at large time.

MS1: Dispersive Nonlinear Vector Waves: Solitons and Shocks

# Vector semi-rational soliton solutions

Antonio Degasperis

Department of Physics, Sapienza University of Rome and INFN, sezione di Roma, Piazzale Aldo Moro 2, 00185 Roma, Italy email: antonio.degasperis@roma1.infn.it

## Abstract

We discuss the Darboux construction of rational solutions of vector integrable wave equations in 1+1 dimensions. Particular examples of such equations which are of interest in applications are considered together with their rational solutions.

#### **References**:

Degasperis A, Lombardo S (2013). Rational solutions of wave resonant interaction models. ( to be submitted )

## Solitary wave solution to the KdV-Burgers-type equation

## Zhaosheng Feng Department of Mathematics, University of Texas-Pan American, Edinburg, TX 78539 email: zsfeng@utpa.edu

#### Abstract

In this talk, we provide a connection between the Abel equation of the first kind, an ordinary differential equation with nth-order in the unknown function, and the Korteweg-de Vries-Burger-type equation, a partial differential equation that describes the propagation of waves on liquid-filled elastic tubes. We present an integral form of the Abel equation with the small initial condition. By virtue of the integral form and the Banach contraction mapping principle we derive the asymptotic expansion of bounded solutions in the Banach space, and apply the asymptotic formula to construct solitary wave solutions to the Korteweg-de Vries-Burgers-type equation.

# Experiments on optical spatial shocks in random media

Neda Ghofraniha, Silvia Gentilini, Viola Folli, Eugenio Del Re and Claudio Conti IPCF-CNR and Department of Physics, La Sapienza University of Rome Piazzale A. Moro 2, 00185-Rome, Italy email: neda.ghofraniha@roma1.infn.it

## Abstract

Dispersive shock waves (DSWs), or undular bores, are observed in nonlinear optics in systems described by universal models, such as the nonlinear Schrodinger equation, when the hydrodynamical approximation holds true. We will report on the experimental investigation of the effect of disorder on the formation and propagation of optical spatial DSWs. We measure the relevant scaling laws relating the shock position with the input power and strength of disorder [1], and the wavevector spectrum generated by the shock and its relation to the underlying physics of the nonlocal nonlinearity [2].

- 1. N.Ghofraniha, S. Gentilini, V. Folli, E. DelRe, and C. Conti, Phys. Rev. Lett. 109, 243902 (2012)
- 2. S.Gentilini, N.Ghofraniha, E.DelRe, and C.Conti, Opt.Exp., 20, 27370 (2012).

MS1: Dispersive Nonlinear Vector Waves: Solitons and Shocks

# Hamiltonian PDEs and dispersive shock waves

Tamara Grava SISSA (International School for Advanced Studies), via Bonomea 265, 34136, Trieste, Italy email: grava@sissa.it

#### Abstract

We review the theory of dispersive shock waves and describe it in detail in the small dispersion limit of the Korteweg-de Vries equation. Then we illustrate some generalizations to two component systems and formulate some conjectures regarding the asymptotic behaviour of the solution.

# Perturbation of the periodic Toda lattice

Spirydon Kamvissis Department of Applied Mathematics, University of Crete 71409 Knossos-Heraklion, Crete,Greece email: spyros@tem.uoc.gr

#### Abstract

We consider the stability of the periodic Toda lattice (and slightly more generally of the algebro-geometric finite-gap lattice) under a "short range" perturbation. We prove that the perturbed lattice asymptotically approaches a modulated lattice that we describe explicitly. Our method relies on the equivalence of the inverse spectral problem to a matrix Riemann-Hilbert problem defined in a hyperelliptic curve and generalizes the so-called nonlinear stationary phase/steepest descent method for Riemann-Hilbert problem deformations to Riemann surfaces.

MS1: Dispersive Nonlinear Vector Waves: Solitons and Shocks

## Internal waves with small dispersion

#### Peter Miller

# Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, U.S.A. email: millerpd@umich.edu

#### Abstract

Dispersive interactions for waves propagating on an interface between two fluids of different density are, by physical nature, long-range. In simple models of weakly nonlinear internal waves, this long-range dispersion shows up as a nonlocal linear term in the equation. One such model is the Benjamin-Ono equation, which is known to possess a Lax pair representation. This talk will review some results on the asymptotic behavior of the solution of the initial-value problem for the Benjamin-Ono equation when the dispersion is small but nonzero. We will first discuss weak convergence results obtained with Z. Xu, and if time remains we will mention ongoing work on strong small-dispersion asymptotics (joint with A. Wetzel).

## Integrable system with peakon, weak kink, and kink-peakon

## interactional solutions

Zhijun (George) Qiao Department of Mathematics, University of Texas-Pan American, Edinburg, TX 78539,U.S.A. email: qiao@utpa.edu

#### Abstract

In this presentation, we present an integrable system with both quadratic and cubic nonlinearity:  $m_t = bu_x + \frac{1}{2}k_1[m(u^2 - u_x^2)]_x + \frac{1}{2}k_2(2mu_x + m_x u), m = u - u_{xx}$ , where  $b, k_1$ 

and  $k_2$  are arbitrary constants. This model is kind of a cubic generalization of the Camassa-Holm (CH) equation:  $m_t + m_x u + 2mu_x = 0$ . The equation is shown integrable with its Lax pair, bi-Hamiltonian structure, and infinitely many conservation laws. In the case of b = 0, the peaked soliton (peakon) and multi-peakon solutions are studied. In particular, the two-peakon dynamical system is explicitly presented and their collisions are investigated in details. In the case of  $b \neq 0$  and  $k_2 = 0$ , the weak kink and kink-peakon interactional solutions are found. Significant difference from the CH equation is analyzed through a comparison. Finally, we also show all possible smooth one-soliton solutions for the system.

- 1. Zhijun Qiao, Baoqiang Xia, and Jibin Li, Integrable system with peakon, weak kink, and kink-peakon interactional solutions, arXiv: 1205.2028v2 [nlin.SI].
- 2. Baoqiang Xia, Zhijun Qiao, and Ruguang Zhou, A synthetical integrable two-component model with peakon solutions, arXiv: 1301.3216 [nlin.SI].

MS1: Dispersive Nonlinear Vector Waves: Solitons and Shocks

# Nonlinear wave propagations in multidimensions

## and wave breaking in Nature

Paolo Maria Santini Department of Physics, Sapienza University of Rome and INFN sezione di Roma, Piazzale Aldo Moro 2, 00185 Roma, Italy. email: paolo.santini@roma1.infn.it

#### Abstract

We make use of the inverse spectral transform for integrable dispersionless PDEs to investigate relevant features of nonlinear wave propagations in multidimensions, in the absence of dissipation and dispersions, and discuss the analytic aspects of the corresponding wave breaking.

# **Observation of the Berezinskii-Kosterlitz-Thouless transition**

# in a photonic lattice

Guohai Situ

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#### Abstract

Phase transitions give crucial insight into many-body systems, as the crossover between different regimes of order are determined by the underlying interactions. These interactions, in turn, are often constrained by dimensionality and geometry. For example, in one- and two-dimensional systems with continuous symmetry, thermal fluctuations prevent the formation of long-range order. Two-dimensional systems are particularly significant, as vortices can form in the plane but cannot tilt out of it. At high temperatures, random motion of these vortices destroys large-scale coherence. At low temperatures, vortices with opposite spin can pair together, canceling their circulation and allowing quasi-long-range order to appear. This Berezenskii-Kosterlitz-Thouless (BKT) transition is essentially classical, arising for example in the traditional XY model for spins, but to date experimental evidence has been obtained only in cold guantum systems. Measurements of superfluid sound speed and critical velocity have been consistent with scaling predictions, and vortices have been observed directly in cold atom experiments. However, the presence of trapping potentials restricts measurement to vortex density, rather than number, and obscures the process of vortex unbinding. Further, atom and fluid experiments suffer from parasitic heating and difficulties in phase recording, leading to results that differ from theory in many quantitative aspects. Here, we use a nonlinear optical system to directly observe the ideal BKT transition, including vortex pair dynamics and the correlation properties of the wavefunction, for both attractive and repulsive interactions (the photonic equivalent of ferromagnetic and antiferromagnetic conditions). The results confirm the thermodynamics of the BKT transition while raising outstanding issues regarding the non-equilibrium approach to it.

## **Optical dispersive shock waves in parametric wave mixing**

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#### Abstract

Recently dispersive shock waves (DSW) have been predicted and observed both in superfluids (Bose-Einstein condensates [1]) and in nonlinear optics [2] for a variety of bright and dark localized wave-packets, i.e. common spatial beams or pulses propagating under the action of weak diffraction or dispersion, respectively. We extend the analysis of DSW to the realm of parametric wave mixing, discussing mainly two settings which involve (i) four-wave mixing occurring via Kerr nonlinearity in optical fibers and (ii) second-harmonic generation in quadratic media, respectively. In the former case we show how DSW develop from an input modulated wave whose evolution is described in the framework of the scalar NLS equation with frequency comb structure. According to the NLS model we predict and observe the occurrence of arrays of twin wave-breaking points which give rises to colliding DSW under a variety of launching conditions. In the second case we highlights experimentally feasible situations where frequency doubling undergoes wave-breaking, discussing in particular how the regularization in terms of DSW can coexist and compete with a different wave-breaking mechanisms based on modulational instability. Such competition between different breaking mechanisms involving either a gradient catastrophe or modulational instability is not present in the scalar NLS equation but seems to be a sufficiently general scenario which can be found also in other models.

- 1. M. A. Hoefer, M. J. Ablowitz, I. Coddington, E. A. Cornell, P. Engels, and V. Schweikhard, 74, 023623 (2006).
- 2. W.Wan, S. Jia, And J. W. Fleischer, Nature Phys. 3, 46 (2007); N. Ghofraniha, C. Conti, G. Ruocco, S. Trillo, Phys. Rev. Lett. 99, 043903 (2007).

# Experiments on NLS solutions in the focusing and

# **Defocusing regime**

Amin Chabchoub Mechanics and Ocean Engineering, Hamburg University of Technology, 21073 Hamburg, Germany email: amin.chabchoub@tuhh.de

#### Abstract

The nonlinear Schrodinger equation (NLS) is a weakly nonlinear evolution equation describing the dynamics of wave packets in nonlinear dispersive media. Recent laboratory experiments on in time and space localized breathers on finite background confirmed the ability of the focusing NLS to describe extreme localization in deepwater. The focusing NLS admits another family of pulsating solutions referred to as breathers on zero background, also known as multi-soliton solutions. Results on laboratory studies on such water waves are reported. A discussion on physical properties related to the evolution dynamics of these solutions, is presented. In addition, first observations of dark soliton solutions of the NLS in the defocusing regime are shown and analyzed.

# Rational solutions of the Boussinesq equation

Peter Clarkson School of Mathematics, Statistics and Actuarial Science,University of Kent, Canterbury, CT2 7NF, UK Email: P.A.Clarkson@kent.ac.uk

## Abstract

In this talk I shall discuss special polynomials associated with rational solutions of the Boussinesq equation, which is a soliton equation solvable by the inverse scattering method. These rational solutions, which are derived through a bilinear equation, are bounded, depend on two parameters and have an interesting structure. Further the rational solutions of the Boussinesq equation have similar appearance to rogue-wave solutions of the nonlinear Schrodinger equation studied in [1]-[5].

- 1. P. Dubard, P. Gaillard, C. Klein and V. Matveev, On multi-rogue wave solutions of the NLS equation and positon solutions of the KdV equation, Eur. Phys. J. Spec. Top., 185, 247-258 (2010).
- 2. P. Dubard and V. Matveev, Multi-rogue waves solutions to the focusing NLS equation and the KP-I equation, Nat. Hazards Earth Syst. Sci., 11, 667-672 (2011).
- 3. P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A, 44, 435204 (2011).
- 4. P. Gaillard, Degenerate determinant representation of solutions of the nonlinear Schrodinger equation, higher order Peregrine breathers and multi-rogue waves, J. Math. Phys., 54, 013504 (2013).
- 5. Y. Ohta and J. Yang, General high-order rogue waves and their dynamics in the nonlinear Schrodinger equation, Proc. R. Soc. Lond. Ser. A, 468, 1716-1740 (2012).

## **Resonant interactions and rogue waves**

## Antonio Degasperis Department of Physics, Sapienza University of Rome, Piazzale Aldo Moro 2, 00185 Roma,Italy email: antonio.degasperis@roma1.infn.it

#### Abstract

Rogue waves in fluid dynamics and optics may be modeled by lump-solutions of nonlinear partial differential equations. Examples of such solutions are displayed and discussed in cases of resonant wave interactions in 1+1 dimensions.

#### **References:**

Baronio F, Degasperis A, Conforti M, Wabnitz S (2012). Solutions of the Vector Nonlinear Schroedinger Equations: Evidence for Deterministic Rogue Waves. Phys. Rev. Lett. vol. 109; p. 044102-044106

## New results on NLS-KP-I correspondence and multiple rogue

## waves events

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#### Abstract

In this talk, we present a 2n-3 parameters family of smooth real rational solutions of the KP-I equation  $(4v_t + 6vv_x + v_{xxx})_x = 3v_{yy}$ , obtained from a 2n-2 parameters family of quasirational solutions of the focusing NLS equation  $iu_t + u_{xx} + 2|u|^2u = 0$  and we investigate the link between these two equations. One of the new results with respect to [1] is an explanation of how to incorporate the higher Peregrine breathers of the NLS equation in the KP-I dynamics. We also give some comments on the large parametric behavior of the multi-rogue wave solutions of the KP-I equation.

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# **Deformations of higher rogue Peregrine breathers**

# and monstrous polynomials

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#### Abstract

We construct a multi-parametric family of solutions of the focusing nonlinear Schrodinger equation (NLS) equation from the known result describing the multi phase almostperiodic elementary solutions given in terms of Riemann theta functions by Its, Rybin and Salle in 1976. In particular we succeeded to find explicit analytical expressions for the  $P_n$  breathers (higher Peregrine breathers of the rank n), for  $n \le 10$  and moreover to describe explicitly quadratic polynomial deformations of  $P_n$  breathers for the ranks  $n \le 9$ . As was pointed out by V.B. Matveev, the large parametric limits of the later produce  $P_{n-2}$  breathers. This was checked first by him and myself initially for the rang  $n \le 5$  and later checked also for n = 6 to 9 in my works. More general results concerning the large parametric limits of non-quadratic quasi-rational deformations were first found by Dubard and Matveev for  $n \le 4$  in [2]. The presented approach allows to study the generic quasi-rational deformations.

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# On the nonlinear stage of modulation instability

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#### Abstract

It is known since 1971 that the Nonlinear Schrodinger (NLSE) is a system completely integrable by the Inverse Scattering Method (ISTM). The NLSE has a simple solution, the monochromatic wave with frequency depended on amplitude - the condensate. The condensate is unstable with respect to modulation instability. There are important question: what is a nonlinear stage of modulation instability? In spatial dimension D = 2; 3, the answer is known-modulation instability leads to formation of finite time singularities-collapses. In dimension D = 1 collapses are forbidden. However in this case development of modulation instability leads to formation of "extreme" (rogue, freak) waves where energy density exceeds the mean level by order of magnitude.

In this join work with professor V. E. Zakharov [1] we study solitonic solutions of the focusing NLSE in the presence of the condensate by using the dressing method. We find a general N-solitonic solution and separate a special designated class of "regular solitonic solutions" that do not disturb phases of the condensate at infinity by coordinate. All regular solitonic solutions can be treated as localized perturbations of the condensate. If we assume that the modulation instability develops from localized perturbation, only regular solution can be used as model for its nonlinear behavior. The central result of our work is following. We find a broad class of "superregular solitonic solutions" which are small perturbations at certain a moment of time. Then they develop into N pairs of different solitons (we call them "quasi-Akhmediev" breathers). These solutions form an infinite-dimensional linear functional space. This describes the nonlinear stage of the modulation instability of the condensate and can be treated as a sort of "integrable turbulence" where local concentration of energy easily exceeds in order of magnitude the energy density in the condensate. Self-consistent analytic theory of this turbulence will be a truly reliable theory of freak waves.

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# New patterns of higher order rogue waves

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## Abstract

Rogue wave is one kind of a localized solution both in x and t directions of many partial differential equations. This solution gives a very good model of extreme short lived event in the physical system. In this talk we shall summary the study of the rogue wave, then provide the outline of the proof about the two conjectures regarding the total number of peaks, as well as a decomposition rule in the circular pattern of an order n rogue wave. We also provide several new pattens of higher order rogue waves.

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## Rogue waves: rational solitons and wave turbulence theory

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#### Abstract

Solitons on finite background (SFB) and particularly rational solitons (RS) are exact analytical solutions of integrable wave equations, and for this reason they may be regarded as a coherent and deterministic approach to the understanding of rogue wave (RW) phenomena. However, RWs events are known to spontaneously emerge from an incoherent turbulent state of the system. It is thus of fundamental importance to study whether RSs can emerge from a turbulent environment (i.e., in more realistic oceanic conditions). To this aim, we first review here our recent experiments based on optical fiber systems to generate the localized nonlinear structures evolving upon a nonzero background plane wave (the Akhmediev, Peregrine and Kuznetsov-Ma solutions) [1-2]. The setup is based exclusively on commercially available telecommunication-ready components and standard silica fiber. It allows us to observe a complete family of SFB solutions to the nonlinear Schrodinger equation. Next we show how a suitably low frequency modulation on a continuous wave can induce higher-order modulation instability splitting. This phenomenon arises from the nonlinear superposition of single breather evolutions [3]. Finally we study the emergence of rogue waves and RSs in a genuine turbulent wave system. In particular we show that the coherent deterministic description of RWs provided by the rational soliton solutions is compatible with an accurate statistical description of the random wave provided by the wave turbulence theory. Furthermore, the simulations reveal that even in the weakly nonlinear regime, the nonlinearity can play a key role in the emergence of an individual rogue wave event in a turbulent environment [4].

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# Algebro-geometric solutions, solitons and breathers of

# **Davey-Stewartson and Nonlinear Schrodinger equations**

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## Abstract

We present a new derivation of solutions to the Davey-Stewartson system and (vector) Nonlinear Schrodinger equations in terms of multi-dimensional theta functions on compact Riemann surfaces. The starting point of the construction of the solutions is a new variant of Fay's trisecant identity. Regularity and reality conditions on the solutions are discussed. In the limit that the Riemann surface degenerates, these solutions reduce to elementary functions describing solitons and breathers.

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## High order rogue waves solutions of NLS and DNLS equations

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#### Abstract

In this talk, firstly we give the generalized Darboux transformation in a heuristic way. Based on this transformation, we give the explicit way to construct the high-order rogue wave solution for NLS and DNLS equations.

# Large parametric asymptotic of the multi-rogue waves

## solutions of the NLS equation and extreme rogue wave

## solutions of the KP-I equation.

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#### Abstract

In this talk, we present some new results [1] concerning the behavior of the multi-rogue waves solutions of the focusing NLS and KP-I equation. These results are based on explicit polynomial formulas obtained from the determinant representations for these solutions found in my works with Philippe Dubard [2]-[3]. In these works the concept of the multiple roque waves solutions both for focusing NLS equation and KP-I equation was first introduced. These works provided an explanation of the fact that the so called higher Peregrine breathers (we'll call them for brevity Pn breathers) with  $n \ge 2$  are not isolated and correspond to particular choice of parameters for the rank n multi-roque wave solution depending on 2n free real parameters. In 2010 only genuine Peregrine breather (i.e. P1 breather), P2-breather (found in 1995 by Akhmediev, Eleonski and Kulagin) and P3 breather (found in 2009 by Akhmediev, Ankiewicz and Soto-Crespo) were known explicitly. The discovery of the multiple roque-waves solutions stimulated the study of their particular cases corresponding to the different choices of parameters revealing quite different symmetric and asymmetric configurations. It seems that a rigorous study of the large parametric behavior of the multi-rogue wave solutions made in [1] was never performed before. In particular, we will show that (at least for small ranks) that all multiple rogue wave solutions of the rank  $m \le n-2$  can be obtained as an appropriately chosen large parametric limits of the rank n solutions.

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## Multi-phase freak waves.

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#### Abstract

There are many reasons to study multi-phase solutions of the nonlinear Schrodinger equation (NLS)

$$i\psi_z + \psi_{zz} + 2|\psi|^2\psi = 0$$
(1)

- One of the possible origins of the freak waves is a synchronous addition of multiphase waves maximums.
- Islas and Schober found in numerical simulations of the NLS equation that rogue waves develop for JONSWAP initial data are "near" 6-phase solutions [1].
- Rational solutions can be obtained using the degeneration of multi-phase solutions.
- Taking all the periods large enough we can make the difference between rational solutions and multi-phase solutions inside of parallelepiped of the periods arbitrarily small.

The results obtained in [2]-[3] for two-phase finite-gap solutions of NLS equation show that these solutions have a relatively simple behavior. In particular, the amplitude of two-phase solution for NLS equation is a doubly-periodic function both with respect to x and z variables:

$$|\psi|(x + X_k, z + Z_k)|\psi|(x, z), k = 1, 2,$$
(2)

i.e. the two-phase solutions have the peaks on the nodes of the parallelogram of the periods. For elliptic two-phase solutions the number of peaks on the parallelogram of periods equals n, where n is a degree of mapping of the spectral curve to the torus. Three-phase finite-gap solutions have a more complicated behavior. They are periodic in 3D-space

$$|\psi|(x + X_k, z + Z_k, t + T_k)|\psi|(x, z, t), k = 1, 2, 3$$
(3)

where t is a modular parameter. During the talk we'll also describe the behavior of finite-gap solutions with higher number of phases in connection with freak waves.

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## Nonautonmous rogue waves and interactions in some

# nonlinear physical models

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#### Abstract

The rogue wave phenomenon appears in many fields of nonlinear science such as ocean, nonlinear optics, Bose-Einstein condensates, and even finance (see, e.g., [1-9]). In this talk, we study self-similar rogue waves and interactions of some nonlinear physical models with varying parameters including nonlinear Schrodinger equation with varying coefficients [10], the three-dimensional Gross-Pitaevskii equation with varying coefficients [11], and the generalized discrete Ablowitz-Ladik model with varying potentials [12]. Some special functions are chosen to illustrate the propagations of these obtained rogue waves.

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## Dynamics of rogue waves in the Davey-Stewartson equations

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#### Abstract

General rogue waves in the Davey-Stewartson-I and Davey-Stewartson-II equations are derived by the bilinear method, and the solutions are given through determinants. It is shown that the simplest (fundamental) rogue waves are line rogue waves which arise from the constant background in a line profile and then retreat back to the constant background again. It is also shown that multi-rogue waves describe the interaction between several fundamental rogue waves, and higher-order rogue waves exhibit di®erent and novel dynamics. Under certain parameter conditions, rogue waves in the Davey-Stewartson-II equation can blow up to infinity in finite time at isolated spatial points, i.e., exploding rogue waves exist in the Davey-Stewartson-II equation. This talk is based on materials in Refs. [1, 2]

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## **Optical tsunami**

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#### Abstract

Extreme or rogue waves have received a great deal of attention recently for their emergence in a variety of applications [1]. The most popular manifestation of a rogue wave is the sudden build-up and subsequent rapid disapperance in the open sea of an isolated giant wave, whose height and steepness are much larger than the corresponding average values of other waves in the ocean. On the other hand, the occurrence of rogue waves in shallow waters has been comparatively much less explored, in particular in the context of nonlinear optics [2]. In this presentation we will point out that extreme waves may be generated in optical fibers in the normal GVD regime of pulse propagation [3-4]. The model linking hydrodynamics with nonlinear optics is provided by the semiclassical approximation to the NLSE, which is known as the nonlinear shallow water equation (NSWE) [5-6]. Although in the normal GVD regime a CW is modulationally stable, extreme waves may still be generated by imposing a suitable temporal pre-chirping or phase modulation [3-4,6], which is analogous to a nonuniform velocity distribution of the propagating water waves, eventually leading to tsunamis. We shall describe first the dynamics of the generation of an intense, at-top, self-similar and chirp-free pulse as a result of the initial step-wise frequency modulation of a CW laser. The intriguing property of such pulses is their stable merging upon mutual collision into either a steady or transient high-intensity wave.

This effect may lead to extreme intensity peaks in optical communication systems whenever various wavelength channels are transported on the same fiber, and to the generation of highly temporally compressed periodic optical pulse trains [3]. Next we will discuss the otical analogy with the shoaling of ocean waves as they run-up to the beach. We consider the propagation of special, input temporally pre-chirped optical pulses with different power profiles. These pulses represent nonlinear invariant solutions of the NSWE (Riemann waves). For such type of chirped pulses, we obtain exact solutions of the optical NSWE, and demonstrate their good agreement with numerical solutions of the NLSE, at least up to the point where a vertical front develops in the power profile. Finally, we discuss how third-order dispersion leads to the occurrence of extreme waves whenever a dispersion tapered fiber is used, in analogy with the dramatic run-up and wave height amplification of a tsunami towards the coast [4].

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## **Coherent wave groups causing rogue waves**

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#### Abstract

The probability of rogue waves deviates significantly from the Rayleigh distribution function due to wave-wave interactions, which make the dynamics of different spectral components coherent. Due to this effect waves may form long-living groups, which manifest themselves through their own dynamics. In the limit of weakly nonlinear modulated waves over deep water the groups are envelope solitons or breathers described by the integrable nonlinear Schrodinger equation. The paper is focused on the question how the long-living nonlinear wave groups behave in the conditions of strong wave nonlinearity, irregular sea states and could be revealed in instrumental measurements of oceanic waves.

The envelope soliton solution of the nonlinear Schrodinger equation is examined by means of fully nonlinear simulations of the Euler equations and also in laboratory experiments in the situation of strong nonlinearity. It is shown that strongly nonlinear effects do not destroy the solitary wavegroup even when the steepness is high.

The attempt to map the weakly nonlinear analytic description of modulated nonlinear waves (due to the Inverse Scattering Technique) to the case of strong nonlinearity is undertaken. Qualitative comprehension of the strongly nonlinear wave dynamics and qualitative description of it would provide elements of short-term forecasting for rogue waves. The accessibility of this goal is discussed.

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# Extreme events in numerical simulations

# of irregular surface water waves

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## Abstract

An approach to study rogue wave occurrence in numerical simulations of unidirectional surface waves is suggested. The spatio-temporal wave data is obtained by means of frequent acquisition of wave fields from the simulated periodic domain. The irregular waves are characterized by the JONSWAP spectrum; and the time-evolution is simulated within the strongly nonlinear solver for the Euler equations (HOSM with M = 3, M = 6) with a short-wave damping. As a result, sheets of irregular wave data of the size 20 min  $\times$  10 km with a good resolution in time and space are obtained. This data is used to visualize the dispersion relation for nonlinear waves, where wave components (free, bound and counter-directional modes) are naturally separated. The data is used for statistical analysis with the particular focus on rogue waves.

Having the complete wave data enables us to capture rogue wave surface shapes, to consider the evolution of rogue waves and to conduct a statistical analysis. Most of the simulated rogue waves have very high crests and shallow long troughs, or sign-variable shapes. "Holes in the sea", characterized by deep troughs, are found in the simulation data as well. We assemble recurrent rogue wave accidents, which are localized close in space and time, into rogue events and then estimate the life-times. Though the life-time of a continuous rogue wave is relatively short, the life time of a rogue event may be significantly longer (dozens of wave periods).

The relation between extreme wave kinematics and kinematics of rogue waves in the stochastic simulation is discussed, and compared versus the third-order Stokes theory. Though in general the significant amount of data well corresponds to each other, some difference between kinematics of extreme waves and extreme kinematics is emphasized. Rogue waves are typically characterized by large values of velocities, but high velocities do not necessarily correspond to rogue waves.

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# **Development of Solid-Gas-Liquid 3-phase calculation system**

# coupling CADMAS-SURF/3D and DEM

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#### Abstract

There are many problems related to destructive phenomena such as failure of structures on land, impacts of drifting objects etc. caused by giant tsunami on coastlines in recent years, which are difficult to clarify by only analyzing simple fluid phenomena. And such phenomena are characterized by extremely strong non - linearity—the impact on structures of breaking waves and other fluids in surge state—thus difficult to analytically unravel.

On the other hand, computers have advanced remarkably, so in recent years, there have been many researches on multi - phase flow calculation by the coupling method, based on calculation methods typified by the particle method (for example, Goto et al., 2008 and Ikari et al. 2009). At the same time, Arikawa and Yamano (2008a) developed a single - phase numerical wave tank based on the VOF method, and confirmed that it is a highly robust system, even in large - scale calculations. And they also showed that structural deformation calculations can be done based on a model coupled with FEM (Arikawa et al., 2009). But when using FEM analysis, it is difficult to calculate after failure, and the mixing of air in breaking waves and surges can become an important element. So this research expanded the numerical wave tank to a gas - liquid 2 - phase model, and also, a system coupled with the distinct element method was developed, with the goal set as building a calculation method which can be applied to various failure phenomena caused by fluid bodies.

The results confirmed that there was less numeric noise than with a single - phase model, and that it can calculate the breaking wave pressure with high precision. Next, a system coupled with the distinct element method was developed. Although qualitative, it conformed with an experiment with large - scale wall failure, confirming the appropriateness of the solid - gas - liquid 3 - phase model.

# The influence of frictional Chezy contributions

## on the shoreline motion

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#### Abstract

The dynamics of wave-forced flows near the shoreline is illustrated and discussed, with specific focus on the influence that frictional Chezy contributions have on the shoreline motion itself [1.]. An asymptotic analysis, valid for vanishing water depths, reveals that the use of such a term leads to a non-receding motion of the shoreline. This phenomenon is induced by a thin layer of water which, because of frictional forces, remains on the beach and keeps it wet seaward of the largest run-up. However, the dynamical influence of such a frictional layer of water on the global wave motion is very weak and practically negligible for most of the swash zone flow. The presence of a non-receding shoreline has called to some clarifications on the role of some ad-hoc tools used in numerical models for the prediction of the wet/dry interface. These will be discussed at the Conference.

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MS3: Tsunami and Coastal Dynamics

## Kinematics of solitary wave runup

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#### Abstract

In Jensen et al. (2003) the dynamics of incident solitary waves on the verge of on shore plunging, on a 10.54° slope, were investigated with the PIV technique. Among other things, acceleration distributions leading to reversal of breaking in overhanging waves were reported. In Jensen et al. (2005) another set of experiments with runup of solitary waves on a 7.18° slope was studied. The velocity and acceleration distri bution in massive onshore plungers are measured with PIV and computed by a VOF technique. This investigation aimed at recognition of cases leading to extreme runup and onshore impact of tsunamis and swells. The present study employs the same incidents waves as Jensen at al. (2005), but the inclination of the beach is lowered to 5.1° Waves may now develop large, but well defined, plungers as opposed to the collapsing breaker that was reported in Jensen et al. (2005). In some experiments a surface - piercing vertical plate, or a circular cylinder, is mounted at the beach. The aim of the these experiments is to correlate the pressure, which is measured at the wall with three probes, with wave kinematics and assess the magnitude of the wave loads.
## Vorticity effect on modulational instability: Application to

#### rogue waves

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#### Abstract

Rogue waves are among the waves naturally observed by people on the sea surface those that represent an real danger. They may occur suddenly without warning and disappear, offshore and in coastal zone as well. The occurrence of this extreme wave event can be fatal for ships and crew due to its suddenness and abnormal features. These waves may be generated by different mechanisms such as wave-current interaction, geometrical and dispersive focusing, modulational instability (Benjamin-Feir instability), soliton collision, crossing seas, etc. In this study we consider rogue wave phenomenon due to modulational instability.

Generally, in coastal and ocean waters, the velocity profiles are typically established by bottom friction and by surface wind stress and so are varying with depth. Currents generate shear at the bed of the sea or of a river. For example ebb and flood currents due to the tide may have an important effect on waves and wave packets. In any region where the wind is blowing there is a surface drift of the water and water waves are particularly sensitive to the velocity in the surface layer.

Firstly, we consider the effect of constant non zero vorticity on the Benjamin-Feir instability. Several studies have been carried out on the computation of surface water waves propagating steadily on a rotational current. Nevertheless, few papers have been published on their stability to modulational perturbations. Very recently, Thomas, Kharif & Manna [1] using the method of multiple scales derived a nonlinear Schrödinger equation in finite depth and in the presence of uniform vorticity. They called this equation the vor-NLS equation and they demonstrated that vorticity modifies significantly the modulational instability properties of weakly nonlinear plane waves, namely the growth rate and bandwidth. Furthermore, it was shown that these plane wave solutions may be linearly stable to modulational instability for an opposite shear current independently of the dimensionless parameter kh, where k and h are the carrier wavenumber and depth respectively. They found that the number of rogue waves due to modulational instability increases for a shear current co-flowing with the waves whereas for a shear current counter - flowing with the worticity.

As an application, we will investigate the effect of the vorticity on the rogue wave properties, namely their lifetime and amplification in arbitrary depth (kh > 1.363). For this pupose, a series of numerical simulations of the vor-NLS equation will be run for several values of the dispersive parameter kh, nonlinear parameter ak where a is the amplitude of the carrier wave and vorticity.

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#### MS3: Tsunami and Coastal Dynamics

### On the multiple steady-state fully resonant water waves

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#### Abstract

The steady-state fully resonant waves in deep and finite water depth governed by exactly nonlinear wave equations are investigated analytically, using the so-called homotopy analysis method (HAM). The multiple steady-state resonant waves are obtained, with time-independent amplitudes and wave spectrum, i.e. without exchange of wave energy between different wave components. Besides, it is found that the resonant wave component may contain rather small proportion of wave energy. Similar conclusions are obtained for the so-called class-I Bragg resonance of progressive waves obliguely propagating over an undulate bed with an infinite number of sinusoidal ripples. It is found that there also exist the multiple, steady-state class-I Bragg resonant waves, whose wave spectrum is time-independent. In addition, the class-I Bragg resonant wave may contain less, equal or more wave energy than the incident wave, corresponding to the reflection coefficient R < 1, R = 1 and R > 1, respectively. Therefore, the multiple steady-state resonant waves exist not only in wave-wave nonlinear interaction but also in wave-bottom nonlinear interaction. All of these might deepen our understanding and enrich our knowledge about the resonance of gravity waves.

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## Impacts of local tsunamis on China coasts

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#### Abstract

Even though the transoceanic Pacific Ocean tsunamis can be ignored for safety assessment of the coasts of China, the effects of local tsunamis on the coasts, potentially caused by the giant ruptures at the deep trenches near the continental shelf, have been received attention after the Indian Ocean tsunami in 2004 and the East Japan tsunami in 2011. It is believed that there are potential tsunami sources in the China Seas due to the fault ruptures along the Okinawa Trench in the East China Sea and the Manila Trench in the South China Sea.

Impacts of the 2011 East Japan tsunami on China coasts will be studied through numerical simulation and the measured data analysis at first. Distribution of the wave height along the China coasts are obtained and discussed. It turns out that the tsunami has almost no impact in the Bohai Sea, Yellow Sea and Liaodong Peninsula, while it generates wave height up to 0.4m in the East China Sea and the South China Sea.

Okinawa Trench in the East China Sea and Manila Trench in the South China Sea are considered to be the region with high risk of potential tsunami induced by submarine earthquakes. Tsunami waves will impact the Southeast Coast of China if tsunami occurs in these areas. The horizontal two-dimensional Boussinesq model is used to simulate tsunami from its generation, propagation, to runup on the shore in a domain with complex geometrical boundaries. The temporary varying bottom boundary condition is adopted to describe the tsunami wave generated by submarine movement. The Indian Ocean tsunami is simulated by this numerical model as a validation case. The time series of water elevation and runup on the beach are validated by the field measurement data. The agreements indicate that this model could be used to simulate tsunami and predict the waveform and runup well. Then, tsunamis in the Okinawa Trench and Manila Trench are simulated by the horizontal two-dimensional Boussinesq model. The arrival time and maximum wave elevation near the cities of Southeast coast were predicted by the model.

Through numerical simulation of the scenarios of the tsunamis potentially motivated in the East China Sea and the South China Sea, it turns out that different wave patterns will appear for tsunami waves moving on continental shelf of different morphology, including N-waves, an undular bore and solitary waves.

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# Evolution of boundary layer flows under a transient long wave

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#### Abstract

Similar to many previously recorded historical tsunamis the 2011 Tohoku tsunamis left behind widespread sediment deposits along the Japanese coast. Knowing the relationship between tsunami sediment deposits and tsunami wave characteristics will allow better estimates of times and recurrence intervals of past tsunamis. This will then enhance tsunami data base and improve the assessment of tsunami risk. Therefore, it is important to understand the sediment transport processes under the leading tsunami waves. Since sediment transport is primarily driven by the near - bed flow turbulence and bottom stresses, we need to enhance our knowledge on the evolution of near - bed flows under the leading waves of tsunamis, which can be characterized as transient long waves.

In a recent study (Sumer *et al.* 2010, JFM) U-tube experiments were performed to investigate boundary layer flows driven by a soliton-like pressure gradient. For Reynolds numbers, defined as  $\text{Re} = aU_{0m}/\nu$ , where  $U_{0m}$  denotes the maximum free-stream velocity, 2a the corresponding maximum fluid particle displacement, and  $\nu$  the fluid viscosity, ranging between 2 x  $10^5$  and 2 x  $10^6$ , the boundary layer flow characteristics transform from being simple laminar uni-directional flows to the appearance of 2D laminar vortex tubes, and to the occurrence of turbulence spots.

In this paper, a high resolution and high accuracy 2D pseudo-spectral numerical model (Diamessis *et al.* 2005) is used to simulate the U-tube experiments and to investigate flow instability characteristics in details for higher Reynolds numbers. Furthermore, we also examine the boundary layer flows under asymmetric wave profile: a steep wave front (acceleration phase) with a longer and flatter tail (deceleration phase), which mimic the real tsunami waves better than solitary waves.

Generally speaking, boundary layer flows under a solitary wave can be divided into laminar, disturbed laminar and turbulent flow regimes depending the Reynolds number. In the laminar flow regime (Re $\leq 10^5$ ) the boundary layer flow is unconditionally stable, in which the growth rate of the shear flow instability is weak, and the time required for a small initial disturbance (say, in order of  $O(10^{-3} \text{ or less})$  to grow into an O(1) quantity is much longer than the time scale of the event (i.e., wave period). Whereas in the disturbed laminar regime  $(10^5 < \text{Re} < 1.8 \times 10^6)$ , the growth rate of the instability is stronger, which enable the shear instability to grow and 2D vortex tubes are shed from the bottom boundary within the time frame of the event. However, this process is constrained by the initial amplitude of the disturbance. For solitary waves with  $Re \geq 1.8 \times 10^6$ , the growth rate of the shear instability is large enough so that the instability will take place even for an infinitesimal small disturbance. We also discover that for boundary layer flows with  $Re \approx 10^7$  a different kind of instability occurs.

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For asymmetric waves, the stability conditions can be classified in a similar way as those for the symmetric waves, except that the Reynolds number is defined by using the time scale for the deceleration phase. In other word, the stability characteristics are not correlated with the acceleration phase. The implications of this feature on the bottom shear stress and sediment transport are investigated.

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# Asymptotic solutions to the Cauchy-Poisson problem in 2D and 3D

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### Abstract

Tsunamis propagating in the deep ocean are basically linear weakly-dispersive transient waves, which develop their characteristic features over long distances. This motivates a study of the classical Cauchy-Poisson problem concerning development of linear waves generated by an impulsive initial condition.

The original work goes back to Poisson (1816), but more recent formulations have been presented by e.g. Wehausen & Laitone (1960), Whitham (1974), LeBlond & Mysak (1978) and Clarisse et al. (1995). The resulting solutions for the surface elevation can be expressed as a Fourier integral in two dimensions and a Fourier-Bessel integral in three dimensions. For large values of time and large distances from the initial disturbance, these integrals can be approximated by asymptotic expansions, which eventually lead to a formulation in terms of Airy functions and their derivatives.

In this work, we consider two different asymptotic approximations to the exact integral formulation for a delta-function initial disturbance. Method 1 is based on Whitham's local solution valid near the front of the wave train, which is equivalent to a KdV-type approximation to the linear dispersion relation. Method 2 is based on a uniform transformation of the variables of integration combined with the method of stationary phase for oscillating integrals and a determination of the resulting Bleistein sequences. Both methods are developed in 2D and 3D, and the corresponding impulse response functions are presented. The two approximative methods are compared to direct numerical integration of the exact formulations.

Finally, we present the convolution formulations in 2D and 3D, which cover the case of general initial conditions. We study the cases of an initial Gaussian disturbance (narrow as well as wide) and an initial rectangular disturbance. The results are compared to numerical simulations using a high - order Boussinesq model.

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## Nonlinear long waves over a muddy beach

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#### Abstract

There are many sea coasts in the world where the seabed is covered with fluid-mud which is a mixture of water, fine and cohesive clay particles, and sand. Dissipation in the mixture is considerably higher than in the pure sea water and causes very effective damping of waves arriving from the open sea. In turn the waves can induce changes of the muddy seabed and alter the coastal morphology in the long run.

For steady flows in muddy rivers numerous experiments have shown that the fluid-mud can be approximated as a Bingham-plastic material. However, for fluid-mud in oscillatory flows rheological experiments are relatively scarce. Limited laboratory tests of natural mud by oscillatory rheometers have however indicated that viscoelastic properties are prevalent. In existing theoretical models different idealizations have been proposed mostly for wave/mud interaction over a horizontal seabed. Some authors have adopted the mathematically convenient Newtonian viscous model with vastly contrasting viscosities. Others have proposed Bingham plastic models based only on rheometer tests of unidirectional flows.

It is known that under waves fluid-mud behaves more as a visco-elastic material. So far simple Kelvin-Voigt models with only two or three constant coefficients have been used. Existing laboratory tests of natural fluid-mud by using dynamic rheometers performing simple harmonic motions have however shown that the constitutive coefficients in the Kelvin-Voigt model depend strongly on frequency. With increasing frequency, the shear modulus increases only mildly, but the viscosity  $\mu$  decreases significantly and monotonically. Hence the Kelvin-Voigt coefficients depend on the motion itself and are not just material properties. A remedy necessary for treating transient but non-sinusoidal problems is to use the generalized viscoelastic model relating the stress  $\tau$  and strain  $\mathcal{E}$ , which reads, in one dimensional motion,

$$\tau + \sum_{n=1}^{N} \alpha_n \frac{\partial^n \tau}{\partial t^n} = \beta_0 \mathcal{E} + \sum_{n=1}^{N} \beta_n \frac{\partial^n \mathcal{E}}{\partial t^n}$$
(4)

The coefficients  $\alpha_n$  and  $\beta_n$  can be chosen to match the data of simple - harmonic tests for the entire range of measured frequencies. In this way the constitutive coefficients depend only on material properties such as the mineral composition, particle size, concentration, etc and not on the frequency. For sinusoidal motions, the formally Kelvin-Voigt model with frequency-dependent coefficients is still meaningful. Using the measured data by Huang & Huhe (1992), Mei, Chan & Liu (2010) examined theoretically the effects of contrasting rheologies of fluid-mud samples from two sites on the eastern

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coast of China on the evolution of narrow-banded waves in water of intermediate but constant depth. Asymptotic equations by Stokes-like expansions are derived for narrow-banded waves in water of finite depth. Damping rate and mud motion are examined at the first order of wave steepness, and long waves induced by radiation stresses at the second order. However, as is known for long waves over a rigid seabed, the Stokes expansion fails in very shallow water.

A prominent feature of a muddy beach is its ability to damp out essentially all incoming waves before they reach the shoreline. Wells (1978) first reported systematic field observations over a mud bank near the mouth of Surinum River, Brazil, that incoming waves were all diminished to naught with no reflection or breaking. This was confirmed by extensive measurements by Matthiew et al (1995) on the Southern Coast of India.

The objective of this work is to develop a nonlinear theory suitable for long waves in a shallow seabed covered by fluid-mud. Since generation of higher harmonics is expected in such a setting, dependence of constitutive coefficients in frequencies is crucial for a realistic model of natural mud and is accounted for. Specifically we aim to examine the physical differences between two natural fluid-muds taken from two field sites on the eastern coast of China. An asymptotic theory extending the Korteweg-de Vries approximation is derived for examining the effects of contrasting rheological mud properties on wave evolution under the influence of nonlinearity, dispersion, dissipation and shoaling. While acoustic streaming is known to occur in an oscillating boundary layer in a Newtonian viscous fluid, as a results of Reynolds stresses, we shall show that an analogous phenomenon exists in a mud layer under periodic waves. Specifically, waves can force a steady displacement in mud, leading to changes of mud depth, which can affect the morphology of the muddy coast.

# Waves due to sub-aerial slides in narrow waterways.

# Experiments and computational challenges.

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### Abstract

In the last decade an unstable slope at Åkneset, in the fjord region of western Norway, has been thoroughly surveyed and it is currently under extensive monitoring. A slide with a volume up to  $80 \,\mathrm{Mm^3}$  will be released some time in the future. Motivated by this event the University of Oslo and the Norwegian Geotechnical Institute have made an combined effort in experimental investigations and modeling of sub-aerial slides into fjords. Similar events may occur in Alaska, Chile, Greenland and alpine lakes.

The experiments have been performed in a 1:500 scale model of the fjord. This has a size  $36 \text{ m} \times 40 \text{ m}$  ( $18 \text{ km} \times 20 \text{ km}$  in full scale), while the size of the slide is  $2.25 \text{ m} \times 0.91 \text{ m} \times 0.16 \text{ m}$  ( $40 \text{ Mm}^3$  in full scale). The impact velocity is estimated to 1.75, m/s (140 km/h in full scale). Surface elevations are measured with resistance and acoustic wave probes, point velocities with acoustic Doppler probes, while shoreline motions and velocity fields are obtained by digital image techniques.

From a modeling point of view such an event inherits important differences from an oceanic tsunami generated by an earthquake. First, a proper description of the generation requires some kind of primitive model with a free surface and moving slide. Next, the propagation of the leading waves in the fjord system is dispersive as well as nonlinear, which suggests a Boussinesq type models with standard or enhanced dispersion properties. Along the fjord the slopes are generally very steep ( $30^{\circ}$  or steeper), which poses extra challenges for long wave models. On the other hand, the exceptional flat regions are the ones that are inhabited and here a robust and efficient inundation model is crucial. Finally, all the models must be combined and compared to measurements.

The group is presently working in parallel on processing and analyzing the experimental data, on the modeling of the slide impact and on propagation models. A number of the state of the art models have been tested and a various strengths and weaknesses have been unraveled. We may mention, for instance, artificial surface boundary layers which are present in some VOF models, while absent in other, and stability problems for high order Boussinesq models on steep slopes. The most relevant findings, in an updated version, will be presented at the conference.

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## Dynamic, tsunami-induced nearshore currents

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#### Abstract

In this talk, the well-established approaches of coupling tsunami generation to seismic seafloor motion and the following trans-oceanic wave propagation will be briefly introduced. The focus of the presentation will be on the complex transformation of the tsunami as it approaches very shallow waters, as well as how these possibly large and fast-moving water waves interact with coastal infrastructure. Examples of coastal impact will be discussed and used to frame the theoretical efforts. The majority of the discussion will focus on tsunami-induced currents in ports and harbors. Tsunamis, or "harbor waves" in Japanese, are so-named due to the common observation of enhanced damage in harbors and ports. However, the dynamic currents induced by these waves, while regularly observed and known to cause significant damage, are poorly understood. We will show that the strongest currents in a port are governed by horizontally sheared and rotational shallow flow with imbedded turbulent coherent structures. Without proper representation of the physics associated with these phenomena, predictive models may provide drag force estimates that are an order of magnitude or more in error (Lynett et al., 2012). Such an error can mean the difference between an unaffected port and one in which vessels 300 meters in length drift and spin chaotically through billions of dollars of infrastructure (e.g. Okal et al., 2006).

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# KdV turbulence in shallow water

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### Abstract

The Korteweg-de Vries (KdV) equation is fully integrable model, and its solution can be obtained for infinity or periodic domain. Meanwhile some statistical properties of the wave field is convenient to find by direct numerical simulation of the KdV equation. There are two kinds of problems. The first is the evolution of the initially random wave field presented by the Fourier superposition of spectral components with deterministic amplitudes and random phases. The properties of the KdV random wave field are analyzed: transition to a steady state, equilibrium spectra, statistical moments, and the distribution functions of the wave amplitudes. Numerical simulations are performed for different Ursell parameters and spectrum width. It is shown that the wave field relaxes to the stationary state (in statistical sense) with the almost uniform energy distribution in low frequency range (Rayleigh-Jeans spectrum). The wave field statistics differs from the Gaussian one. The growing of the positive skewness and non-monotonic behavior of the kurtosis with increase of the Ursell parameter are obtained. The probability of a large amplitude wave formation differs from the Rayleigh distribution.

The second one is the study of the turbulence of soliton gaz. The characteristics of the solitons are not changed due to integrability of the KdV equation. But the statistical characteristics and the distribution functions of the wave field include extreme distribution vary with time. It is demonstrated that two soliton interaction decreases the third and forth moments characterized the skewness and kurtosis. Increasing of the density of solitons leads to the normalization of random processes.

The influence of the variable depth on the KdV turbulence is also considered. Our publication in this field [1-4].

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## MS4: Nonlinear Optical Waves and Novel Phenomena Self-accelerating beams through spectrum-engineering

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#### Abstract

We introduce the concept of spatial spectral phase gradient, and demonstrate, both theoretically and experimentally, how this concept could be employed for generating single- and multi-path self-accelerating beams. In particular, we show that the trajectories of the accelerating beams are determined a priori by different key spatial frequencies through direct spectrum-to-distance mapping. In the non-paraxial regime, our results clearly illustrate the breakup of Airy beams from a different perspective, and demonstrate how circular, elliptic or hyperbolic accelerating beams can be created by judiciously engineering the spectral phase. Furthermore, we found that the accelerating beams still follow the predicted trajectory also for vectorial wavefronts. Our approach not only generalizes the idea of Fourier-space beam engineering along arbitrary convex trajectories, but also offers new possibilities for beam/pulse manipulation not achievable through standard direct real-space approaches or by way of time-domain phase modulation. In addition, by introducing the spectral amplitude modulations in our methods, zigzag beams along convex trajectories are realized in both paraxial and non-paraxial conditions.

## Nonlinear waves emitting resonant radiation: new scenarios

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#### Abstract

Perturbed solitons are well-known to emit Cherenkov-like radiation owing to a mechanism of resonance with linear waves. Many recent experimental results have widened this area considerably, showing the possibility to extend radiative phenomena in several directions involving, e.g. new spectral regions (far in the normal dispersion regime but even involving the anomalous dispersion regime) in fibers [1], novel arrangements such as second-harmonic generation [2] or semiconductor photonic crystal waveguides [3], or new paradigms such as the negative-frequency resonant radiation [4]. Inspired by these results, we discuss what we found to be the most interesting aspects of the theory behind these new scenarios, showing in particular that the radiative emission does not require a soliton but only two coexisting mechanisms, namely a considerable spectral broadening and a definite velocity of the wave-packet. As such, resonant radiation can efficiently occurs also for dispersive shock waves [5] with a variety of possible interesting scenarios.

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## Navigating accelerating beams along arbitrary trajectories

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#### Abstract

Self-accelerating beams have stimulated growing research interest since the concept of Airy wave packets was introduced from quantum mechanics into optics. As exact solutions of the paraxial wave equation (which is equivalent to the Schrödinger equation), Airy beams propagate along parabolic trajectories and are endowed with useful properties such as self-accelerating and self-healing, promising for many applications such as guiding micro-particles, producing curved plasma channels, and dynamically routing surface plasmon polaritons. In this talk, we will provide a brief overview of our work on optimal control of linear and nonlinear self-accelerating beams, including generation and application of optical bottle beams and auto-focusing beams, self-accelerating Bessel-like and vortex beams along arbitrary trajectories, and recent work on nonparaxial accelerating beams that could bend into large angles along various trajectories.

# Shaping and twisting light beams using nonlinear holograms

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#### Abstract

Recent developments in quadratic nonlinear photonic crystals enable to convert fundamental Gaussian beams into beams of arbitrary shapes at a new optical frequency, by implementing holographic techniques in nonlinear optics [1-3]. The methods that we use are nonlinear optical implementations of techniques that were developed 50 years ago in the field of computer generated holography. Moreover, the properties of the generated beams can be all-optically controlled through the nonlinear process.

Specifically, we demonstrate nonlinear conversion of a fundamental Gaussian beam into high order Hermite-Gauss beams, Laguerre-Gauss beams or Airy beams at the second harmonic. For the case of Laguerre-Gauss beams, the angular momentum of the pump beam, second harmonic beam and the nonlinear crystal follow a quasi-angular momentum conservation law [2], in a similar fashion to the well known momentum conservation law of quasi phase matching. Furthermore, the concept of nonlinear holography can be extended from the spatial domain to the spectral domain, thereby enabling to shape the spectrum and temporal shape of nonlinearly generated light pulses [4].

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# Vortex knots in self-trapped laser beams

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## Abstract

We demonstrate theoretically the spontaneous nucleation and knotting of optical vortex lines in the filed of a spinning spatial soliton. The physical mechanism responsible for spontaneous knotting is the combination of nonlinear self-phase modulation of an elliptic laser beam and its spatial twist introduced by optical orbital angular momentum.

# Nonlinear dynamics of multiwave mixing in optical microcavities

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#### Abstract

The generation of optical Kerr frequency combs by microresonators has attracted much interest in the last few years [1]. The equidistant and highly resolved spectral lines of these combs are expected to help facilitate numerous applications such as optical clocks, sensing and spectroscopy. To date, the theoretical descriptions of microresonator frequency combs was mostly carried out using a modal expansion approach, which describes the slow evolution of the comb spectrum using time-domain rate equations [2]. An alternative description of microresonator frequency combs was recently proposed [3] that allows for a time-domain description of the comb as mode-locking of soliton solutions of the driven and damped nonlinear Schrödinger equation (NLSE), which was previously applied for the description of CW-driven fiber-loop soliton memories [4-5].

In this presentation, we apply the driven and damped NLSE model to demonstrate that comb generation can be given a simple interpretation in terms of modulational instability (MI) of the CW pump field [4]. We present a theoretical stability analysis applicable to microresonators combs, which provides analytical expressions for the conditions under which stable comb generation may occur. Although MI of the CW solutions may also occur in the normal dispersion regime [4], we show that the generated combs tend to have a finite lifetime in this case. On the other hand, we predict that stable comb generation and trains of mode-locked dark soliton pulses may be generated in optical microresonators by using a dual-frequency input pump [6]. Important qualitative insights into the nonlinear dynamics of the comb generation in both normal and anomalous cavity dispersion regimes can be obtained by using truncated three or four wave models. The results of finite mode truncations are compared and found to be in good agreement with the full numerical simulations.

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# Observation of diffraction-free submicrometer visible laser beam

# propagation through nanodisordered ferroelectrics

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## Abstract

We report the observation of the propagation of visible beam with an intensity full-width-at-half-maximum of 0.8 micrometers without diffraction through 6 millimeters of nano-disordered potassium-tantalateniobate. The effect is observed after the photorefractive ferroelectric is subject to a rapid temperature hump below its Curie point and is attributed to the emergence of scale-free optical propagation. The result amounts to extending the depth of focus of a high-aperture microscope through the crystal length, permitting the transfer of high resolution pixel visible images without distortion.

# Binary plasmonic waveguide arrays: energy localizarion,

# modulational instability and gap solitons

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### Abstract

We first obtain solitary-wave solutions of a model describing light propagation in binary (linearly and nonlinearly) waveguide arrays. This model describes energy localization and transport in various physical settings, ranging from metal-dielectric (i.e., plasmonic) to photonic crystal waveguides. The solitons exist for focusing, defocusing, and even for alternating focusing-defocusing nonlinearity. We also consider a model consisting of two subsystems coupled exclusively by nonlinear terms. We show the existence of bright-dark gap solitons of both the discrete system and its continuous long wavelength limit, in spite of the absence of a gap in the linear (i.e. plane wave) spectrum. We find that these solitons are always modulationally unstable in the continuous limit, whereas they can be stable in the discrete system if the amplitude of the background component exceeds a certain threshold.

# Slow and fast light bullets and vortices in coherent atomic systems

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### Abstract

In recent years, much attention has been paid to the study of light-wave propagation in coherent atomic systems via electromagnetically induced transparency (EIT). One-dimensional slow-light solitons in such systems have been widely investigated. In this talk, we shall report the results of our recent studies on high-dimensional slow and fast light solitons (alias optical bullets) and vortices in resonant atomic gases through EIT-realated quantum coherence. In addition, we also present our study on the Stern-Gerlach effect of slow-light bullets in a cold atomic system.

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# Parametric amplification and cascaded FWM in optical fibers

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### Abstract

Four wave mixing in optical fibres and other waveguide structures has been studied for several decades since low attenuation optical fibres were first manufactured. The process is responsible for a variety of nonlinear optical phenomena including Modulation Instability, Phase Conjugation (or two pump parametric amplification) and Bragg Scattering (or wavelength exchange). Four wave mixing can be utilised to generate optical gain via parametric amplification using either one or two pumps, and such parametric amplifiers have the potential to compete with other optical amplifers which are currently in use commercially. Unlike the amplifiers based on fiber Raman gain or rare earth doped fibres, the parametric amplifier is inherently a phase sensitive amplifier. It also has the advantage, in common with the Raman amplifier that it can potentially operate at any wavelength. The major limitations on the operation of the amplifier are the availability of suitable pumps and the uniformity of the optical fibre.

Groups in different laboratories have recently demonstrated significant improvements in the operation of fiber optical parametric amplifiers and oscillators, using novel pump sources and fibers. The main fiber advances which have contributed to this performance improvement are the availability of photonic crystal fibers (PCF) which have led to optical parametric amplifiers and oscillators in the visible, and highly nonlinear fibers (HNLF) which have contributed to improved performance in the infra red. It has also become apparent in both HNLF and PCF based amplifiers that the major limitation to the performance is now the uniformity of the fiber itself. Whilst in principle these amplifiers could be further improved, this would require very strict limits to be placed on the diameter fluctuations of the fiber itself[1].

Optimising the design a single pump oscillator in our laboratories has led to the demonstration of a multi Watt level tuneable optical fiber parametric oscillator with a tuning range in excess of 500nm (70THz) in the infra red using a HNLF. A tuning range of 180nm (160THz) has also been demonstrated in the visible using a PCF.

The nonlinear interaction of two pumps, separated by a frequency  $\Delta$ , propagating in an optical fiber, generates two sidebands themselves detuned from the two pumps by  $\Delta$ . At sufficiently high power this process can continue as a cascade with the nth sideband detuned by  $(n + 0.5)\Delta$  from the center frequency. Strong amplification of the nth sideband is possible provided that the sum of the wavevector mismatches of the n elementary FWM processes required to generate the nth sideband is zero [2]. This has recently been demonstrated experimentally for cascades designed to phase match high order (the n = +6 and n = -4) sidebands. It has been found that the phase-matched

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frequency shift of the nth sideband corresponds exactly to the frequency shift of a dispersive wave emitted by a soliton centered at the center frequency of the two pumps [3]. A careful analysis of this process has enabled cascaded FWM to be identified as the nonlinear mechanism that drives dispersive wave emission.

In addition, this frequency domain theory also predicts that a pump propagating in the normal dispersion regime can emit a dispersive wave in a manner entirely analogous to the emission of a dispersive wave by a soliton [4]. Other experiments have unequivocally demonstrated this emission and a similar process involving cascaded nonlinear Bragg scattering. This cascaded theory will play an important role in the future development of optical frequency combs in both optical fibers and high-Q micro-resonators.

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### Sub-wavelength plasmonic lattice solitons

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#### Abstract

When the size of conventional optical circuits is reduced to the nanoscale, the propagation and the concentration of light are inherently limited by diffraction. One effective approach to overcome this major challenge is to use surface plasmon polaritons (SPPs), which are evanescent waves trapped at the interface between a dielectric material and a conductor such as a metal in visible frequency range. The electric field of SPPs is significantly enhanced at the metallic surfaces and thus it is natural to exploit the nonlinear effects of SPPs, which enables the nonlinear optics at the deep-subwavelength scales.

We study the spatial lattice solitons in nanostructures composed of periodic arrangement of metallic layers or metallic nanowires which are embedded in an optical medium with Kerr nonlinearity. The spatial extent of plasmonic lattice solitons (PLSs) can be significantly smaller than the wavelength, and their propagation can be steered actively, for example, by a power-controlled mean.

In this talk, we present three types of PLSs: 1) PLSs in one-dimensional and two-dimensional arrays of coupled metallic nanowires, including fundamental solitons and vortical solitons; 2) Surface PLSs at the boundary of semi-infinite metallic-dielectric periodic nanostructures; 3) Vector PLSs originating from different transmission bands.

The theoretical analysis is based on an extension to the nonlinear case of a coupled-mode theory (CMT), which captures the full vectorial character of the propagating modes of the metallic nanowires. Rigorous solutions based on full-set Maxwell equations will be also presented and compared with the CMT results.

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# Nonlinear photonic crystal for optical switching and logic

# functionality

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#### Abstract

Nonlinear photonic crystals (NPCs) made from materials of high Kerr nonlinearity offer a promising way to build ultrafast and low-power optical switching devices. When photonic crystals are built from materials with Kerr effect, the bandgap or defect state frequency can be controlled effectively by external pump light with a remarkable shift, leading to optical switching effect [1]. In this talk we present our recent works on exploiting ultrafast optical switching and logic functionality by using polystyreneNPC, which is a polymer material with a very large Kerr nonlinearity and extremely fast optical response speed (down to several femtoseconds). First we will show that ultrafast optical switching with a response time down to 10 fs can be achieved by polystyrene NPC when the pump light is high - intensity ultrafast laser pulse with a duration of several femtoseconds [2]. Second we will present the concept of hybrid polymer-silicon NPC [3,4] and discuss a versatile technique based on nano-imprint lithography to fabricate high-guality silicon-polystyrene compound nonlinear photonic crystal slabs [5]. The hybrid photonic crystal structures can incorporate both advantages of ultrafast and low power nonlinear optical effects. The versatile method can be expanded to make general semiconductor-polymer hybrid optical nanostructures, and thus it may pave the way for reliable and efficient fabrication of ultrafast and ultralow power all-optical tunable integrated photonic devices and circuits. Finally we will discuss optical logic gate based on the scheme of two switchable optical cavities in NPC [6].

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# High-power fiber optical frequency combs

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#### Abstract

This talk will discuss recent developments of fiber optical frequency combs. A robust feed-forward compensation scheme was used to suppress phase noises in nonlinear polarization rotation mode-locked Yb-doped fiber lasers and large-mode-area photonic crystal fiber amplifiers, resulting in all-fiber femtosecond optical comb up to 100W. This facilitated efficient generation of UV and XUV and ultra-broadband optical frequency combs, as well as synchronized multi-color femtosecond combs.

MS5: Quantum Degenerate Systems

## Holography and turbulence

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#### Abstract

Holographic duality provides a systematic new approach to studying quantum turbulence in which the dynamics of the quantum liquid are *encoded in the dynamics of a black hole horizon in classical gravity.* This talk will introduce the basic ideas of holographic duality and then use the gravitational description to numerically construct turbulent ows in a holographic superuid in two spatial dimensions. The resulting superuid kinetic energy spectrum obeys the Kolmogorov -5/3 scaling law in the regimes studied. We trace this scaling to a direct energy cascade by injecting energy at long wavelengths and watching it ow to a short-distance scale set by the vortex core size, where dissipation by vortex annihilation and vortex drag becomes efficient. The power of the holographic dual, all dynamics are perfectly Lagrangian; dissipation derives from the fact that uctuations which fall behind the black hole horizon are effectively lost forever. The horizon thus gives us a precise local probe of energy dissipation.

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# Lieb's soliton-like excitations in harmonic trap

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### Abstract

We study the solitonic Lieb II branch of excitations in one-dimensional Bose gas in homogeneous and trapped geometry. Using Bethe *ansatz* equations we calculate the "number of particles" and the "effective mass" of a soliton. The frequency of oscillations in a harmonic trap is calculated. It changes continuously from its "soliton-like" value  $\omega_h/\sqrt{2}$  in the high density mean field regime to  $\omega_h$  in the low density Tonks-Girardeau regime with  $\omega_h$  the frequency of the harmonic trapping. The phase jump of the order parameter is calculated.

## Inverse energy cascade in two-dimensional quantum

## vortex turbulence

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#### Abstract

We demonstrate an inverse energy cascade in a minimal model of forced 2D quantum vortex turbulence. We simulate the Gross-Pitaevskii equation for a moving super fluid subject to forcing by a stationary grid of obstacle potentials, and damping by a stationary thermal cloud. The forcing injects large amounts of vortex energy into the system at the scale of a few healing lengths. A regime of forcing and damping is identified where vortex energy is efficiently transported to large length scales via an inverse energy cascade associated with the growth of clusters of same-circulation vortices, a Kolmogorov scaling law in the kinetic energy at the scale of the system size. Our results provide clear evidence that the inverse energy cascade phenomenon, previously observed in a diverse range of classical systems, can also occur in quantum fluids.

# Dark solitons in superfluid Ferml gases: Dispersion relation

# and snaking instability

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### Abstract

A great puzzle is presented by recent experimental results on the oscillation frequency of dark solitons in a trapped superuid Fermi gas [1] that differ by an order of magnitude from earlier theoretical predictions [2,3]. This highlights the problem of describing a strongly correlated quantum uid by mean-field theory. In this talk we present theoretical studies of the dispersion relation and the transverse snaking instability of dark solitons for a superuid Fermi gas with tunable interaction strength in the crossover regime between Bose-Einstein condensation of atom pairs and BCS-type superfluidity.

For the strongly interacting unitary Fermi gas, scaling arguments permit the derivation of very general and exact statements about the soliton dispersion relation that link several experimentally accessible observables [2]. In a second part of the talk, we report numerical results using Bogoliubov-de Gennes mean field theory for the dispersion relation and discuss the implications for the soliton oscillations in a trapped gas [2] and a high-velocity instability [4]. In order to study the dynamical instability to transverse modulation (snaking instability) that leads to the decay of dark solitons into vortices, we have implemented a calculation of the linear-response function in the RPA approximation. Complex poles of the response function provide information about the time scale of the snaking instability and give an indication about a suppression of the snaking instability for narrow transverse samples. Again, our numerical results indicate significant deviations from experimental observations reported in Ref.[1] and highlight the puzzle of understanding a strongly correlated quantum fluid.

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## Towards experimentally testing the paradox of black hole

## information loss

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#### Abstract

In 1976, Hawking pointed out that information about the collapsed matter in a black hole will be lost since there is no correlation among the thermal Hawking radiations. In 2000, Parikh and Wilczek discovered that Hawking radiations are slightly derived from exact thermality when back reaction is considered. With the nonthermal spectrum of Parikh and wilczek, we recently showed that Hawking radiations can carry off all information about the collapsed matter in a black hole [1,2]. In this talk, I will discuss the spectroscopic features of Hawking radiation from a Schwarzschild black hole, contrasting the differences between the nonthermal and thermal spectra. We found that the energy covariances of Hawking radiations for the thermal spectrum are exactly zero, but the energy covariances are nontrivial for the nonthermal spectrum. Thus, the nonthermal spectrum can be distinguished from the thermal one by counting the energy covariances of Hawking radiations, which presents an avenue towards experimentally resolving the paradox [3]. Finally, I will discuss the possibility of testing the paradox by counting Hawking radiations from small manmade black holes such as predicted to appear in LHC experiments or discussed with optical, acoustic, and cold-atomic systems.

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#### MS5: Quantum Degenerate Systems

## Macroscopic quantum tunneling in Bose-Einstein condensates

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#### Abstract

I present three studies on macroscopic quantum tunneling of Bose-Einstein condensates. First, I show how even at the simplest mean field level already the problem of escape through a barrier has new features compared to single particle physics: the tunneling time is not the inverse of the rate and interactions allow one to tune from bound to quasi-bound to unbound states freely [1]. Second, I show how tunneling in a double-well system leads to Josephson junction and Schrodinger cat (NOON state) physics. I demonstrate that although a very small bias or tilt in the potential can destroy Cat-like states, by intentional use of bias the many body wavefunction can be used to protect such states from destruction (or internal decoherence) [2]. Third, I present a full many-body calculation of entangled guantum dynamics of the escape problem [3], exploring entanglement, number correlations, and other features not accessible by instanton or other methods. I show that the tunneling process is non-smooth, and actually occurs in bursts. When approximately half the particles have tunneled out of the well, the particles remaining are maximally entangled with the escaped portion. Number fluctuations greatly modify the escape time beyond mean field predictions. Preliminary indications are that the time derivative of number fluctuations serves as a witness to the spatial entropy of entanglement. In summary, these three views on macroscopic quantum tunneling show that even a weakly interacting many body system already has significantly different features on all levels as compared to the single particle problem.

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# Nonequilibrium superfluidity and internal convection

# in finite temperature Bose gases

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### Abstract

Classical-field methods provide powerful tools for the non-perturbative simulation of weakly interacting Bose systems at finite temperatures, in both equilibrium and non-equilibrium regimes [1,2]. Here we describe some of our recent work on the development and application of the stochastic Gross-Pitaevskii equation to such systems.

We describe how the calculation of the anomalous correlations of the classical-field facilitates the determination of the quasiparticle mode structure of the finite-temperature Bose gas, and how this can be used to identify the onset of condensation and superfluidity. This is particularly valuable in low dimensional systems where condensation is a finite-size effect absent in the thermodynamic limit. We also consider finite-temperature superfluid flow in a quasi-two-dimensional torus, and demonstrate the characterisation of the inhomogeneous, non-equilibrium superfluid density.

Finally, we study a degenerate Bose gas coupled to two spatially separated heat reservoirs held at different temperatures, and simulate the onset of heat transport and superfluid internal convection [3]. We further consider the prospects for observing thermal-superfluid counterflow turbulence in this system.

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# Ultracold collision in the presence of synthetic

# spin-orbit coupling

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### Abstract

We present an analytic description of ultracold collision between two spin -  $\frac{1}{2}$  fermions with isotropic spin-orbit coupling of the Rashba type. We show that regardless of how weak the spinorbit coupling may be, the ultracold collision at sufficiently low energies is significantly modified, including the ubiquitous Wigner threshold behavior. We further show that the particles are preferably scattered into the lower-energy helicity states due

to the break of parity conservation, thus establishing interaction with spin-orbit coupling as one mechanism for the spontaneous emergence of handedness. The theory is

applicable both to elementary spin -  $\frac{1}{2}$  fermions such as electrons in condensed matter,

and to spin -  $\frac{1}{2}$  atoms such as  $\,^6\text{Li}$  in its ground hyperfine state.

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## Interactions of bright matter-wave solitons with a

## potential defect

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#### Abstract

Non-dispersive solitary waves (solitons) are ubiquitous in nature. Ultracold atomic systems allow us to study solitons and their interactions in a controlled environment. In our experiments [1, 2] we form bright matter-wave solitons in Bose-Einstein condensate of <sup>7</sup>Li atoms. We use the broad Feshbach resonance of <sup>7</sup>Li in the  $|1,1\rangle$ state to tune the scattering length through zero to small negative values. When the inter-atomic interactions become attractive a bright soliton can form close to the critical number for collapse. We excite the collective dipole mode of the soliton in a weakly confining axial harmonic potential created by a single focused laser beam, and study its interaction with either a narrow repulsive barrier or an attractive well. Both the barrier and well are formed by a near-resonant, cylindrically focused laser beam that perpendicularly bisects the trapping beam at its focus. Through adjustment of the barrier potential height, the soliton can either be split in two, transmitted, or reflected. When the barrier produces splitting, the fragments will undergo a second interaction at the barrier thus realizing the ingredients of a Mach-Zender type interferometer. We have attempted to observe coherent recombination. A potential well is created by detuning the laser red of the atomic resonance, for which quantum reflection [3] can occur. Through varying the well depth, we investigate reflection, transmission and trapping in the well [4], in which the soliton may exhibit small-amplitude oscillations inside the well.

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# Correlated metallic few-particle bound states in quasiperiodic

# potentials

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#### Abstract

Single particle states in a chain with a quasiperiodic potential show a metal-insulator transition upon the change of the potential strength. We consider two particles with local interaction in the single particle insulating regime. The two particle states change from being localized to delocalized upon an increase of the interaction strength to a non-perturbative finite value [1]. At even larger interaction strength the states become localized again. This transition of two particle bound states into a correlated metal is due to a resonant mixing of the noninteracting two particle eigenstates. In the discovered correlated metal states two particles move coherently together through the whole chain, therefore contributing to a finite conductivity.

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MS5: Quantum Degenerate Systems

## Nonlinear wave dynamics of cold quantum matter

## near atom chips

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#### Abstract

In this talk, I will consider the nonlinear dynamics of ultracold atoms near atom-chip structures that are designed to enable near-surface (<  $1\mu$ m) trapping and thereby offer fine spatial control of the potential landscape and atom density profile.

Firstly, I will consider the properties of the chips themselves and show, for example, how opening and closing a single quantized conductance channel in a quantum wire can split and remerge an atomic condensate.

Next, I will focus on the spatial and spin dynamics of the ultracold atom clouds and highlight the importance of the interplay between these two degrees of freedom, which can trigger the formation of complex spin textures.

# Kibble-Zurek scaling and its breakdown for spontaneous generation

# of Josephson vortices in Bose-Einstein condensates

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### Abstract

Atomic Bose-Einstein condensates confined to a dual-ring trap support Josephson vortices as topologically stable defects in the relative phase. We propose a test of the scaling laws for defect formation by quenching a Bose gas to degeneracy in this geometry. Stochastic Gross-Pitaevskii simulations reveal a -1/4 power-law scaling of defect number with quench time for fast quenches, consistent with the Kibble-Zurek mechanism. Slow quenches show stronger quench-time dependence that is explained by the stability properties of Josephson vortices, revealing the boundary of the Kibble-Zurek regime. Interference of the two atomic fields enables clear long-time measurement of stable defects, and a direct test of the Kibble-Zurek mechanism in Bose-Einstein condensation.

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# Solitons in armchair and zigzag geometries in the nonlinear dirac

## equation

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#### Abstrct

We present solitons which solve the one-dimensional (1D) zigzag and armchair nonlinear Dirac equation (NLDE) for a Bose-Einstein condensate (BEC) in a honeycomb optical lattice [1], where the two types of NLDEs correspond to the two independent directions in analogy to the narrowest of graphene nanoribbons. We analyze the solution space of the 1D NLDE by finding fixed points, delineating the various regions in solution space, and through a conservation equation which we obtain as a first integral of the NLDE. For both the zigzag and armchair geometries we obtain soliton solutions using five different methods: by direct integration; through the conservation equation; by parametric transformation; a series expansion; and by the method of numerical shooting. We interpret our solitons as domain walls in 1D which separate distinct regions of pseudospin-1/2 with  $S_z = \pm 1/2$ , where the domain wall is topologically protected. By solving the relativistic linear stability equations (RLSE) we obtain the low-energy spectrum for excitations in the bulk region far from the soliton core and for bound states in the core. We find that excitations occur as guadrupolar pseudospin waves and as a Nambu-Goldstone mode. For a BEC of <sup>87</sup>Rb atoms, we find that our soliton solutions are stable on time scales relevant to experiments [2]

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## Fractional quantum Hall phases of two-component ultracold

### bosonic gases

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#### Abstract

Artificial gauge fields for cold atoms are tools for producing topological quantum states. In spinless or spin-polarized systems, cold bosons are known to support the incompressible phases from the Read-Rezayi series, containing also the famous Laughlin and Moore-Read states with anyonic or even non-Abelian quasiparticle excitations [1,2,3,4]. Here we show that in the case of a pseudospin -1/2 Bose gas a generalization of this series, the so-called non-Abelian spin singlet (NASS) series, describes well the ground states at different filling factors [5]. We have also investigated the scenario where, in addition to an external magnetic field, the gauge field mimics an intrinsic spin-orbit coupling of the Rashba type [6]. We find a variety of different phases, which can be controlled by the strength of this coupling.

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## Controlling phase separation of a two-component Bose-Einstein

### condensate by confinement

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#### Abstract

We point out that the widely accepted condition  $g_{11}g_{22} < g_{12}^2$  for phase separation of a two-component Bose-Einstein condensate is insufficient if kinetic energy is taken into account, which competes against the inter-component interaction and favors phase mixing. Here  $g_{11}$ ,  $g_{22}$ , and  $g_{12}$  are the intra- and inter- component interaction strengths, respectively. Taking a *d*-dimensional infinitely deep square well potential of width *L* as an example, a simple scaling analysis shows that if d = 1 (d = 3), phase separation will be suppressed as  $L \rightarrow 0$  ( $L \rightarrow \infty$ ) whether the condition  $g_{11}g_{22} < g_{12}^2$  is satisfied or not. In the intermediate case of d = 2, the width *L* is irrelevant but again phase separation can be partially, or even completely suppressed even if  $g_{11}g_{22} < g_{12}^2$ . Moreover, the miscibility-immiscibility transition is turned from a first-order one into a second-order one by the kinetic energy. All these results carry over to *d*-dimensional harmonic potentials, where the harmonic oscillator length  $\xi_{ho}$  plays the role of *L*. Our finding provides a scenario of controlling the miscibility-immiscibility transition is transition is transition of a two-component condensate by changing the confinement, instead of the conventional approach of changing the values of the g's.

# The Bose gas as a simulator of interacting gauge theories

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### Abstract

In this talk, we show how under proper conditions conveniently engineered laser fields similar to those employed in Refs. [1-3] can induce an effective density-dependent vector potential in a weakly-interacting ultracold Bose gas [4]. Such a gas constitutes the semiclassical limit of an interacting gauge theory for bosons. When the system is tightly confined such that it forms a quasi-one-dimensional gas, we show that the density-dependent gauge field leads to a number of counterintuitive consequences, including density-dependent persistent currents in ring geometries, drifts in the free expansion dynamics, and chiral solitons in a Bose-Einstein condensate (BEC).

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# Critical velocity and current-phase relation

# of dilute ultracold bosonic atoms

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#### Abstract

Dilute Ultracold Bosonic Gases are optimal systems where to study the fundamental aspects of superfluidity. They are indeed well described by the Gross-Pitaevskii equation which contains all the essential ingredients characterising a superfluid, like the existence of persistent currents below a finite critical velocity, Josephson effects, or quantized vortices. In this talk, we focus on critical velocities and propose a possible general criterion determining the maximal speed with which the system can flow past an obstacle. This criterion is closely related to the Josephson critical current characterising the current-phase relation of the weak-link formed through the obstacle.

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# Heavy solitons in a Fermionic superfluid

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### Abstract

Topological excitations are found throughout nature, in proteins and DNA, as dislocations in crystals, as vortices and solitons in superfluids and superconductors, and generally in the wake of symmetry-breaking phase transitions. In fermionic systems, topological defects may provide bound states for fermions that often play a crucial role for the system's transport properties. Famous examples are Andreev bound states inside vortex cores, fractionally charged solitons in relativistic quantum field theory, and the spinless charged solitons responsible for the high conductivity of polymers. However, the free motion of topological defects in electronic systems is hindered by pinning at impurities. We have created long-lived solitons in a strongly interacting fermionic superfluid by imprinting a phase step into the superfluid wavefunction, and directly observe their oscillatory motion in the trapped superfluid. As the interactions are tuned from the regime of Bose-Einstein condensation (BEC) of tightly bound molecules towards the Bardeen-Cooper-Schrieffer (BCS) limit of long-range Cooper pairs, the effective mass of the solitons increases dramatically to more than 200 times their bare mass. This signals their filling with Andreev states and strong quantum fluctuations. For the unitary Fermi gas, the mass enhancement is more than fifty times larger than expectations from mean-field Bogoliubov-de Gennes theory. Our work paves the way towards the experimental study and control of Andreev bound states in ultracold atomic gases.

# Macroscopically superposed ground states of a dipolar Bose gas in a

## double-well potential

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#### Abstract

In this talk, I will discuss the ground state properties of a quasi-two-dimensional dipolar gas trapped in a double-well potential. Using the theory of the multiconfigurational time-dependent Hartree for bosons, we obtain the ground state wave function of the system. We show that the interplay between the anisotropic dipolar interaction and the trapping potential gives rise to a rich variety of quantum phases. Of particular interest, in certain parameter regime, the ground state of the system is a macroscopic superposition state of the bosons localized in the left and right potential wells. I will also analyze the origin of the macroscopically superposed ground states.

# Bright solitons in spin-orbit coupled Bose-Einstein condensates

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#### Abstract

We study bright solitons in a Bose-Einstein condensate with a spin-orbit coupling that has been realized experimentally. Both stationary bright solitons and moving bright solitons are found. The stationary bright solitons are the ground states and possess well-defined spin parity, a symmetry involving both spatial and spin degrees of freedom; these solitons are real-valued but not positive-definite and the number of their nodes depends on the strength of spin-orbit coupling. For the moving bright solitons, their shapes are found to change with velocity due to the lack of Galilean invariance in the system.

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MS5: Quantum Degenerate Systems

# Dynamics of spin-orbit coupled quantum gases

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### Abstract

In this talk I will discuss several dynamical phenomena in spin-orbit coupled quantum gases, for instance, collective mode of spin-orbit coupled bosons and kicked rotor dynamics with spin-orbit coupling.

# Low-dimensional spin-orbit-coupled BECs:

# Inspirations from semiconductor spintronics

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### Abstract

The recent experimental realisation [1] of synthetic gauge fields in ultra-cold atom gases [2] has opened up the possibility for studying a host of interesting magnetic-field and spin-orbit-coupling effects for macroscopic quantum states [3]. My presentation will be focussed on two-component (pseudo-spin-1/2) systems that are subject to (pseudo-)spin-dependent gauge fields. As is familiar from semiconductor spintronics [4], the interplay of spin-orbit coupling and confinement results in unusual single-particle properties. Using grey solitons in a ring-trapped BEC as an example, the additional presence of the (in general, pseudospin-dependent) contact interactions in cold-atom gases is shown to enable new types of spin-dependent phenomena [5].

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### Nonlinear modes of the nonlinear Schrödinger equation with

complex periodic potential

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#### Abstract

In the talk, we address the problem of description of nonlinear states for the nonlinear Schrödinger (NLS) equation with a complex periodic potential W(x) = U(x)+iV(x) and repulsive (defocusing) nonlinearity.

$$-\Psi_t = -\Psi_{xx} + W(x)\Psi + |\Psi|^2\Psi \tag{1}$$

Nonlinear states for Eq.(1) are of the form  $\Psi(x,t) = \rho(t) \exp\{-i(\omega t + \phi(x))\}$ . We study the dynamics of Poincaré map T generated by the system of ODE in 3D space  $(\rho, \rho_x, \theta), \theta = \phi_x$  for fixed value of the frequency  $\omega$ . An important peculiarity of the system is that for a wide class of the potentials W(x) the "most part" of its solutions collapse (i.e. tend to infinity at some finite value  $x \in R$ ). Thus, the dynamics generated by the map T can be described in terms of T-iterations of specific sets  $u_{\pi}^+$  and  $u_{\pi}^-$ . These sets consist of the points from  $(\rho, \rho_x, \theta)$  which have T-image (the set  $u_{\pi}^+$ ) or T-pre-image (the set  $u_{\pi}^-$ ) and must be found numerically. The main attention is focused on the cases when W(x) is real potential,  $\mathcal{PT}$  -symmetric potential, and more general case, including "nearly- $\mathcal{PT}$ "- symmetric potentials.

## Bose-Einstein condensates in $\mathcal{PT}$ -symmetric double-well

## and multi-well setups

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#### Abstract

In recent publications it has been shown that  $\mathcal{PT}$ -symmetric solutions of the nonlinear Gross-Pitaevskii equation describing Bose-Einstein condensates do exist [1,2]. Following a suggestion by Klaiman et al. [3] we first investigate theoretically a Bose-Einstein condensate in a double-well setup. When particles are removed from one well and coherently injected into the other the external potential is  $\mathcal{PT}$  symmetric.We solve the underlying Gross-Pitaevskii equation and show that the  $\mathcal{PT}$  symmetry of the external potential is preserved by both the wave functions and the nonlinear Hamiltonian as long as eigenstates with real eigenvalues are obtained. The linear stability analysis and the temporal evolution of condensate wave functions demonstrate that the  $\mathcal{PT}$  symmetric condensates are stable and should be observable in an experiment [4]. We then extend the setup by introducing additional wells and discuss methods of embedding an effectively open double well in a closed multi-well structure.Suggestions for experimental realizations are presented.

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# **Eigenstates of lattices with PT-symmetic defects**

#### JENNIE D'AMBROISE

#### Abstract

This talk will report on recent work with P. Kevrekidis and S. Lepri. We consider the eigenvalue problem for a one-dimensional linear Schrödinger lattice with an embedded dimer or trimer defect (more generally, an oligomer). We begin by constructing the general solution on the zero potential portions of the lattice. Working our way toward the impurity sites using the restraints of the discrete Schrödinger equation, we find that appropriately defined portions of the solution must satisfy a polynomial equation. This approach allows us to semi-analytically compute the spectrum in the linear case and the linearization spectrum around stationary solutions in the nonlinear case. Our approach is general enough to apply to both real valued Hamiltonians and purely complex parity-time symmetric potentials.

# Mean-field approximation for non-Hermitian and

## $\mathcal{PT}$ -symmetric many-boson systems

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### Abstract

The mean-field description of many-boson systems can be formulated as the effective dynamics resulting from a constraint to fully condensed states. For Hermitian systems of weakly interacting particles this yields the Gross-Pitaevskii equation for the dynamics of the effective single-particle wave function. It has recently been shown that this approximation is modified in the presence of non-Hermitian terms in the many-particle Hamiltonian, for the example of a Bose-Hubbard dimer with comple on-site energies. Here we review the generalised mean-field approximation for non-Hermitian systems, and compare the resulting dynamics to those arising from a Gross-Pitaevskii equation with complex on-site energies. Further we consider the case of an additional complex particle interaction term, modelling particle losses due to interaction. The latter leads to a complex nonlinear term in the mean-field approximation in contrast to the linear term arising from complex on-site energies. The interplay between non-linearity and non-Hermiticity modifies the self-trapping transition and leads to new bifurcation scenarios that are analysed in detail.

The talk is based on joint work with Hans-Jürgen Korsch (TU Kaiserslautern, Germany), Astrid Nniederle (Saarland University, Germany) and Chiara Liverani (Imperial College London)

## **Competing PT-potentials and re-entrant PT-symmetric phase**

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#### Abstract

The competition between nonlinear interaction and dispersion leads to remarkable solutions of the wave equation such as solitons. In recent years, the competition between dispersion and local, PT-symmetric, non-Hermitian interaction and its consequences have been extensively explored in tight-binding lattice models. These lattice models are motivated by coupled optical waveguide arrays. In this talk, I will present a simple model with two PT-potentials, one of which is short-ranged and the other is broad, and discuss PT-symmetry breaking consequences. I will then discuss the effect of nonlinearities in such a system focusing on solitonic solutions. I will conclude with predictions regarding the time-evolution of a suitably localized wavepacket in the presence of nonlinearities.

# Nonlinearities supporting localized modes in

## $\mathcal{PT}\text{--symmetric lattices}$

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#### Abstract

Nonlinear  $\mathcal{PT}$ -systems occupy an intermediate position between Hamiltonian systems allowing for existence of families of nonlinear solutions and dissipative systems in which balance between the dissipation and gain results in isolated (i.e. "fixed point") solutions. The way of how a  $\mathcal{PT}$ -system behaves depends on the type of its nonlinearity. In the talk we address the effect of the type of the nonlinearity on existence of families of the nonlinear modes in  $\mathcal{PT}$ -symmetric lattices. More specifically, we describe bifurcations of the nonlinear modes from the linear limit, considering the cases of nondegenerate and degenerate spectra, as well as exceptional points of the underlying linear systems [1-3]. On the other hand, we also consider the effect of the nonlinearity type on the possibility of analytical continuation from the anticontinuum limit [4].

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# Spectral singularities, unidirectional invisibility and PT-symmetry

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### Abstract

This talk consists of two parts. In its first part I offer a general discussion of spectral singularities of complex scattering potentials and their physical meaning. Then I discuss PT-symmetric and non-PT-symmetric self-dual spectral singularities whose optical realizations correspond to coherent perfect absorbing lasers. In the second part of the talk, I describe the PT-symmetric nature of unidirectional invisibility and survey PT-symmetric and non-PT-symmetric invisible configurations for a simple two-layer optically active system. I will conclude giving some preliminary results on the generalizations to a class of nonlinear waves.

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# Nonlinear dynamics of wave packets in $\mathcal{PT}$ -symmetric

## optical lattices

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#### Abstract

Nonlinear dynamics of wave packets in PT-symmetric optical lattices near the phasetransition point are analytically studied in one- and two- dimensional systems. We show analytically that when the strength of the gain-loss component in the PT lattice rises above the phase-transition point, an infinite number of linear Bloch bands turn complex simultaneously. For the 1D case, a nonlinear Klein-Gordon equation is derived for the envelope of wave packets. A variety of novel phenomena known to exist in this envelope equation are shown to also exist in the full equation including wave blowup, periodic bound states and solitary wave solutions. In 2D a novel equation is derived for the envelope dynamics, and wave packets exhibit pyramidal diffraction.

## PT-symmertic wave dynamics in photonic molecular systems

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### Abstract

The property of non-Hermitian Hamiltonians satisfying parity-time (PT) symmetry has been widely investigated, for the alleviation of restricted Hermitian conditionand for the application of their unusual phenomena related to complex potentials. Among those phenomena, non-reciprocal wave dynamics around the onset of PT symmetry breaking has been a hot topic in optics, for the realization of non-reciprocal devices without magneto-optical effects.

In this talk, we explore the non-reciprocity in photonic molecular systems, focusing on the phase of eigen-spectra. We show the unconventional phase dependency of non-reciprocal dynamics, contrary to the intuitive expectation.Exploiting this phase-dependent dynamics, we also demonstrate the novel phenomenon of 'reversible non-reciprocity', based on the local inversion of PT symmetry. The separation of exceptional points, related to spatio-spectral dispersion of photonic molecular systems, also will be presented.

## Nonlinear stationary states in PT-symmetric lattices

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### Abstract

We will present results on the linear and nonlinear properties of two related PT-symmetric systems of the discrete nonlinear Schrödinger (dNLS) type.

First, we examine the parameter range, for which the finite chains have real eigenvalues and PT-symmetric linear eigenstates, as well as the nonlinear stationary states. We develop a systematic way of analyzing the nonlinear states with the implicit function theorem at an analogue of the anti-continuum limit for the dNLS equation.

Secondly, we consider the case of such finite PT-symmetric chains embedded as defects on the infinite dNLS lattice. In that case, we show that the PT-symmetry phase transitions are upshifted. We also prove existence of localized stationary states (discrete solitons) in the analogue of the anti-continuum limit. Numerical computations illustrate the existence, as well as the stability of such discrete solitons.

The presentation is based on the two recent works with P. Kevrekidis [1,2].

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# Spatial solitons in defocusing Kerr media with

**PT**-symmetric potentials

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#### Abstract

We provide a brief review of recent studies of spatial solitons in defocusing Kerr media with PT-symmetric potentials. Bright, dark and gray solitons all exist in the system. For bright solitons, an exact one-dimensional solution and a closed two-dimensional solution are found. The degree of nonlocality can influence the stability of bright solitons. Both a symmetry-breaking bifurcation destabilizing the dark solitons that leads to nonstationary dynamics, as well as a nonlinear analog of the PT transition that eventually terminates both the ground state and the dark soliton branch, yielding purely gain-loss dynamics. In addition, simulated results show that there are two kinds of gray solitons, the dip-shaped gray solitons and the hump-shaped solitons, and they can be stable in certain conditions.

## PT symmetry with a system of three-level atoms

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### Abstract

We show that a vapor of multilevel atoms driven by far-o® resonant laser beams, with possibility of interference of two Raman resonances, is highly e±cient for creating parity-time (PT) symmetric profiles of the probe-field refractive index, whose real part is symmetric and imaginary part is anti-symmetric in space. The spatial modulation of the probe-field susceptibility is achieved by proper combination of standing-wave strong control fields and of Stark shifts induced by far-off resonance laser fields. As particular examples we explore a mixture of isotopes of Rubidium atoms and design a PT-symmetric lattice and a parabolic refractive index with a linear imaginary part.

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## Nonlinear wave packets in deformed honeycomb lattices

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#### Abstract

The spectrum of a Schrödinger operator with a perfect honeycomb lattice potential has special points, called Dirac points where the lowest two branches of the spectrum touch. Deformations can result in the merging and disappearance of the Dirac points and the originally intersecting dispersion relation branches separate. Corresponding to these deformations, nonlinear envelope equations are derived and their dynamics are studied. In the region where Dirac points exist a maximally balanced equation is derived which has limits to a nonlinear Schrödinger-Kadomtsev-Petviashvili type equation and its dispersionless reduction. When the Dirac points disappear and a gap opens a different derived maximally balanced equation is which has the nonlinear Schrödinger-Kadomtsev-Petviashvili type and a one dimensional nonlocal evolution equation as limits. When the gap is sufficiently wide a nonlinear Dirac equation with nonzero mass and a nonlinear Schrödinger focusing-defocusing system are found. The latter two equations admit nonlinear localized modes. Typical dynamical behaviors of the effective envelope equations are presented.

## Solitons in PT -symmetric nonlinear lattices

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#### Abstract

We discuss the existence of localized nonlinear modes in PT -symmetric nonlinear lattices. The system considered reveals a number of unusual properties. Thus, unlike other typical dissipative systems it possesses families (branches) of solutions, which can be parameterized by the propagation constant. Relatively narrow localized modes appear to be stable, even when the conservative nonlinear lattice potential is absent, while broad modes may be unstable. Finally, the system supports stable multipole solutions despite the fact that refractive index is uniform.

## Stability of symmetric and antisymmetric solitons in

### *PT***-symmetric couplers**

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#### Abstract

Dual-core *couplers*, i.e., systems of parallel waveguides (*cores*) linearly coupled by tunneling of light between them, play a fundamental role in optics, as well as in other physical applications. In case the cores carry the intrinsic Kerr nonlinearity, the coupler gives rise to (temporal or spatial) solitons, which remain symmetric at small values of the energy (power), and undergo spontaneous symmetric breaking, i.e., the destabilization of symmetric solitons and a transition to stable asymmetric ones, at a critical value of energy. In fact, the spontaneous symmetry breaking of solitons in couplers is one of basic manifestations of this effect, which occurs in a broad variety of nonlinear symmetric systems [1].

Recently, a model of a  $\mathcal{PT}$ -symmetric nonlinear coupler, with linear gain ( $\gamma$ ) applied to one core, and the balancing loss acting in the mate one, was proposed [2,3]. In the general form, this model is based on the following system of propagation equations for amplitudes  $\psi_{1,2}$  of electromagnetic waves in the linearly coupled cores, written in terms of the spatial-domain propagation:

$$i(\psi_1)_z + (\psi_1)_{xx} + (|\psi_1|^2 + \chi |\psi_2|^2) \psi_1 + \psi_2 = i\gamma\psi_1,$$
(5)

$$i(\psi_2)_z + (\psi_2)_{xx} + (\chi|\psi_1|^2 + |\psi_2|^2)\psi_2 + \psi_1 = -i\gamma\psi_2,$$
(2)

where the linear-coupling coefficient is scaled to be 1, and real coefficient  $\chi$  accounts for the possible nonlinear coupling between the cores. Symmetric and antisymmetric soliton solutions of this system are available in an analytical form [4]. The stability boundary for the symmetric solitons against symmetry-breaking perturbations in the case of  $\chi = 0$  was recently found in an exact form [1,2]. We here report exact results for the stability boundary of symmetric and antisymmetric solitons in the general model (1),(2). The main finding is that the stability region for the squared amplitude of the solitons, known in an analytical form in the conservative model ( $\gamma = 0$ ), is reduced by factor  $\sqrt{1-\gamma^2}$ . The stability region shrinks to nil at  $\gamma = 1$ , when the the gain/loss coefficient is equal to the constant of the inter-core linear coupling, making the system "supersymmetric"[5] at this point

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## Finding the stability of modelocked pulses over a broad

### parameter range

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#### Abstract

The study of passively modelocked lasers has undergone a renaissance in recent years with the development of a host of new lasers and applications [1]. A key issue in the design of passively modelocked lasers is determining the parameter ranges in which they can operate stably. To date, these studies have been carried out either in limited situations in which the pulse stability can be determined analytically [2] or using computational methods in which an initial pulse shape or initial computational noise are assumed [3]. The former approach cannot be used, except to give qualitative insights, with real-world systems, while we have found that the latter approach is inefficient and often unreliable.

The goal of our work is to develop computational analogs of the analytical methods [2] that are sufficiently powerful and efficient to be applied to real-world systems.

To date, we have applied our approaches to studies of the cubic-quintic modelocking equation with a fast saturable gain and a slow saturable absorber [4]. We have found a rich dynamical structure that includes the following regimes: (1) A regime in which radiation modes are unstable. (2) Two regimes in which a single pulse shape is stable. (3) A regime in which two different pulse shapes are simultaneously stable. (4) A regime in which a shelf instability occurs.

We will discuss how our work to date can be extended to realistic systems that include discrete components and solutions that are only periodically stationary.

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## From rogue waves to randomness in nonlinear

## supercontinuum generation

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#### Abstract

The noise properties of supercontinuum generation continue to be a subject of wide interest within both pure and applied physics. Aside from immediate applications in supercontinuum source development, detailed studies of supercontinuum noise mechanisms have attracted interdisciplinary attention because of links with extreme instabilities in other physical systems, especially the infamous and destructive oceanic rogue waves. The instabilities inherent in supercontinuum generation can also be interpreted in terms of natural links with the general field of random processes. In this contribution we describe recent work where we interpret supercontinuum intensity and phase fluctuations in terms of the physics of rogue wave generation. We present both experimental and numerical results.

# Frequency comb injection locking of mode locked lasers

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### Abstract

I will show that injection locking of a mode locked laser to an external pulse train is a two-frequency synchronization problem, where the source and target frequency combs are distinguished by the spacing and offset frequency mismatches. Injection-locked steady states are characterized by a fixed source-target time and phase shifts. The solution of the synchronization problem in the weak injection regime will be presented in the form of locking diagrams where the region of stable injection locking is identified and mapped by the curves of constant source-target time shifts and phase shifts. While the analysis suggests that in application to mode locked fiber lasers active offset mismatch stabilization will be needed to injection lock source and target frequency combs, the synchronization properties are significantly improved with shorter pulses and stronger dispersion.

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# Nonlinear fiber optics with ultraviolet light and high fields

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### Abstract

Kagomé-style hollow-core photonic crystal fiber, filled with noble gases, is transforming nonlinear fiber optics by permitting operation in hitherto inaccessible parameter regimes [1]. The hollow core system can handle extremely high intensities (up to  $10^{15}$  W/cm<sup>2</sup> has been reached without damage), and offers both ultrabroadband transmission from the IR to the vacuum ultraviolet and pressure-tunable group velocity dispersion, allowing exploration of soliton-driven nonlinear effects.

By adjusting the pressure so that the group velocity dispersion is anomalous at the pump wavelength (800 nm) but normal for shorter wavelengths, soliton-driven dispersive waves can be generated from the visible down to 180 nm in the vacuum UV. Conversion efficiencies as high as 8% have been demonstrated [2].

Alternatively, by lowering the pressure, the dispersion can be made anomalous at all wavelengths longer than e.g. 300 nm, allowing solitons to self-compress to durations of only a few optical cycles, yielding intensities sufficiently high to ionize the gas. This opens up the exciting new opportunity of studying soliton propagation in the presence of a light-induced plasma, and has led to the first observation of a soliton self-frequency blue-shift akin, though opposite in sign, to the well known Raman-driven soliton self-frequency red-shift [3,4].

In the talk recent developments in the field will be reviewed, particular attention being paid to the generation of short wavelengths and high intensities.

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## Bright optical solitons from defocusing nonlinearities

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#### Abstract

The talk aims to review recently obtained results which demonstrate that self-defocusing cubic [1-3], as well as quintic [4] media, with a spatially inhomogeneous nonlinearity, whose strength grows from the center to periphery faster than  $r^{D}$  in the *D*-dimensional space (D = 1, 2, 3), where r is the radial coordinate  $(r \equiv |x| \text{ in 1D})$ , can support a variety of stable *bright* solitons in all the three dimensions, including 1D fundamental and multi-node modes, 2D fundamental and vortex solitons with an arbitrary topological charge, and also 3D fundamental solitons (the latter case is relevant not to optics, but to BEC). Solitons initially shifted from the center maintain their coherence while oscillating in the effective nonlinear (pseudo-) potential as robust quasiparticles [1-4]. In addition to numerically found soliton families, many particular solutions are found in an exact analytical form, and accurate approximations for the entire families of fundamental solitons are developed by means of variational and Thomas-Fermi approximations [1-4]. It should be stressed that these stable solitons exist without any support from a linear potential, i.e., they have nothing in common with the gap solitons, which may exist in a self-defocusing medium combined with a linear periodic potential (grating). Very recently, it has been demonstrated that a similar mechanism supports, as well, stable fundamental and twisted solitons in a 1D nonlocal self-defocusing medium (modeling the thermal nonlinearity in optics), with the local strength of the interaction growing faster than |x|. Qualitatively similar numerical and analytical results demonstrate the existence of stable dissipative solitons in media with the uniform linear gain and nonlinear loss whose local strength grows toward the periphery faster than  $r^{D}$  [5].

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# Nonlinear optical phononics: Harnessing stimulated Brillouin

## scattering in nanoscale circuits

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#### Abstract

I will review our recent progress in developing nanoscale optical-phononic circuits that use nonlinear optical effects to efficiently interact phonons (sound) with photons (light), creating a new paradigm in information processing. The innovation is to exploit stimulated Brillouin scattering, the strongest nonlinearity for parametric coupling of light and sound, in highly nonlinear nanoscale circuits, in which electrostriction efficiently couples photons and phonons. The new level of control over the interaction between light and sound will result in optical-phononic circuits in which light can be spectrally controlled, amplified and processed in ways currently unachievable.

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# Similaritons and chirped fibre lasers

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#### Abstrct

Self similar solutions of nonlinear partial differential equations are chirped solutions which maintain their mathematical form, whilst being scaled in time or amplitude. Standard techniques for finding such solutions have been developed with applications in nonlinear acoustics, plasma physics and other areas. Recently these techniques have been applied to locate new solutions of the nonlinear Schrödinger equation (NLSE), which are finding increasing applications in high power amplifiers and mode locked fibre lasers. Of particular interest are the solitary pulse solutions which are known as similaritons. The various different similariton solutions applying to pulse propagation under the influence of the NLSE and the LSE provide a route to the development of stable pulsed lasers, with the potential to generate much higher power pulse energies than are available through soliton laser systems.

There are three different experimental regimes areas where self similar propagation is important. These are propagation in the linear regime, and nonlinear propagation in the normal and the anomalous dispersion regimes. In the normal dispersion regime the NLSE supports the development of expanding parabolic similaritons as an asymptotic solution, whilst in the anomalous dispersion regime it has an exact compressing hyperbolic secant similariton solution.

The first demonstration of the existence of parabolic pulses was made using a high power amplifier at  $1\mu$  wavelength, using a seed pulse of about 200fs duration [1]. The asymptotic nature of the solution ensures that it forms regardless of the input pulse shape given sufficient gain and propagation length, but a short unchirped seed pulse facilitates rapid evolution to the asymptotic form. It is important for the fidelity of the similariton, that the amplifier has sufficient gain bandwidth to support the steadily increasing bandwidth of the developing pulse, but both Raman and rare earth doped fibre amplifiers have been shown to support their development experimentally[2]. Early on, it was clear that the possibility of constructing a laser yielding the parabolic shaped chirped output pulse was attractive, instead of developing it in a single pass amplifier from a separate seed pulse, although this has proved challenging.

Recently, similariton lasers of varying design have been constructed by different groups. In our laboratories the construction of these lasers has benefited materially from the development of a full theoretical model of their operation. Since the evolution of the similariton in the high gain region involves extensive spectral and temporal reshaping, the main challenge is to recover a suitable seed pulse from the output parabolic pulse, which has a wide bandwidth and strong linear chirp.

### MS7: Forontiers of Nonlinear Waves in Optics

We have constructed fibre lasers which produce both parabolic and non parabolic output pulses depending upon the details of the laser design [3,4]. The lasers which produce a non parabolic pulse output are related to so called "ANDi" and "giant chirp" lasers which are the subject of much development currently, and whilst the output pulse does not have a parabolic shape, the propagation within some sections of the laser is still self similar.

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### Logarithmic scaling in the catastrophic self-focusing (collapse)

of a laser beam in Kerr media

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#### Abstract

We study the catastrophic stationary self-focusing (collapse) of laser beam in nonlinear Kerr media described by the nonlinear Schrödinger equation (NLSE) for the amplitude  $\psi$  in dimension two. The width L(z) of a self-similar solutions near collapse distance  $z = z_c$  obeys a  $(z_c - z)^{1/2}$  scaling law with the well-known, leading order modification of loglog type  $\propto (\ln |\ln(z_c - z)|)^{-1/2}$ . We show that the validity of the loglog modification requires double-exponentially large amplitudes of the solution  $\sim 10^{10^{100}}$ , which is unrealistic to achieve in either physical experiments or numerical simulations. We derive a new equation for the adiabatically slow parameter which determines the system self-focusing across a large range of solution amplitudes. Based on this equation we develop a perturbation theory for scaling modifications beyond the leading loglog. We show that for the initial pulse with the optical power N moderately above ( $\sim 20\%$ ) the critical power  $N_c$  of self-focusing, the new scaling agrees with numerical simulations beginning with amplitudes around only three times above of the initial pulse amplitude.



Figure 1: Dependence of *L* on  $z_c - z$  obtained from numerical simulations of the NLSE (solid lines) and from new analytical scaling (dashed lines) for different initial conditions. Each pair of closely spaced solid and dashed lines corresponds to the same initial condition. The curves are labeled by the optical power  $N = \int |\psi|^2 d\mathbf{r}$  normalized to the critical value  $N_c = 11.7008965...$  The inset shows L(z) starting from the beginning of the beam propagation. Dotted line shows the standard loglog scaling.

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# Lugiato-Lefever model for optical frequency comb generation based

### on monolithic whispering gallery mode resonators

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#### Abstract

We show that Kerr comb generation in whispering gallery mode resonators can be modeled by a variant of the Lugiato-Lefever equation that includes higher-order dispersion and nonlinearity. This model is equivalent to a modal expansion model that was previously derived, and these two models provide complementary information about the generation of Kerr combs. The modal expansion is most useful when the number of modes is small, while the Lugiato-Lefever equation is most useful when there are a large number of modes that interact cooperatively to produce wide-span Kerr combs. We also show that when the dispersion is anomalous, Kerr comb generation can arise as a consequence of Turing patterns leading to the formation of multiple free spectral range combs, or to the formation of dissipative cavity solitons, thus leading to wide-span combs with small pumping.

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### Modeling of frequency domain mode-locked fiber lasers

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#### Abstract

Frequency domain mode locked (FDML) fiber lasers is a type of wavelength sweep lasers in which the sweeping period of the narrowband scanning filter equals to the cavity round trip time [1]. As a result, the speed of the scanning filter can be much higher than conventional wavelength sweep lasers because the scanning speed is no longer limited by the laser buildup time. FDML fiber lasers find applications in optical coherency tomography and many other sensing systems. It is challenging to numerically simulate FDML fiber lasers because of the large time-bandwidth product,  $\sim 10^8$ . Recently, using the sweeping filter as the frame of reference, a theoretical model was derived to characterize the laser dynamics of FDML fiber lasers [2,3]. However the model remains computational intensive and cannot be used for analysis and optimization of the FDML fiber laser performance. We observe that further simplification of the model is possible because some of the physical effect such as dispersion, nonlinearity can be neglected in some physically important parameter regimes. Without the time-consuming Fourier transforms, the simulation speed can be increased by two order of magnitude. It is then possible to determine point spread functions which is important for the design and optimization of the FDML fiber laser systems. In this paper, we report a systematic study of the FDML fiber lasers in different parameter regime. We will investigate the impact of different physical parameters including the linewidth enhancement factor of the amplifier on the FDML fiber laser dynamics.

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### Linear and nonlinear frozen light in waveguides

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#### Abstract

We show theoretically that high-order stationary points (SPs), where  $\Delta \omega \propto \Delta k^m$  for m > 2 (see Fig. 1a), can be created in the dispersion relation of optical waveguides using a (near) periodic refractive index perturbation. Near such SPs light can be coupled efficiently into low and even zero group velocities modes, leading to high field strength ("frozen light"). Stationary solutions to the nonlinear coupled mode equations for these systems, including the effects of a Kerr nonlinearity, are discussed.

When low-intensity light is tuned in the bandgap near a regular, quadratic band edge the field decays exponentially and so the energy in the structure is low and most incident light is reflected. At high intensities, the (positive) nonlinearity shifts the band features to lower frequencies, changing the transmission [1]. Because the field inside the structure is weak, this requires high intensities. In contrast, near a high-order stationary SP the structure has strong evanescent modes [2,3], which allow for the high coupling efficiency. As a consequence, even in the linear limit, the field inside the structure is substantial, and so increasing the input power causes a much larger shift, reducing the switching threshold. We modeled the switching in such structures and find regimes where the switching is stable and does not exhibit a bistability (see Fig.1(b)). Higher input intensities lead to pulse trains due to modulation instability (Fig.1(c)) [4]. These structures have multiple input channels and their exploitation requires the relative amplitude and phase of the two channels to be carefully chosen.



Figure1: (a) Schematics of quadratic (blue), cubic (black) and quartic (red) stationary points. (b) Input and output intensities normalized to the nonlinearity versus time for a waveguide with quartic SP, showing stable all-optical switching. (c) Modulation instability in a quartic SP waveguide at moderately high intensities.

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# Nonlinearity engineering in photonics:

### from mode-locked lasers to nonlinear laser lithography

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#### Abstract

Emergent systems based on an interplay of nonlinear feedback mechanisms are ubiquitous in nature. Examples of such phenomena in photonics include mode-locking of lasers and a broad class of fractal optics, including self-similarity. We are interested in such phenomena in photonics since they often share important features with vastly different physical systems, including those that are notoriously difficult to experiment on. An excellent is example the rouge waves in photonics, which shed light on rogue ocean waves [1]. Here, we will present two very different physical systems, which nevertheless exhibit analogous dynamics, including self-similarity. The first system is a new femtosecond laser design that combines two distinct regimes of pulse propagation, soliton and similariton evolution, in a single cavity [2]. Both of these regimes are nonlinear attractors, showing that multiple nonlinear attractors can co-exist in a single cavity, if transitions between them can be arranged. The second system is self-organised formation of metal-oxide nanostructures under femtosecond laser irradiation. Laser-induced formation of micro- and nano-scale surface structures is almost as old as the history of the laser [3], but this technique has suffered to date from a stubborn lack of long-range order. We have formulated an approach through which we demonstrate unprecedented levels of uniformity (~1 nm) over indefinitely large areas (few-mm range) by simply scanning the laser beam over the surface [4]. Robustness, which refers to the ability of a system to preserve its core functionality in the presence of large perturbations, is thought to be an intrinsic property of complex dynamical systems. Both of these systems clearly exhibit robustness, as will be discussed. In a broader context, these are demonstrations of our approach, which we refer to as nonlinearity engineering, which aims to exploit complex nonlinear systems to achieve advanced technological functionalities that would be extremely difficult or even impossible to obtain from their linear counterparts.

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### Discrete spectrum of nonlinear modes in weakly nonlocal

### problems: a mechanism to emerge

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#### Abstract

We discuss a hypothesis on existence of a countable set of heteroclinic orbits connecting saddle-center points (also called embedded solitons" in some applications). In short, it can be described as follows. Let a system of differential equations depend on some external parameter " that defines a singular perturbation. Assume that the system has a heteroclinic orbit for  $\varepsilon = 0$  and that the corresponding solution can be analytically extended into upper complex half-plane with the closest to the real axis singularities given by a pair of points  $z = \pm a + ib$ . Then there is a countable set of heteroclinic orbits for the singularly perturbed system corresponding to the discrete set of values  $\varepsilon$ ,  $\varepsilon = \varepsilon_1, \varepsilon_2, \varepsilon_3, \cdots$  such that  $\varepsilon_n \to 0$  as  $n \to \infty$  and

$$\varepsilon_n \sim \frac{b}{\pi/2 + n\pi + \phi_0}$$

where  $\phi_0$  is a constant. We illustrate this statement by numerical results for several nonlinear and weakly nonlocal problems of various physical origin.

### Stability of solutions to a non-local Gross-Pitaevskii equation

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#### Abstract

The Gross-Pitaevskii equation is a common model in physics, but it only takes local interactions into account. This talk demonstrates the validity of using a nonlocal formulation as a generalization of the local model. A large class of nonlocalities and potentials is studied. We then establish the orbital stability of a class of parameter dependent solutions to the nonlocal problem. Numerical results corroborate the analytical stability results.

# Stability of water waves in the presence of surface tension

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### Abstract

The aim of the project is to examine the perturbative effect of surface tension on the slow-growing oscillatory instabilities that are present with emphasis of stability in finite depth and possible change in stability behaviour in shallow water, specifically on the bubble instabilities found by Deconinck and Oliveras [5]. Recently, Ablowitz, Fokas and Musslimani presented a reformulation of the surface water wave problem [1]. This formulation uses only surface variables, allowing for greatly improved computational efficiency. Building on previous work [2,4], we use this formulation to compute stationary periodic solutions of the one-dimensional problem, in the presence of small surface tension. The spectral stability of these solutions is examined, using Hill's method [3,4].

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# Localized modes in nonlocal nonlinear Schr odinger equation

# with PT-symmetric parabolic potential

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### Abstract

We consider localized solutions of the nonlinear Schödinger equation with nonlocal nonlinearity and the complex *PT*-symmetric potential  $x^2 - 2i\alpha x$ . We identify families of localized modes bifurcating from the eigenstates of the underlying linear problem and numerically extend the solutions in the region of strong nonlocal nonlinearity. We also examine stability of the found solutions and discuss interesting differences between properties of our model and cases with the real parapolic potential and local nonlinearity studied before [1]. Several types of the nonlocal nonlinearity will be addressed.

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# Applications of Painlevé functions to

### nonlinear wave equations

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### Abstract

The solution to a nonlinear wave equation with given Cauchy data often displays two or more qualitatively different behaviors in different space-time regions, such as having oscillatory and non-oscillatory zones. The boundaries between these regions may become well-defined in certain limits (such as long time or small dispersion), making it natural to consider the transition behavior between the two regions. It has recently been discovered that certain transition regions for solutions of nonlinear wave equations (such as KdV, focusing NLS, and Camassa-Holm) can be universally described for wide classes of initial conditions in terms of Painlevé functions [1,2,4]. These functions, which are solutions of nonlinear ordinary differential equations, play a role for nonlinear equations analagous to the role played by the classical special functions for linear equations. We will present our recent result with P. Miller [3] establishing Painlevé-type asymptotics in solutions of the semiclassical sine-Gordon equation.

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# Painlevé equations - nonlinear special functions

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### Abstract

In this talk I shall give an overview of the Painleve equations and discuss some of their properties. In particular I shall discuss some appl

# Solution of the generalized nonlinear Schrödinger equation in

# the semiclassical limit and Painlevé equations

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### Abstract

We study the solution of the Schrödinger equation in the semiclassical limit and consider the behaviour of the solution near the formation of singulariies of the equations describing the semiclassical limit. Such behaviour is described in terms of Painlevé transcendents.

### Solutions of ODEs with movable algebraic singularities

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#### Abstract

The main aspect of this work is to show that for certain classes of equations, all movable singularities of all solutions are at most algebraic branch points [1], extending a result in [2] on Liénard type equations. Around any movable singularity  $z_{\infty}$ , the solution can be represented by a Laurent series expansion with finite principle part in a fractional power of z- $z_{\infty}$ . This property is a weakening of the Painlevé property,under which all solutions of a differential equation are single-valued in the complex plane. Whereas the Painlevé equations are in some sense understood to be completely integrable, the equations considered here are in general non-integrable, it is however believed that a simple singularity structure indicates some nice properties of the equation. In fact, our method to prove that the solutions of the equations considered here have only algebraic movable singularities, originates in a certain type of proof that the Painlevé equations possess the Painlevé property [3,4]. The solutions of our equations extend, although locally only finitely-branched, over a Riemann surface with an infinite number of sheets. Another aspect of this work is to give conditions under which the equations admit solutions that are globally finitely-branched.

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### Solitary waves dispersive shock waves in

### colloidal media

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#### Abstract

The diffractive resolution of a discontinuity at the edge of an optical beam in a colloidal suspension of spherical dielectric nanoparticles by a collisionless shock, or dispersive shock wave, is studied. The interaction of the nanoparticles is modelled as a gas with the Carnahan-Starling formula used for hard-sphere the gas compressibility. The governing equation is a focusing nonlinear Schrödinger-type equation with an implicit nonlinearity. It is found that the discontinuity is resolved through the formation of a dispersive shock wave which forms before the eventual onset of modulational instability. A semi-analytical solution is developed in the (1+1) dimensional case by approximating the dispersive shock wave as a train of uniform solitary waves. A semi-analytical solution is also developed for a (2+1) dimensional circular dispersive shock wave for the case in which the radius of the bore is large. Depending on the value of the background packing fraction, three qualitatively different solitary wave amplitude versus jump height diagrams are possible. For large background packing fractions a single stable solution branch occurs. At moderate values an S-shaped response curve results, with multiple solution branches, while for small values the upper solution branch separates from the middle unstable branch. Hence, for low to moderate values of the background packing fraction the dispersive shock bifurcates from the low to the high power branch as the jump height, the height of the input beam's edge discontinuity, is increased. These multiple steady-state response diagrams, also typically found in combustion applications, are unusual in applications involving solitary waves. The predictions of the semi-analytical theory are found to be in excellent agreement with numerical solutions of the governing equations for both line and circular dispersive shock waves. The method used represents a new technique for obtaining semi-analytical results for a dispersive shock wave in a focusing medium.

### Wave fronts and soliton webs-exact solutions

### of the 2D Volterra system

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#### Abstract

Two dimensional periodic Volterra system (with the period N  $\geq$ 3) has an interesting and rather unusual set of exact soliton solutions. In particular, it has wave front solutions, which represent non-stationary interfaces between non-linear periodic waves of different periods. Another class of exact soliton solutions in the space-time domain looks like a web of solitons. We compare multi-soliton solutions of rank one with one soliton solutions of higher rank. The famous Kadomtsev-Petviashvili equation is a continuous limit (N  $\rightarrow\infty$ ) of the two dimensional Volterra system.

### Asymptotics of rational solutions of the inhomogeneous

# **Painlevé-II equations**

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#### Abstract

The inhomogeneous Painlevé-II equation with constant parameter  $\alpha$  has a unique rational solution exactly when  $\alpha$  is an integer. This talk concerns the asymptotic behavior of this rational solution in the limit of large  $|\alpha|$ . Numerical calculations of Clarkson and Mansfield have shown that the poles and zeros of these rational functions seem to be confined to a triangular region in the complex plane, forming a regular lattice pattern within this region. Using the representation of the rational solutions in terms of a matrix Riemann-Hilbert problem and applying the Deift-Zhou steepest descent method to the latter problem, we prove the existence of an asymptotically confining region for the poles and zeros, a region which is not exactly a triangle but is rather a curvilinear triangle with equal corner angles of  $2\pi/5$ . We also can give asymptotic formulae for the rational solutions in all regions of the complex plane: inside the region, outside the region, near one of the edges of the region, and near one of the corners of the region. Asymptotics in the interior are given in terms of elliptic functions, similar to the Boutroux ansatz that holds for general (non-rational) solutions of Painlevé-II for large arguments. Near the corners, the Hamiltonian of the tritronguée solution of the Painlevé-I equation describes the asymptotic behavior of the rational functions. This is joint work with R. Buckingham.

# Numerical calculation of finite random matrix statistics,

# and the onset of universality

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### Abstract

The presence of universality as the dimension of random matrices tends to infinity for unitary invariant (UI) ensembles (amongst others) has been well established. For example, the largest eigenvalue tends to the Tracy-Widom distribution for generic UI ensembles. However, the statistics can differ drastically for finite matrices. In this talk, we present an algorithm based on a corresponding Riemann-Hilbert formulation for calculating such finite dimensional statistics. This approach continues to work for non-generic UI ensembles, which do not follow the Tracy-Widom distribution.

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# **Riemann-Hilbert problems in Hardy spaces**

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### Abstract

We consider the (non)existence of solutions of the Riemann-Hilbert problem in Hardy spaces, and show how the behavior of the symbol near its zeros affects the existence of solutions. We mainly focus on the class of scalar-valued continuous symbols, but may also mention other classes of (matrix-valued) symbols. Some applications to spectral theory of Toeplitz operators and hydrodynamics will also be mentioned.

# Discretization of a nonlocal nonlinear wave equation

# preserving its variational structure

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### Abstract

We consider a nonlocal second order nonlinear wave equation, which describes discrete breathers (Feng-Doi-Kawahara, 2006). In this talk, we show that by slightly modifying the energy function, this equation can be formulated as a Hamiltonian PDE. Then we discuss its structure-preserving discretization, based on the variational structure.

# Structure-preserving discretizations for Ostrovsky-type

# nonlocal nonlinear wave equations

### Yuto Miyatake

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### Abstract

We consider numerical discretizations for the Ostrovsky equation, which was derived by Ostrovsky as a generalization of the well-known KdV equation. The Ostrovsky equation describes a lot of physical phenomena, and correspondingly contains several important sub-classes, which we refer to as the Ostrovsky-type equations. However, the numerical treatment of such equations is not straightforward due to a nonlocal operator. In this talk, we discuss structure-preserving discretizations of the equations based on their geometric structures.

# An attempt to design a fast and structure preserving

# scheme for Feng equation

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#### Abstract

We have studied how to design structure preserving schemes for partial differential equations and obtained some results about Feng equation. But the obtained scheme is fully implicit and it means that we have to use some nonlinear solver to obtain numerical solutions. To overcome this nonlinearity, linearization technique, which is a kind of relaxation of discrete approximation, may be available and we have tried to use it for Feng equation.

# **Development of a Hamiltonian-conserving combined compact**

# difference scheme for simulating CH equation at different

# initial conditions and investigating head-on collision of solitons

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### Abstract

In this paper a three-step scheme is proposed to solve the Camassa-Holm equation, which contains only the first-order derivative terms, in conservative form. Reduction of the differential equation order facilitates us to develop scheme in a stencilof comparatively fewer points. For accurately predicting the unidirectional propagation of shallow water wave, the modified equation analysis is conducted to eliminate several leading discretization error terms and the Fourier analysis is performed to minimize errors of the wave-like type. In this study, the three-point seventh-order spatially accurate combined compact upwind scheme is developed for the first-order derivative term. For the purpose of preserving Hamiltonian and multisymplectic geometric structures in Camassa-Holm equation, the time integrator should conserve symplecticity. One emphasis of this study of Camassa-Holm equation is to shed light on the conserved Hamiltonian property by the u- P-alpha formulation in simulating the peakon-antipeakon and soliton-anticuspon head-on collision problems. Another aim is to reveal the propagation wave natures of the Camassa-Holm equation subject to different initial wave profiles, whose peaks take the smooth, peakon, and cuspon forms. The transport phenomena for the cases with/without inclusion of the linear first-order advection term in the CH equation will be specifically addressed.

### Structure-preserving numerical integrators for

### peakon b-family equations

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#### Abstract

In this talk, we study the numerical methods for Degasperis{Procesi (DP) equation with two kind of solutions: peakon solution and shock peakon solution. As an integrable system, the DP equation has the bi-Hamiltonian structure. Based on the Hamiltonian formulation, we simulate the peakon solution by combining the symplectic method in time and Fourier pseudospectral method in space. Though the DP equation and the Camassa-Holm (CH) equation both belong to a family of dispersive nonlinear equations, they have the di\_erent physical nature: the DP equation has the shock peakon solution while the CH equation does not have. As the solution has the discontinuous property, we use the operator splitting technique presented by Feng and Liu (2009) in [1] to decompose the DP equation as two subsystems: Burgers' equation and the BBM equation. The resulting numerical discretization is derived by applying the WENO scheme of order 5, Euler mid-point rule in space and the corresponding structure-preserving method in time. The numerical comparison of the new derived method and the numerical method constructed in [1] is also provided.

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# Self-adaptive moving mesh methods for a class of nonlinear wave

### equations

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### Abstract

In the present talk, we will firstly construct integrable discretizations for several nonlinear wave equations with hodograph transformation such as the short pulse (SP) equation and the coupled short pulse equation etc. Then we will show that these integrable discretizations can be successfully used as self-adaptive moving mesh methods for the numerical simulation of these equations. Various numerical experiments including loop, breather and loop-breather interaction reveal very good results when compared with exact solutions. In the last, we generalize the proposed moving mesh method to a class of nonlinear wave equations irrespective of the integrability of the original nonlinear wave equation or its discrete analogue.

This is a joint work with Dr. Ohta at Kobe University and Dr. Maruno at the University of Texas-Pan American.

# The generalizations of the peakon's systems

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#### Abstract

We present three different methods of the generalization of the peakon's systems. The first based on the generalizations of the hamiltonian structures of the Camassa - Holm and Degasperis-Procesi equation. The next one uses the Lax representation for the generalization while in the last we consider the higher order non-linearity in the Pearson's equations. We present two different vectorial generalization of the cubic non-linearity in the peakon system.

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### Calogero-type models with maximally extended

#### superconformal symmetry

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#### Abstract

We overcome the barrier of constructing N = 4 superconformal models in one space dimension for more than three particles. The  $D(2,1;\alpha)$  superalgebra of our systems is realized on the coordinates and momenta of the particles, their superpartners and one complex pair of harmonic variables. The models are determined by two prepotentials, F and U, which must obey the WDVV and a Killing-type equation plus homogeneity conditions. We investigate permutation-symmetric solutions, with and without translation invariance. Models based on deformed  $A_n$  and  $BCD_n$  root systems are constructed for any value of alpha, and exceptional  $F_n$ -type and super root systems admit solutions as well. Translation-invariant mechanics occurs for any number of particles at  $\alpha = -1/2$ (osp(4|2)) invariance as a degenerate limit) and for four particles at arbitrary alpha (three series).

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# Hamiltonian flows from Bäcklund transformations: an

# application to the Ablowitz-Ladik hierarchy.

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### Abstract

There exists a fairly large number of papers explicitly or implicitly related to the Bäcklund transformations for the Ablowitz-Ladik model. Usually they are presented as local implicit maps [1], [2]. Under certain conditions, for lattice models, is however possible to obtain explicit parametric transformations. This observation leads to the identification of a Hamiltonian flow defined by the transformations themselves: these indeed represent the integral curves of a non-autonomous Hamiltonian system of equations, the time being represented by the parameter associated to the "spectrality property" of the maps [3]. Furthermore this flow possesses the same conserved quantities as the original lattice model and allows to find an explicit formula for the Hamiltonian interpolating the ow given by iterations of the Bäcklund maps. As an application, one can obtain an exact discretization for the Ablowitz-Ladik hierarchy.In this talk we will discuss the previous points, referring both to analytic and numerical results.

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# On the (non)removability of spectral parameters in $Z_2$ -graded

### zero-curvature representations and its applications

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### Abstract

We present our alternative solution to P. Mathieu's (1989) Gardner deformation problem for the integrable N=2 supersymmetric a=4 Korteweg-de Vries equation. Our method is based on the switch from a representation of Lie superalgebras in spaces of square matrices to a representation of those Lie superalgebras in term of vector fields. Then we apply homological techniques[1,2] for deformations of nonlocal structures over partial differential equations. The new approach can be successfully applied to solution of similar deformation problems for other integrable supersymmetric models. Besides, we generalise a standard method for inspecting the (non)removability of spectral parameters in zero-curvature representations to the case of  $Z_2$ -graded partial differential equations. In particular, we illustrate a link between deformation techniques for two types of flat structures over such  $Z_2$ -graded equations, namely, their zero-curvature representations and construction of their parametric families by using Frölicher--Nijenhuis' bracket.

This talk is based on two recent papers [3,4].

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# Euler equations related to the (generalized) Neveu-Schwarz algebra

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### Abstract

In this talk, we report some results about Euler equations related to the (generalized) Neveu-Schwarz algebra, including supersymmetric Euler equations and bisuperhamiltonian Euler equations.

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# Super extension of Bell polynomials with applications to

### supersymmetric equations

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#### Abstract

We generalize classical Bell polynomials into a super version, which are found to be effective in systematically constructing super bilinear representation, bilinear Backlund transformation, Lax pair and infinite conservation laws of supersymmetric equations. We take supersymmetric KdV equation and supersymmetric sine-Gordon equation to illustrate this procedure.

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### Partial difference schemes and commuting operators on the lattice

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#### Abstract

After discussing the construction of difference schemes for ordinary difference equations we present the problems connected with the construction of partial difference schemes. In particular we discuss the discrete analog of the Clairaut-Schwarz-Young theorem and its consequences.

# Nonlocal symmetries, conservation laws and related reduction

# solutions of integrable systems

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#### Abstract

Infinitely many nonlocal conservation laws related to nonlocal symmetries are explicitly given for many integrable continuous and discrete systems with helps of infinitely coupled Lax pairs. The nonlocal conservation laws and symmetries of the original systems can be changed to local ones for suitable prolonged systems.

### Peakons and stability of the modified $\mu$ -Camassa-Holm equation

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#### Abstract

In this talk, we study dynamical stability of periodic peaked solitons for the modified  $\mu$ -Camassa- Holm equation with the cubic nonlinearity. The equation is a  $\mu$ -version of the modified Camassa-Holm equation and is integrable with the Lax-pair and bi-Hamiltonian structure. The equation admits the periodic peakons. It is shown that the periodic peakons are dynamically stable under small perturbations in the energy space. This is a joint work with Y. Liu, X.C. Liu and Y. Zhang.

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# Bilinear identity for an extended KP hierarchy

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#### Abstract

In this talk, we construct the bilinear equations for an extended Kadomtsev-Petviashvili (KP) hierarchy (introduced by the authors [1]). By introducing an auxiliary parameter, whose flow corresponds to the so-called squared eigenfunction symmetry of KP hierarchy, we find the tau-function for this extended KP hierarchy. It is shown that the bilinear equations generate all the Hirota's bilinear equations for the zero-curvature forms of the extended KP hierarchy, which includes two types of KP equation with self-consistent sources (KPSCS) [2]. It seems that the Hirota's bilinear equations obtained in this paper for KPSCS are in a simpler form by comparing with the existing results. (This is a joint work with Xiaojun Liu and Yunbo Zeng [3].)

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### Integrable twisted hierarchies with D2 symmetries

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#### Abstract

Two new integrable twisted hierarchies with D2 symmetries are constructed via the loop algebra factorization method. The splitting type factorization yields the generalized sinh-Gordon equation, this result justifies some far-reaching generalizations of the well-known connection between the sine-Gordon equation, the Backlund transformation, and surfaces with curvature -1. The non-splitting type factorization yields the Gerdjikov-Mikhailov-Valchev equation which is an anisotropic multicomponent generalization of the classical Heisenberg ferromagnetic equation and is one of the simplest twisted integrable systems.

Special analytical features in the associated inverse scattering theory are discussed to solve the Cauchy problem of these twisted ows.

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# Solitons and dynamic properties of the coupled

### Semidiscrete Hirota equation

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#### Abstract

In this talk, we will give the coupled bright-bright and the coupled dark-dark soliton solutions for the integrable coupled Hirota equation (ICDHE) using bilinear method. These solutions have both singular and nonsingular behaviors depending upon the choice of parameters. The distinguishing feature of the ICDHE is that the singular solution can possess some nonsingular characteristics in the discrete space at some time. The asymptotic behavior of these coupled bright-bright and coupled dark-dark solitons is discussed in order to explain their collision properties. This is a joint work with Dr. Hai-qiong Zhao.

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# A maple package for generating one-dimensional optimal

# systems of finite dimensional Lie algebra

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### Abstract

We present a Maple computer algebra package ONEOptimal which can calculate onedimensional optimal system of finite dimensional Lie algebra for nonlinear equations automatically based on Olver's theory. The core of this theory is viewing the Killing form of the Lie algebra as an invariant for the adjoint representation. Some examples are given to illustrate the validity of the program.
# Integrable semi-discretizations for the short wave limit of the

## **Degasperis-Procesi equation**

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### Abstract

Based on our previous work to the short wave limit of the Degasperis-Procesi equation [1], we construct its integrable semi-discretizations. Since the short wave limit of the Degasperis-Procesi equation have two forms which are derived from two different bilinear equations, two semi-discrete analogues are constructed possessing the same N-loop soliton solution. A relation between these two versions of semi-discretizations is also clarified.

This is a joint work with K. Maruno at UTPA and Y. Ohta at Kobe University of Japan.

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## Behavior of Hamiltonian structures under supersymmetric

## reciprocal transformations

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### Abstract

For a Hamiltonian system, the behavior of the Hamiltonian structures under variable transformations is an interesting problem. In the case of changing dependent variables, the formula is well known. A less known result, due to Peter Olver, is the general case involving change variables of both dependent and independent variables. In this talk, we will generalize Olver's theorem to the supersymmetric case and furthermore show that it is very useful in the study of supersymmetric systems.

### Lattice solitons in cubic-saturable media with external potentials

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#### Abstract

In this talk, I will discuss the existence of lattice solitons supported by cubic-saturable nonlinerarity, in the framework of nonlinear Schrödinger equation with external potentials such as periodic, Penrose and PT symmetric lattices by using the pseudospectral renormalization method. The idea behind the spectral renormalization method is to transform the governing equation into Fourier space and find a nonlinear nonlocal integral equation coupled to an algebraic equation and determine a convergence factor based upon the degree (homogeneity) of a single nonlinear term. The convergence factor can not found explicitly from the governing equation for saturable case by the use of the spectral renormalization method . In order to find the convergence factor has to be used the root finding code such as the Newton method but if we use the pseudo-spectral renormalization method the convergence parameter is found from the governing equation explicitly. The pseudo-specral renormalization method can efficiently be applied to a large class of problems including higher order nonlinear terms with different homogeneities. In first part of my talk, I will present the pseudo-spectral renormalization method for cubic-saturable nonlinearity. The propagationn of light beams along the z- axis of the medium composed from alternating domains with cubic and saturable nonlinearities is described by nonlinear Schrödinger equation (NLSE) with the external potential

$$i\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{|u|^2 u}{1 + \alpha |u|^2} + V(x, y)u = 0$$

Saturation parameter  $\alpha = 0$  and  $\alpha = 1$  correspond to the cubic domain and the saturable domain respectively. I will consider two different configuration. In the second part of my talk, I will investigate the linear and nonlinear stability properties of the lattice solitons centered on the maximum of the periodic, Penrose and PT symmetric lattices in two different configuration. It is found that for the periodic, Penrose and PT symmetric lattices the solitons suffer collapse in the cubic domain. I will discuss the stability properties of the lattice solitons in the saturable domain with the external potentials such as periodic, Penrose and PT symmetric lattices.

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## Vortex and dipole solitons in defect lattices

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### Abstract

In this talk, band gap structures and nonlinear stability properties of localized optical solitons that arise in the solution of the nonlinear Schrödinger (NLS) equation with irregular lattice-type potentials will be presented.

The governing equation for the physical model is defined as the nonlinear Schrödinger (NLS) equation with an external potential in Eq.(1)

$$iu_z + \Delta u + |u|^2 u - V(x, y)u = 0.$$
 (1)

In optics, u(x, y, z) corresponds to the complex-valued, slowly varying amplitude of the electric field in the xy plane propagating in the z direction,  $\Delta u \equiv u_{xx} + u_{yy}$  corresponds to diffraction, the cubic term in u originates from the nonlinear (Kerr) change of the refractive index and V(x, y) is an external optical potential that can be written as the intensity of a sum of N phase-modulated plane waves,

$$V(x,y) = \frac{V_0}{N^2} \left| \sum_{n=0}^{N-1} e^{i\vec{k}_n \cdot \vec{x} + i\theta_n(x,y)} \right|^2$$
(2)

where  $V_0 > 0$  is the peak depth of the potential, i.e.,  $V_0 = max_{x,y}V(x,y)$ ,  $\vec{x} = (x,y)$ ,  $\vec{k}_n$  is

a wave vector defined by  $(k_x^n, k_y^n) = [K\cos(2\pi n/N), K\sin(2\pi n/N)]$ , and  $\theta(x, y)$  is a phase

function through which irregularities are introduced.

As external potential, we consider a lattice with an edge dislocation and a lattice with a vacancy defect. These lattices obtained from Eq. (2), and given by Eq. (3) and Eq. (4) respectively.

$$V(x,y) = \frac{V_0}{25} [2\cos(k_x x + \theta(x,y)) + 2\cos(k_y y) + 1]^2$$
(3)

with the phase-dislocation function  $\theta(x, y) = \frac{3\pi}{2} - tan^{-1}(\frac{y}{x})$ .

$$V(x,y) = \frac{V_0}{25} \left| 2\cos(k_x x) + 2\cos(k_y y) + e^{i\theta(x,y)} \right|^2$$
(4)

with the phase function  $\theta(x,y) = tan^{-1}(\frac{y-y_0}{x}) - tan^{-1}(\frac{y+y_0}{x})$ .

In this study, solution of the NLS equation with an external potential is obtained by using the spectral renormalization method.

In the final part of the presentation, effect of the defects on the first nonlinear band gap structures, and the nonlinear stability properties of the dipole and vortex solitons in defect lattices will be discussed.

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### Solitons and wave collapse in nonlocal nonlinear Schrödinger

### type equations

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#### Abstract

In this talk, fundamental solitons in nonlocal nonlinear Schrödinger (NLS) systems with external potentials (lattices) possessing crystal and quasicrystal (e.g. Penrose) structures will be presented.

Such systems can be written in the following non-dimensional form,

$$iu_{z} + \frac{1}{2}(u_{xx} + u_{yy}) + |u|^{2}u - \rho\phi u - V(x, y)u = 0$$
  
$$\phi_{xx} + \nu\phi_{yy} = (|u|^{2})_{xx}$$
(1)

where u is the normalized amplitude of the envelope of the optical beam and  $\phi$  is the normalized static field,  $\rho$  is the coupling constant which comes from the combined optical rectification - electro optic effect,  $\nu$  is the asymmetry parameter comes from the anisotropy of the material and V(x, y) is an external optical potential that can be written as the intensity of a sum of N phase-modulated plane waves,

$$V(x,y) = \frac{V_0}{N^2} \left| \sum_{n=0}^{N-1} e^{i(k_x x + k_y y)} \right|^2$$
(2)

where  $V_0 > 0$  is constant and corresponds to the peak depth of the potential.

In this study, the fundamental solitons of NLSM system with the periodic and quasicrystal lattices are computed by using the spectral renormalization method. In current literature, in (2+1)D, solitons of NLSM systems (without external potential) and solitons of NLS equation (with or without external potentials) are shown to suffer from wave collapse. Therefore, understanding/arresting collapse in such systems is an important challenge.

In the second part of my talk, I will discuss the nonlinear stability properties of the optical modes (solitons) of NLSM systems for a parameter space, and focus on the effect of the external potential as a collapse arrest mechanism.

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# Super-focusing and mirroring of Airy pulses propagating in fibers

## with third order dispersion

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### Abstract

Truncated Airy pulses [1-3] were launched in an optical fiber close to its zero dispersion point allowing the effect of third order dispersion (TOD) to play a dominant role in the dynamics of pulses. When the truncated Airy pulse propagates in the presence of pure quadratic dispersion it lives for a finite distance until it reaches its divergence area. However when the pulse dynamics is governed by a pure TOD with positive sign, we found a somehow surprising result. The pulse reaches the focal point fast, then undergoes a mirror transformation and continues to propagate accelerated in the opposite direction. At the focal point all the pulse energy is concentrated in a very short temporal slot, featuring a very interesting pulse compression technique. Such a transformation was not observed when the sign of the TOD was negative [4]. When both dispersion terms act on the pulse, the focal point extends to a finite area of truncated Airy pulse "non-existence". The size of the area depends on the relative strength of the third order dispersion term with respect to its second order counterpart. After this area, the pulse reemerges again being mirror-transformed and continues its evolution. Such a propagation behavior is also realizable under the action of negative TOD by reversing the initial acceleration direction of the Airy pulses. For each value of pulse width there is a minimal value corresponding to the ratio between the third and second order dispersion strengths starting from which we can observe the mirror effect. Below this value the pulse vanishes. In a strongly nonlinear regime we expect to observe soliton shedding out of the Airy pulse structure [5], where the frequency of the ejected soliton is controlled by the TOD strength.

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# Newton cradle in optics. Understanding the fission of higher-order

# solitons under the action of the higher-order dispersion

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### Abstract

The fission of N-solitons fission is a key mechanism leading to supercontinuum generation (SC) and creation of ultra-compressed pulses and solitons featuring strong frequency tuning [1-3]. Surprisingly, the mechanism per se was not studied in detail, beyond the approach treating higher-order dispersive and nonlinear terms as small perturbations to the nonlinear Schroedinger equation (NLS) [4]. This approach has little validity when these terms, in particular, the higher-order dispersion (HOD), dominate the dynamics of femtosecond pulses. We demonstrate that a universal scenario of the HOD-induced fission of higher-order solitons into chains of the fundamental ones proceeds via formation of the soliton Newton's cradle (NC) [5]. After the compression of the initial N-soliton into a chain of fundamental quasi-solitons and dispersion waves, the tallest soliton runs along the chain through consecutive collisions with other solitons, and then escapes, while the remaining pulse peaks stay in a bound state. Detailed spectral analysis demonstrated that at high values of the dominant third-order dispersion (TOD), the pulse chain mostly resides in the normal-dispersion domain, while, with the decrease of the TOD, the chain shifts towards the anomalous dispersion. At higher values of N, the soliton-NC scenario recurs several times. After the release of the first soliton, the cradle retains enough power and momentum to push additional solitons all the way through the chain and eject them, which is accompanied by strong emission of radiation, resulting in a broadband supercontinuum. The mechanism reported is valid in simulations, including dispersive terms up to the eight's order, the Raman and shock terms, thus proving its universality.

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## Finite size effects in wave packet scattering off a dispersion band gap.

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### Abstract

The wave vector of a wave packet propagating in a periodic medium subject to an external force follows the band dispersion until it reaches the gap. Then it can either reflect and perform Bloch oscillations or make a Zener transition to the neighboring band [1,2]. The reflection and transition amplitudes can be approximated by the Landau-Zener formula, which depends only on the bandgap and the wave vector acceleration, because it is based on the universal quantum normal form of a Hermitian operator in the neighborhood of a conical crossings. While the Landau-Zener formula is exact only in the non-physical limit of infinitely slow forcing and vanishing gap, Colin de Verdiere [3] has shown that for a smooth dispersion the quantum normal form can be constructed to arbitrary order in the gap size.

Using quantum normal theory I will derive the leading finite-speed corrections to the transition amplitudes, showing that they are completely determined by the local behavior of the dispersion, and express them in terms of the curvature of the band dispersion at its extremum. I will use numerical analysis to estimate the domain of applicability of the quantum normal form transformation and the validity of the asymptotic expansion of the transition amplitudes.

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### Nonlinear pulse propagation in optical fibers with randomly varying

### birefringence

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#### Abstract

The Manakov PMD equation was derived from the Maxwell equations to study light propagation over long distances in optical fibers with randomly varying birefringence [1]. We denote by  $L_B$  the beat length and  $l_c$  the correlation length. The lengths  $l_{nl}$  and  $l_d$ are related respectively to the Kerr effect and to the chromatic dispersion. Considering the following regime  $L_B \ll l_c \ll l_{nl} \sim l_d$ , we introduce a dimensionless parameter  $\epsilon > 0$ , given by the ratio of these lengths. The slowly varying envelope X has then the following evolution

$$izX(z) + \frac{ib'}{\epsilon} \left(\nu(z)\right) tX(z) + \frac{d_0}{2} 2tX(z) + \nu(z)X(z) = 0,$$
(1)

where  $\nu$  is a stochastic process,  $d_0$  is the group velocity dispersion, b' is the frequency derivative of the birefringence strength b and  $\nu(t)X(t)$  is a cubic nonlinearity. In this talk, I will explain why the asymptotic dynamics (when  $\epsilon$  goes to 0) is described by a stochastic perturbation of the Manakov equation [2,3]

$$i\partial_z X(z) + \frac{d_0}{2} 2t X(z) + F(X(z)) + i\sqrt{\gamma} \sum_{k=1}^3 \sigma_k t X(z) \dot{\xi}_k(z) = 0.$$
<sup>(2)</sup>

The positive constant  $\gamma$  is a small positive parameter given by the physics of the problem,  $\dot{\xi} = (\dot{\xi}_1, \dot{\xi}_2, \dot{\xi}_3)$  is a white noise in time. The nonlinear interaction term in Equation (2) is given by  $F(X(z)) = \frac{8}{9} |X|^2 X(z)$  and the nonlinear PMD effects have been averaged out to zero. I will also display numerical simulations of soliton's propagation subject to PMD and statistics of the PMD.

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## Solitons of cubic-quintic nonlinear Schrödinger equation with various

### external potentials

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#### Abstract

Solitons are nonlinear wave structures that are widely present in nature. In nonlinear optics, localized solitary waves are usually called solitons and their existence in homogeneous and/or periodic systems is shown experimentally. In recent years, optical lattice solitons in Kerr media are deeply analyzed and it is shown that they suffer from collapse due to self-focusing or diffraction.

In this study, the existence and stability of optical solitons on periodic and certain type of quasicrystal lattices (Penrose -5 and Penrose -7) are investigated. The governing equation for the physical model that has been used is the cubic-quintic nonlinear Schrödinger equation (CQNLS) with an external potential in (2+1)D space:

$$iu_z + u_{xx} + u_{yy} + \alpha u|u|^2 + \beta u|u|^4 + Vu = 0.$$
<sup>(1)</sup>

The solution to the CQNLS equation with an external potential is obtained by spectral methods. The investigated potentials are of the following form:

$$V_N(x,y) = \frac{V_0}{N^2} \left| \sum_{n=0}^{N-1} e^{i(x\cos\frac{2\pi n}{N} + y\cos\frac{2\pi n}{N})} \right|^2.$$
 (2)

As the external potential V, periodic and quasicrystal lattices corresponding to N = 4, N = 5 and N = 7 are considered. In particular, quasicrystals for N = 5 and N = 7 are called Penrose type lattices.

For the numerical solution of Eq.(1), a Fourier iteration method, namely the spectral renormalization method is employed.

In this work, the numerical existence of fundamental solitons on the periodic and quasicrystal lattices is shown and the band-gap structures are found for varying parameters  $\alpha$ ,  $\beta$  and  $V_0$ .

Next, the nonlinear stability properties of the solitons are investigated and the effect of the external potential as a collapse arrest mechanism is discussed.

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## Wave propagations in periodically curved waveguide arrays

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### Abstract

Arrays of coupled waveguides which can effectively discretize the light propagation and in turns give rise to many interesting phenomena have been considered as a powerful tool to control the fundamental properties of light propagation and allow people to study many useful optical analogies with other fields, such as the physics of solid state. In particular, light propagation in photonic lattices can be effectively controlled by applying periodical modulations, resembling the motion of electrons in a crystalline within an external potential. Thus, by investigating the light propagation in modulated photonic lattices, people can study a variety of phenomena which have been proposed originally for quantum electron system but difficult to be monitored. Here, we study light propagation in photonic lattices with periodically curved waveguides modelled by parametrically driven coupled nonlinear Schrodinger equations. For two periodically curved waveguides, we investigate the dynamics of coupled wavefields in a periodically oscillating double-well potential which is analogous to horizontally shaken Bose-Einstein condensates in a double-well magnetic trap. Specially, we study the persistence of equilibrium states of the undriven system due to the presence of the parametric drive and find that such parametric drives may stabilize or destabilize the continuations of equilibrium time-independent states that are respectively unstable or stable in the undriven case. For many periodically curved waveguide arrays, we show that discrete surface solitons persist, but the threshold power to support their existence is altered by the drive. There are critical drives at which the threshold values vanish. We also show that parametric drives can create resonance with a phonon making a new barrier for discrete solitons. By calculating the corresponding Floquet multipliers, we find that the stability of symmetric and antisymmetric off-side discrete surface solitons switches approximately at the critical drives when the threshold values vanish.

## Optical solitary waves in thermal media with non-symmetric

## boundary conditions

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### Abstract

In this talk, optical spatial solitary waves are considered in a nonlocal thermal focusing medium with non-symmetric boundary conditions. The governing equations consist of a nonlinear Schrodinger equation for the light beam and a Poisson equation for the temperature of the medium. Three numerical methods are investigated for calculating the ground and excited solitary wave solutions of the coupled system. It is found that the Newton conjugate gradient method is the most computationally efficient and versatile numerical technique. The solutions show that by varying the ambient temperature, the solitary wave is deected towards the warmer boundary. Solitary wave stability is also examined both theoretically and numerically, via power versus propagation constant curves and numerical simulations of the governing partial differential equations. Both the ground and excited state solitary waves are found to be stable. The Newton conjugate gradient method should also prove extremely useful for calculating solitary waves of other related optical systems, which support nonlocal spatial solitary waves, such as nematic liquid crystals.

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### Domain walls and dissipative solitons in polariton systems

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#### Abstract

The formation and the dynamics of dissipative domain walls and solitons in semiconductor microcavities with coherent optical pump are considered for the case when strong coupling between the excitons and the photons generates hybrid light-matter excitations called polaritons. To describe the dynamics of the photons and the excitons in mean field approximation we use the standard mathematical model consisting of two coupled equations for the photon E and the exciton  $\psi$  fields

$$\partial_t E - i(\partial_x^2 + \partial_y^2)E + (\gamma_1 - i\Delta_1)E = i\psi + E_p \tag{1}$$

$$\partial_t \psi - i\sigma (\partial_x^2 + \partial_y^2)\psi + (\gamma_2 - i\Delta_2 + i|\psi|^2)\psi = iE,$$
(2)

where  $\gamma_1$  and  $\gamma_2$  are the losses for the photon and the exciton subsystems,  $\Delta_1$  and  $\Delta_2$  are the detunings of the photon and exciton resonances from the pump frequency,  $E_p$  is the pump filed,  $\sigma$  is the diffraction coefficient for the exciton field.

The waves in the linearized system (1)-(2) can be described by the dispersion characteristics having two branches. In recent years a number of papers were published reporting dissipative structures nestling on the lower polariton mode [1]-[3]. In this talk we address a new kind of optical domain walls having two very different characteristic scales. The motion of the kinks, their pinning on inhomogeneities and their mutual interaction are discussed. The formation and the stability of the bound states of the domain walls (dissipative solitons) are also considered.

Special attention is paid to two-dimensional case. It is shown that for some parameters the axially symmetric dissipative solitons are linearly stable. It is also demonstrated by direct numerical simulation that the development of the radial and azimuthal instabilities can result in the formation of very interesting objects like dissipative breathers.

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# Inter-soliton interactions mediated by dispersive waves

# in optical fibers

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### Abstract

In this talk we discuss how quasi-linear waves bouncing between optical soliotons in fibers with high order dispersion can lead to strong interaction between the solitons at the distances much greater than the characteristic width of the solitons. This effect was first reported in [1], however no theory explaining the effect was suggested.

To describe the dynamics of the light we use the standard approach based on the generalized nonlinear Schrodinger equation with high order dispersion. We show that the dispersive waves can get trapped between the solitons due to the cascaded resonant four-wave mixing [2] of the dispersive waves and the solitons. Under certain conditions the resonant scattering results in nearly perfect reflection of the dispersive waves off the solitons and so a pair of the solitons can form a soliton Fabry-Perot-like resonator. It is important to notice that each act of the reflection of the dispersive waves affects the parameters of the solitons [2, 3] resulting in the effective attraction between them. In its turn the change of the soliton frequencies leads to the modification of the spectrum of the trapped dispersive waves. In the present talk we develop the theory of the phenomenon and discuss the analogy between this effect and Casimir force. The importance of the reported effect for optical supercontinuum generation is also discussed.

AVY acknowledges support of the FCT grant PTDC/FIS/112624/2009 (Portugal). **References:** 

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## Study of the scattering problem of Camassa-Holm equation

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### Abstract

Similar to KdV equation, the solutions of Camassa-Holm (CH) equation can be constructed by the direct and inverse scattering problem through the isospectral problem of the corresponding Lax pair. The scattering data can be used to derive the expressions of the long time asymptotic behaviors of solutions of CH equation. In this talk we consider some special initial conditions and then the corresponding scattering data can be obtained. The explicit relations between the initial conditions and scattering data can be found.

# The apprpximate method and its application for the AKNS-type

# linear scattering problem by descritizing the initial wave packet

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## Abstract

The Zakharov–Shabat eigenvalue problem for the AKNS form[1]

$$\begin{pmatrix} \Psi_{1x} \\ \Psi_{2x} \end{pmatrix} = \begin{pmatrix} \xi & q(x,t) \\ r(x,t) & -\xi \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$
(1)

is a universal framework to describe many soliton equations which appear in various fields of physics. These equations are integrable, however, the analysis of their initial value problems beyond formal solutions is hopeless. For general initial pulses that deviate from pure solitons, it is at least desirable to obtain some asymptotic  $(t \rightarrow \infty)$  information of the solution. Recently, we have found an asymptotic method for this problem, in which we divide the initial wave packet into many small intervals and consider the transfer matrices connecting each of them. We can extract the asymptotic information of the wave packet starting from arbitrary initial conditions. We have also applied our method to the nonlinear Schrödinger equation and found the non monotonic increase of the number of asymptotic solitons as a function of initial amplitude for double box type initial conditions. Moreover, using the Darboux transformation, we analytically show the class of initial profile in which the asymptotic solitons share the same amplitude and width. The possibility of classifying the set of initial conditions into equivalence classes represented by each asymptotic solitons is addressed.

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# Integrable discretisations of the nonlinear Schrödinger equation

# on Grassmann algebras

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## Abstract

Integrable discretisations for a class of coupled nonlinear Schrödinger (NLS) type of equations are presented. The class corresponds to a Lax operator with entries in a Grassmann algebra.

Elementary Darboux transformations are constructed. As a result, Grassmann generalisations of the Toda lattice and the NLS dressing chain are obtained.

Furthermore, the compatibility (Bianchi commutativity) of these Darboux transformations leads to integrable Grassmamm generalisations of the difference Toda and NLS equations.

The resulting discrete systems will have Lax pairs provided by the set of two consistent Darboux transformations. The corresponding Bäcklund transformations represent symmetries of the discrete (difference systems) and formal diagonalisations of the Darboux transformations provide generating functions of integrals of motion.

# Lattice modified KdV hierarchy from a Lax pair expansion

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### Abstract

A hierarchy of integrable equations is produced by systematically adding terms to the Lax pair for the lattice modified KdV equation. The equations in the hierarchy are related to one another by recursion relations. These recursion relations are solved explicitly so that every equation in the hierarchy along with its Lax pair is known.

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## Long-time asymptotics for the defocusing

### integrable discrete nonlinear Schrödinger equation

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#### Abstract

We study the long-time behavior of the defocusing integrable discrete nonlinear Schrödinger equation introduced by Ablowitz-Ladik on the doubly infinite lattice (i.e.  $n \in \mathbb{Z}$ )

$$i\frac{d}{dt}R_n + (R_{n+1} - 2R_n + R_{n-1}) - |R_n|^2(R_{n+1} + R_{n-1}) = 0.$$
(1)

It is an integrable discretization of

$$u_t + u_{xx} - 2u|u|^2 = 0.$$
<sup>(2)</sup>

It is known that (1) is the compatibility condition of the following AKNS pair:

$$X_{n+1} = \begin{bmatrix} z & R_n \\ R_n & z^{-1} \end{bmatrix} X_n,$$
  
$$\frac{d}{dt} X_n = \begin{bmatrix} iR_{n-1}\bar{R}_n - \frac{i}{2}(z-z^{-1})^2 & -i(z\bar{R}_n - z^{-1}\bar{R}_{n-1}) \\ i(z^{-1}R_n - zR_{n-1}) & -iR_n\bar{R}_{n-1} + \frac{i}{2}(z-z^{-1})^2 \end{bmatrix} X_n.$$

We have obtained the long-time asymptotics of (1) by using the nonlinear steepest descent method of Deift-Zhou. Roughly speaking, the result is as follows. If |n/t| < 2, there exist  $p_j = p_j(n/t), q_j = q_j(n/t) \in \mathbf{R}$  and  $C_j = C_j(n/t) \in \mathbf{C}$  (j = 1, 2) such that

$$R_n(t) = \sum_{j=1}^2 C_j t^{-1/2} e^{-i(p_j t + q_j \log t)} + O(t^{-1} \log t) \quad \text{as } t \to \infty.$$
(3)

The quantities  $q_j$  and  $C_j$  are defined in terms of *the reflection coefficient* that is associated with the potential  $\{R_n(0)\}_n$ . Each term in the sum exhibits the behavior of *decaying oscillation* of order  $t^{-1/2}$ . Notice that in the case of (2) the asymptotic behavior is expressed by a single term.

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## Rogue wave solutions of a three-component coupled

# nonlinear Schrödinger equation

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### Abstract

In this talk, we investigate rogue wave solutions in a three-component coupled nonlinear Schrödinger equation. With the certain requirements on the backgrounds of components, we construct a new multi-rogue wave solution that exhibits a structure like a four-petaled flower in temporal-spatial distribution, in contrast to the eye-shaped structure in one-component or two-component systems. The results could be of interest in such diverse fields as Bose-Einstein condensate, nonlinear fibers and super fluid.

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C3: Genaral aspects of nonlinear waves

## Nonlineariry and constrained quantum motion

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#### Abstract

A general prescription for the treatment of constrained quantum motion, leading to nonlinear quantum dynamics of various types, is outlined. As an extension of the formalism, the dynamical equation satisfied by the general density matrix when a quantum system is subjected to one or more constraints arising from conserved quantities is derived. The resulting nonlinear evolution of the density matrix has the property that it is independent of the specific composition of the pure - state mixture generating the initial state of the system. As an illustrative example, evolution of a quantum state that is constrained to a coherent - state subspace of the quantum state space is worked out.

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# Nonlinear effects on the focussing of tsunami due to

## underwater lenses

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### Abstract

In [1] the linear focussing of tsunami by underwater lenses was considered using diffraction theory. This proposed that under appropriate conditions underwater lenses could increase the intensity of tsunami three-fold. In this talk we consider the nonlinear propagation towards underwater lenses in the context of the variable coefficient Kadomtsev-Petviashvili (vKP) equation derived in [2]. Using the linear version of this equation we are able to construct equivalent asymptotic expressions for the focussing which qualitatively agree with [1], although with significantly smaller increase in the intensity. Numerical simulations of the nonlinear propagation then demonstrate that the focussing can be eliminated due to the acceleration of solitary waves over the underwater lenses. Furthermore, the vKP equation enables the downstream corrections due to finite width of the lenses to be investigated.

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## Numerical generation of traveling wave profiles with the

# Petviashvili method

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### Abstract

The purpose of this talk is the analysis of the Petviashvili method as an iterative technique for the numerical resolution of a system of nonlinear equations. The origin of the method is in [3], to generate lump solitary wave profiles of the Kadomtsev-Petviashvili I (KPI) equation, as an alternative technique to the fixed point algorithm, which is divergent. From this original work, some studies of convergence of the method have been made, concerning particular cases of its main application, which is the numerical approximation of traveling wave profiles or ground states in nonlinear dispersive systems [1,2,4]. In this talk, some more general convergence results are derived and applied to the discretization of several systems for traveling wave generation. Acceleration techniques are also discussed.

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## Autoresonant wave dynamics in Vlasov-Poisson systems

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### Abstract

Autoresonance is an important nonlinear phenomenon, where a driven system is captured into resonance and stays phase-locked continuously despite variation of system's parameters. Since the first use in particle accelerators, new studies of autoresonance emerged recently in a broad range of fields in physics, few examples being plasmas, nonlinear optics, planetary dynamics, and superconducting Josephson junctions (see [1] for a short review). While the transition to autoresonance in an oscillatory one degree of freedom nonlinear systems is well understood, there remain many unsolved questions in autoresonant many degrees of freedom systems, the main issues being the thermal spread and collective phenomena. In this context, resonant phase space dynamics and space charge effects will be analyzed in chirped frequency driven, charged ensembles of particles (plasmas) described by the Vlasov-Poisson system of equations for particle distributions and self-electric fields. It will be shown that different nontrivial coherent nonlinear structures in this system can be conveniently formed and controlled by slow chirped frequency perturbations. In particular, for a range of parameters one can excite large amplitude fluid-type waves. In the small wave vector limit these waves are described by the KdV equation, so the aforementioned fluid modes comprise a realistic generalization of autoresonant KdV solutions [2,3]. In addition, by reversing the direction of variation of the driving frequency, one can form dissipationless nonlinear kinetic excitations (Bernstein-Green-Kruscal modes) associated with the holes in particles phase space. A simplest problem of this type can be dealt with by modeling the particle distributions via the water bag model. This case allows analysis of the autoresonant Vlasov-Poisson dynamics via the Whithams variation principle [4]. More general particle distributions can be treated within the multi-water bag model. Formation of different autoresonant coherent structures in the system will be illustrated in numerical simulations and the associated theory will be discussed.

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# Modulation theory for the steady fKdVB equation- constructing

## periodic solutions

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### Abstract

The forced Korteweg-de Vries (fKdV) equation describes near resonance, weakly non-linear and weakly dispersive long waves propagating in various mediums with external forcing. With the addition of a linear damping term, it becomes the fKdV-Burgers (fKdVB) equation. We present a multiple-scale perturbation technique for obtaining asymptotic solutions to the steady fKdVB equation. The first order solution in the perturbation hierarchy is the modulated cnoidal wave equation. From the second order equation in the hierarchy, we find a system of odes governing the modulation of the properties of the cnoidal wave. Using this, we then construct periodic solutions and examine the stability of these solutions.

## Directional flow of solitons with asymmetric potential wells:

## Soliton diode

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## Abstract

We study the flow of bright solitons through two asymmetric potential wells. The scattering of a soliton by certain type of single potential wells, e.g., Gaussian or Rosen-Morse, is distinguished by a critical velocity above which solitons can transmit almost completely and below which solitons can reflect nearly perfectly. For two such wells in series with certain parameter combinations, we find that there is an appreciable velocity range for which solitons can propagate in one direction only. Our study shows that this directional propagation or diode behavior is due to a combined effect of the sharp transition in the transport coefficients at the critical velocity and a slight reduction in the center-of-mass speed of the soliton while it travels across a potential well.

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# Legendre pseudospectral method for solving three-dimensional

## non-linear hyperbolic partial diffential equations

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### Abstract

In this talk, numerical solutions of three-space non-linear hyperbolic partial differential equations will be presented by using Legendre pseudospectral method. The discretization of the spatial derivatives of the problem have been solved by using Legendre pseudospectral method. A system of non-linear ordinary differential equations is generated. The values of unknown function *u* can be found by using kronecker product. The representation of this kind of product can easily be extended to higher dimensions. The numerical results are obtained and compare with exact solutions to validate the high precision of the Legendre pseudospectral method.

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# A dispersively accurate compact finite difference method for

# the Degasperis-Procesi equation

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### Abstract

In this paper we are aimed to solve the non-dissipative Degasperis-Procesi equation based on the u - P formulation. To resolve the computational difficulty at the wave crest where the first-order derivative may diverge and the shockpeakon solution may form, the first-order spatial derivative term in the two-step equations will be approximated in a conservative form. The resulting equations will be approximated by the symplecticity-preserving time-stepping scheme and the spatial discretization scheme that can optimize the numerical wavenumber for the first-order spatial derivative term. This scheme will be developed in a three-point grid stencil with the accuracy order of seventh within the combined compact finite difference framework. Besides the validation of numerical accuracy, we will in particular address the discrete conservation of Hamiltonians even when peakon collides with antipeakon and generates, as a result, a shockpeakon. We will also demonstrate the capability of applying the proposed numerical method to sharply resolve some important features of the third-order dispersive DP equation.

**Keywords:**non-dissipative;Degasperis-Procesiequation;shockpeakon; symplecticity-preserving; Conservation of Hamiltonians.

# Two-dimensional vector field visualization Of Gursey instantons

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### Abstract

It is known that instantons are classically topological solitons. The spinor type instanton solutions are found in four-dimensional conformally invariant pure spinor Gursey model

[1] with nonlinear  $(\overline{\psi}\psi)^{\frac{4}{3}}$  self-coupled spinor term by the spontaneous symmetry breaking of the conformal invariance of  $\psi$  spinor field, i.e.  $\langle 0 | \overline{\psi}\psi | 0 \rangle \neq 0$  [2]. In this work, we examine the vector fields of model to provide a better understanding the dynamic of spinor type Gursey instantons in phase.

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# Generation of ultra-compressed solitons with a high tunable

## wavelength shift in Raman-inactive hollow-core photonic

## crystal fibers

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### Abstract

The fission of N-solitons is a key mechanism leading to the supercontinuum generation and creation of hyper-compressed pulses and solitons featuring strong wavelength tuning [1, 2]. Recent advances in manufacturing PCFs filled with Raman-inactive gases [3] provide a strong motivation to focus the studies on the fission driven by the third-order dispersion (TOD) [4], and potential applications of this setting to photonics. In the absence of the Raman-induced self-frequency shift and the Raman-associated noise, fission-produced strongly compressed solitons, once generated, may propagate keeping constant internal frequencies. A higher-order N-soliton,  $u = N\sqrt{P_0} sech(T/T_0)$  is launched into the fiber, where  $T_0$  and  $P_0$  are width and peak power of the corresponding fundamental soliton. If the TOD dispersion  $\beta_3$  is very small, it can be considered as a perturbation added to the second-order dispersion,  $\beta_2$ . In this case, the peak power of each fundamental soliton emerging after the splitting of the N-soliton is given by the classical result,  $P_i = P_0(2N - 2j + 1)^2$  [5]. If  $\delta_3$  is larger, it leads to a significant increase in the largest fundamental-soliton's peak power and compression degree, along with the increase of the wavelength shift. However, further increase of  $\beta_3$  leads to a loss of the peak-power enhancement. For optimal pulse compression and wavelength conversion, universal optimal value of TOD strength parameter  $\delta_3 \equiv \beta_3/(6\beta_2 t_0)$  was found. This optimal value is valid for any pulse duration, second- and third-order dispersion coefficients, depending solely on order N of the injected soliton . The optimal pulse-compression degree significantly exceeds the well-known analytical prediction [5]. On the contrary to the insensitivity of the solitons' peak powers to the signs of  $\delta_3$ , the wavelength upshift and downshift significantly differ for the opposite signs of  $\delta_3$ . The downshift of strongest solitons wavelength is opposed by the proximity to the zero dispersion point of PCF, beyond which solitons cannot propagate. The highest order in Fig.1 is N =15. For still larger N, one can achieve even higher peak-power ratio and wavelength shift, but, due to interaction between multiple generated solitons and trapped dispersive waves [6, 7], the control over parameters of the emerging solitons deteriorates. The physical mechanism beyond the phenomenon, which is valid also in the presence of the self-steepening effect, is the power and momentum absorption by

the strongest newly born soliton in the course of inelastic interactions with weaker pulses after the decomposition of the initial N-soliton [4,8].

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# On axisymmetrical boundary problem of unsteady

# motion of micropolar fluid in the half-space

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### Abstract

The Newtonian relationships cannot be characterized to explain the behaviour of materials in shear. Eringen [1] proposed the basics of the theory of micropolar fluids display the effects of couple stresses, body couples and local rotary inertia. This theory might serve as a satisfactory model for describing the flow properties of polymeric fluids, liquid crystals which are made up of dumbbell molecules, animal blood and fluids containing certain additives.

El-Sirafy [2] generalized the results of the solution of the homogeneous Navier-Stokes equations in the half-plane for the slow motion of viscous incompressible fluids to the class of the micropolar fluids for the case of the given shear stresses on the boundary.

The aim of this paper is developing an exact solution for the problem of axisymmetrical flow of unsteady micropolar fluid in the upper half-space  $y \ge 0 | t > 0$  when the shear stresses are given while each of normal velocity and microrotation vanishes on the boundary. We assume that the components of the vector of velocity and microrotation vanish initially and also at large distance from the boundary of the upper half-space. The Laplace-Hankel transform technique is used to solve this problem. Some physical quantities such as velocities, microrotation and pressure are obtained in a closed form and illustrated numerically as a function of y at different values of time. This problem could be met in the study of the flow near a boundary which is not wet by the given fluid.

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# Deformations of higher Peregrine breathers, multi-rogue waves and

# their analytical expressions.

Pierre Gaillard

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### Abstract

Here we show how to extend the results of recent articles [1], [2] which were dealing with two parametric ( quadratic ) deformations of higher order Peregrine breathers. The strong point of these works was an easy way to describe explicitly not only higher Peregrine breathers but also their quadratic deformations. Here, we show how to extend an approach of [1], [2] in order to get the generic rational deformations of higher order Peregrine breathers thus providing alternative approach to the construction of generic multi rogue waves solutions of NLS equation.

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# Superregular solitonic solutions in nonlinear schrödinger equation

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### Abstract

It is known since 1971 that the Nonlinear Schrödinger (NLSE) is a system completely integrable by the Inverse Scattering Method (ISTM). The NLSE has a simple solution, the monochromatic wave with frequency depended on amplitude - the condensate. The condensate is unstable with respect to modulation instability. The first solitonic solution of NLSE in the presense of condensate was fount in 1977 by E.A. Kuznetsov. This is a standing soliton which oscillate in time. Since this moment different types of solitonic solutions were found such as Akhmediev breather and Peregrine soliton. Now general *N*-solitonc solution is known.

In terms of ISTM solitonic solution is characterized by location of poles in the plane of spectral parameter. In the case of condensate the plane has a cut from -*A* to *A*, where *A* is the condensate amplitude. This fact essentially complicate properties of solitonic solutions thus not all soliton's "species" are studied still. There are also important question: what is a nonlinear stage of modulation instability? In spatial dimension D = 2, 3, the answer is known - modulation instability leads to formation of finite time singularities - collapses. In dimension D = 1 collapses are forbidden. However in this case development of modulation instability leads to formation of "extreme" (rogue, freak) waves where energy density exceeds the mean level by order of magnitude.

We study solitonic solutions of the focusing NLSE in the presence of a condensate by using the dressing method. We find a general *N*-solitonic solution and separate a special designated class of "regular solitonic solutions" that do not disturb phases of the condensate at infinity by coordinate. All regular solitonic solutions can be treated as localized perturbations of the condensate. If we assume that the modulation instability develops from localized perturbation, only regular solution can be used as model for its nonlinear behavior.

The central result of our work is following. We find an important class of "superregular solitonic solutions" which are small perturbations at certain a moment of time. Then they develop into pairs of different solitons. This describes the nonlinear stage of the modulation instability of the condensate and can be treated as a sort of "integrable turbulence" appearing as a result of nonlinear development of the modulation instability.

## Non-paraxial traveling solitary waves

## in layered nonlinear media

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### Abstract

The dominant underlying model governing Solitary Wave (SW) formation and propagation in spatially inhomogeneous structures is the NonLinear Schrödinger (NLS) equation with spatially varying coefficients. Although this model is analytically tractable, it is a result of an important assumption with far-reaching consequences. The NLS equation is derived from the Maxwell's equations under the paraxial approximation, that restricts drastically the domain of applicability of the NLS equation excluding various applications where either the dimensions of the inhomogeneous structures are small in comparison to the wavelength or the angle of incidence is large. Although there exist a huge number of studies on NLS soliton formation and dynamics in inhomogeneous media, the investigation of the respective phenomena and the exploration of novel features of SW dynamics, is yet to be followed up in the non-paraxial regime. [1,2,3]

For the case of layered media, the original Maxwell's equations, under no approximation, lead to a scalar NonLinear Helmholtz (NLH) equation with coefficients that are piecewise constant functions of the transverse coordinate and an intensity-dependent refractive index, when the electric field is assumed to be monochromatic and linearly polarized. The NLH equation has fundamental mathematical differences with NLS, resulting in the description of qualitatively different phenomena of wave propagation. In this work, we derive a large class of exact analytical traveling SW solutions in a variety of inhomogeneous structures consisting of linear and nonlinear layers. The solutions are related to a spatial resonance condition and describe reflectionless and radiationless SW propagation for arbitrary angles and spatial widths in the nonparaxial regime. The generality of the results facilitates their experimental observation of the solutions in planar dielectric structures having the form of finite or infinite waveguide arrays for layer dimensions ranging from several wavelengths to sub-wavelength. Finally, these solutions are applicable and provide physical intuition for the formation of traveling SW in layered media occurring in other branches of physics beyond nonlinear optics.

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## Block algebra in two-component BKP and D type

# **Drinfeld-Sokolov hierarchies**

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#### Abstract

In this talk, we construct generalized additional symmetries of a two-component BKP hierarchy defined by two pseudo-differential Lax operators. These additional symmetry flows form a Block type algebra with some modified (or additional) terms because of a B type reduction condition of this integrable hierarchy. Further we show that the D type Drinfeld-Sokolov hierarchy, which is a reduction of the two-component BKP hierarchy, possess a complete Block type additional symmetry algebra. That D type Drinfeld-Sokolov hierarchy has a similar algebraic structure as the bigraded Toda hierarchy which is a differential-discrete integrable system.

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# Rogue waves of the Hirota and the Maxwell-Bloch equations

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## Abstract

In this poster, we derive a Darboux transformation of the Hirota and the Maxwell-Bloch(H-MB) system which is governed by femtosecond pulse propagation through an erbium doped fibre and further generalize it to the matrix form of the *n*-fold Darboux transformation of this system. This *n*-fold Darboux transformation implies the determinant representation of *n*-th new solutions of  $(E^{[n]}; p^{[n]}; \eta^{[n]})$  generated from known solution of  $(E, p, \eta)$ . The determinant representation of  $(E^{[n]}; p^{[n]}; \eta^{[n]})$  provides soliton solutions, positon solutions, and breather solutions (both bright and dark breathers) of the H-MB system. From the breather solutions, we also construct bright and dark rogue wave solutions for the H-MB system, which is currently one of the hottest topics in mathematics and physics. Surprisingly, the rogue wave solution for *p* and  $\eta$  has two peaks because of the order of the numerator and denominator of them. Meanwhile, after fixing time and spatial parameters and changing other two unknown parameters  $\alpha$  and  $\beta$ , we generate a rogue wave shape for the first time.

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# Virasoro type algebraic structure and the analyticity of the

# constrained discrete KP hierarchy

## Maohua Li

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#### Abstract

In this talk, the algebraic structure and the analyticity of the constrained discrete KP hierarchy will be analyzed. We construct the additional symmetries of one-component constrained discrete KP (cdKP) hierarchy, and then prove that the algebraic structure of the symmetry flows is the positive half of Virasoro algebra. The gauge transformation of the constrained discrete KP hierarchy is constructed explicitly by the suitable choice of the generating functions. Under the *m*-step successive gauge transformation  $T_m$ , we give the transformed (adjoint) eigenfunctions and the  $\tau$ -function of the transformed Lax operator of the cdKP hierarchy. We also analyze the Wronskian solution of the eigenfunction of the cdKP hierarchy.

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# Matter-wave solitons supported by field-induced dipole-dipole

# repulsion with a spatially modulated strength

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## Abstract

In this talk, we demonstrate the existence of one and two-dimensional (1D and 2D) *bright* solitons in the Bose-Einstein condensate (BEC) with *repulsive* dipole-dipole interactions (DDIs) induced by a polarizing field, oriented perpendicular to the plane in which the BEC is trapped, whose strength grows from the center to periphery. Accordingly, the 2D setting is isotropic. These systems support stable 1D and 2D fundamental solitons, twisted solitons in 1D, and solitary vortices in 2D. Scaling properties of the soliton families are explained in an analytical form. The Thomas-Fermi approximation (TFA) is elaborated too, for fundamental solitons. The mobility of the solitons is limited to a vicinity of the central point. The setting with a 1D double-well modulation function is considered too. Stable even and odd solitons are found in it, along with regimes of Josephson oscillations.

# (2+1)-dimensional analytical solutions of the combining cubi-quintic

# nonlinear Schrödinger equation

## Ji Lin Department of physics, Zhejiang Normal University, China

## Abstract

We investigate analytical solutions of the (2+1)-dimensional combining cubic-quintic nonlinear Schrödinger (CQNLS) equation by the classical Lie group symmetry method. We not only obtain the Lie-point symmetries and some (1+1)-dimensional partial differential systems, but also derive bright solitons, dark solitons, kink or anti-kink solutions and the localized instanton solution.

# The supersymmetric Burgers equation: Bosonization and

# exact solution

Bo Ren<sup>1</sup>, Xiao-Nan Gao<sup>2</sup>, Jun Yu<sup>1</sup>

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#### Abstract

Using bosonization approach, the *N*=1 supersymmetry Burgers system is changed to a system of coupled bosonic equations. And the difficulties caused by intractable anticommuting fermionic fields can be effectively avoided with the approach. The traveling wave solutions and the similarity reduction solutions of the model are obtained with the mapping and deformation method and the Lie point symmetry theory, respectively. Besides, the richness of the localized excitations of the supersymmetric integrable system is discovered.

# Search of chaos in Bose-Einstein condansate in tilted

# bichromatic potential

## Eren Tosyali and Fatma Aydogmus Department of Physics, Istanbul University, Istanbul, Turkey email: erentosyali@gmail.com

#### Abstract

Many theoretical studies have been performed on nonlinear properties in Bose-Einstein Condansate (BEC) for different optical lattice potential [1,2,3]. Very recently, supper lattice potential has been fulfilled chaotic behavior in BEC [4,5]. In this study, we present the dynamics of 1D Gross-Pitaevskii equation (GPE) for tilted bichromatic potential

 $(V(x) = V_1 Cos^2(w_1 x) + V_2 Cos^2(w_2 x) + Fx)$ . We show that density of flow (J) affects

behavior of BEC for different potential depths. In addition, for regular case with a number of density of flow under the bichromatic potential exhibits similar behavior with Wainner Stark lattice potential [2]. Finally we investigate the existence of chaos with different parameter values for our system.

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## Darboux transformation of the Sasa-Satsuma equation: New solitons

## and resonant interaction

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#### Abstract

The Sasa-Satsuma equation (SSE) is an integrable version of the higher-order nonlinear Schrödinger model which contains the third-order dispersion, self-steepening and stimulated Raman scattering terms. In this paper, we construct the N-th iterated Darboux transformation of the SSE. With the plane-wave solution as the seed, we obtain two families of "bright"-type soliton solutions on a nonzero background, and a family of resonant soliton solutions with three arms. In addition, we derive a family of rogue wave solutions with some specific value of the spectral parameter. We hope that the resonant phenomenon of bright solitons will be observed in the femtosecond fiber experiment.

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# Bosonization of the supersymmetric Ito equation

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## Abstract

Based on the bosonization approach, the N =1supersymmetric Ito system is changed to a system of coupled bosonic equations. The approach can effectively avoid difficulties caused by intractable fermionic fields whichare anticommuting. By solving the coupled bosonic equations, the traveling wave solutions of the slto system are obtained with the mapping and deformation method. Some novel types of exact solutions for the supersymmetric system are constructed with the solutions and symmetries ofthe usual Ito equation. In the meanwhile, the similarity reduction solutions of the model are also studied with the Lie point symmetry theory.

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Matsuo, T.	MS10	S			
Matveev, V. B.	MS2			Yamane,H.	C2
Mei,C. C.	MS3	Santini,P.	MS1	Yan,Z. Y.	MS2
Mei,F. D.	MS4	Scott,D.	MS6	Yang, J. K.	MS2
Menyuk,C.	MS7	Sergeeva,A.	MS2,3	Yu,C. H.	C3
Mikhailov, A. V.	MS9,K	Sheu,T.	MS10	Yu,J.	Р
Miller,P.	MS1,9	Shi,Z. W.	MS6	Yulin,A.	C1
Miyatake,Y.	MS10	Situ,G H.	MS1		
Modotto,D.	MS4	Slunyaev,A.	MS2		
Mostafazadeh,A.	MS6	Smirnov,A.O.	MS2	Z	
		Sommer,A.	MS5		
		Sterke,M.	MS7		
Ν		Su,Y.	MS5		

Zeng,H. P.	MS4	Zhu,Y.	MS6	Zuo,D. F.	MS11
Zezyulin,D.	MS6,8	Zhu,Z. N.	MS11		
Zhai,H.	MS5	Zuelicke,U.	MS5		
Zhao,L. C.	C2	Zullo,F.	MS11		



**:北京世纪金源香山商旅酒店** Fragrant Hill Empark Hotel, Beijing, China 北京市海淀区北正黄旗 59 号 North Yellow Flag No.59, Haidian district, Beijing

The red 'A' is the location of the hotel.

The black ring is the location of the Capital International Airport.

We suggest conference participants take taxi to the hotel. It takes about one hour from the airport to the hotel and the taxi fee is about 120 Yuan.

# 北京世纪金源香山商旅酒店

(Fragrant Hill Empark Hotel)



# 首都机场至北京世纪金源香山商旅酒店:

方案一:从首都机场乘出租车至北京世纪金源香山商旅酒店(酒店位 于香山公园东门外),打车费用120元。

Plan A: Take a taxi to the Fragrant Hill Empark Hotel (The hotel is located outside the east gate of the Fragrant Hills Park) from the Beijing Capital International Airport ,the taxi charges RMB120.

方案二:从首都机场成机场快轨至东直门地铁站,换乘地铁2号线(西 直门方向),在西直门站换乘地铁4号线至地铁北宫门站C口出,乘 公交563或696至香山公园站下车,步行300米即到。 Plan B: From the Beijing Capital International Airport express to Dongzhimen subway station,take Metro Line 2(towards Xizhimen),change to the Metro Line 4 in Xizhimen subway station to the beigongmen station exit C,take bus 563 or 696 to Fragrant Hill Park Station and get off,walk 300 meters to the Fragrant Hill Empark Hotel.

### 南苑机场至北京世纪金源香山商旅酒店:

从南苑机场乘出租车至北京世纪金源香山商旅酒店(酒店位于香山公园东门外),打车费用120元。

Take a taxi to the Fragrant Hill Empark Hotel (The hotel is located outside the east gate of the Fragrant Hills Park)from the Nanyuan airport, the taxi charges RMB120.

## 北京站至北京世纪金源香山商旅酒店:

方案一:从北京站乘出租车至北京世纪金源香山商旅酒店(酒店位于香山公园东门外),打车费用78元。

Plan A: Take a taxi to the Fragrant Hill Empark Hotel(The hotel is located outside the east gate of the Fragrant Hills Park) from the Beijing Railway Station ,the taxi charges RMB78.

方案二:从北京站乘地铁2号线,在西直门站换乘地铁4号线至地铁北 宫门站C口出,乘公交563或696至香山公园站下车,步行300米即到。

Plan B: From Beijing Railway Station take the subway Line 2, transfer to Metro Line 4 at Xizhimen Station to Beigongmen station exit C, take bus 563 or 696 to the Fragrant Hills Park Station, walk 300 meters to the Fragrant Hill Empark Hotel.

北京北站至北京世纪金源香山商旅酒店:

方案一:从北京北站乘出租车至北京世纪金源香山商旅酒店(酒店位 于香山公园东门外),打车费费用大约44元。

Plan A: Take a taxi to the Fragrant Hill Empark Hotel(The hotel is located outside the east gate of the Fragrant Hills Park) from the Beijing North Railway Station ,the taxi charges RMB44.

方案二:在西直门地铁站乘地铁4号线至地铁北宫门站C口出,乘公交 563或696至香山公园站下车,步行300米即到。

Plan B: Take subway in Xizhimen subway station subway line 4 to the Metro Beigongmen station exit C, take bus 563 or 696 to the Fragrant Hills Park Station, walk 300 meters to the Fragrant Hill Empark Hotel.

北京西站至北京世纪金源香山商旅酒店:

方案一:在北京西站乘出租车至北京世纪金源香山商旅酒店(酒店位 于香山公园东门外),打车费用大约50元。

Plan A: Take a taxi to Fragrant Hill Empark Hotel(The hotel is located outside the east gate of the Fragrant Hills Park)from the Beijing West Railway Station, taxi costs about RMB50.

方案二: 在北京西站地铁站乘坐9号线至国家图书馆站, 换乘地铁4 号线至地铁北宫门站C口出, 乘公交563或696至香山公园站下车, 步 行300米即到。 Plan B: On Beijing West Railway Station subway station, take Line 9 to the National Library station, transfer to Metro Line 4 to Subway Beigongmen station exit C, take the bus 563 or 696 to the Fragrant Hills Park Station, walk 300 meters to the Fragrant Hill Empark Hotel.

酒店联系人: 王连峰(Spyker) 13311258188

酒店总机: 010-59898888