Scientific Achievements of Professor Yu-Hong Dai

Prof. Dai works on numerical methods for optimization with applications. He has made outstanding contributions on nonlinear optimization, eigenvalue problems, integer programming, and applied optimization. He has already published one monograph and over 130 research papers; many of his papers were appeared in international journals such as *Math. Program.*, *SIAM J. Optim.*, *Numer. Math.*, *Math. of Comp.*, *IMA J. Numer. Anal.*, *SIAM J. Matrix Anal.*, *SIAM J. Imaging Sci.* and *IEEE T. Signal Proces.*

As one of the leading experts on nonlinear optimization in the world, Prof. Dai is well known for his systematical studies on the convergence properties of classical nonlinear conjugate gradient methods, convergence properties of general nonlinear conjugate gradient methods and construction of nonlinear conjugate gradient methods for nonconvex optimization. Dai and Yuan(1999,2001) were the first to show that the standard Wolfe line search is sufficient for the global convergence of conjugate gradient methods. Dai(2011) provided sufficient and necessary conditions for the global convergence of general conjugate gradient methods. Remarkably, jointly with Prof. Ya-xiang Yuan, he proposed a new nonlinear conjugate gradient method, which is widely called the Dai-Yuan method by the community and is regarded as one of the four leading nonlinear conjugate gradient methods (the other three are classical methods given by Fletcher-Reeves(1964), Polak-Ribiére-Polyak(1969) and Hestenes-Stiefel(1952). These authors are very famous; for example, Hestenes and Stiefel once gave 45-minute and 1-hour invited talks on ICM1954, respectively). The Dai-Yuan method has received more than five hundreds sci citations and was regarded one of "most successful recent methods"; "possess attractive theoretical and computational properties" (see the survey in *Acta Numerica* (2005) by Gould, Orban and Toint and the book *Numerical Optimization* (2006) by Nocedal and Wright, respectively). The Dai-Yuan method has also been used in various softwares including SEISCOPE (see *Geophysics, 81:2, F11-F25, 2016*) and MANOPT (see *Expert Systems with Applications, 90, 127-145, 2017*). By making the conjugate gradient direction closest to some quasi-Newton direction, Dai and his Ph.D. student Kou (2013) proposed an efficient nonlinear conjugate gradient scheme, which is being utilized in the two state-of-the-art conjugate gradient solvers, CGOPT by themselves and CG_DESCENT by Hager and Zhang.

His researches on quasi-Newton methods are very impressive. The BFGS method is generally believed to be the most efficient quasi-Newton method. However, it is not known whether the BFGS method with the standard Wolfe line search converges for general nonconvex functions. This open problem was regarded as "one of the most fundamental questions in the theory of unconstrained optimization" (see Nocedal (1991)); "a challenging and intriguing question remained open along these forty years" (see Mascarenhas (2004)). After hard working, and encouraged by M.J.D. Powell, Dai (2002, 2013) gave a negative answer to the open problem. Specifically, Dai (2013) constructed a perfect example for the nonconvergence of BFGS, where the objective function is a polynomial of order 38, the unit stepsize is always taken and satisfies various line search conditions, and each line search function is convex (although the objective function is not itself). As commented by the articles in *Mathematical Programming* (2004, 2014), "Finally, in [1], Yu-Hong Dai showed that the BFGS method may fail..."; "In [4] you will find the end result of skillful and hard work."

Prof. Dai is also a leading expert on spectral gradient methods and famous for the convergence analysis of spectral gradient methods, construction of nonmonotone and monotone gradient methods for smooth optimization, extension of spectral gradient methods for stochastic optimization and other gradient methods. The Barzilai-Borwein (BB) method is a simple and nonmonotone spectral gradient method, which is significantly faster than the classical steepest descent method by Cauchy (1847). Dai and Liao (2002) established the R-linear convergence of the BB method, which was regarded as "another recent theoretical development" by Roger Fletcher (2005). Then Roger Fletcher invited Dai to visit Dundee for one year. They further proved that the BB method is likely to be R-superlinearly convergent for 3-dimensional quadratic functions and the cyclic steepest descent method can be R-superlinearly convergent in the general n-dimensional case provided that the cyclic time $m \geq \frac{n+1}{2}$ (see Dai and Fletcher (2005)). They also proposed a new projected spectral gradient method for support vector machine (see Dai and Fletcher (2006)), which was called as the Dai-Fletcher method by Zanni (2006) and "has shown a better convergence rate and, consequently, a significant time reduction (over the BB method)" in his software parallel PGDT. Together with Yuan, Prof. Dai proposed the Dai-Yuan gradient method, the first monotone gradient method which can beat the BB method for quadratic optimization. Together with collaborators, he gave the first successful try in (2016) to extend the spectral gradient method for stochastic optimization, a hot topic in machine learning.

For eigenvalue problems, together with his Ph.D. students, Prof. Dai proposed the first unconstrained optimization model for calculating a part of extreme eigenvalues of a symmetric matrix independently (see Jiang, Cui and Dai (2014)). He developed the first algorithm which can calculate all real eigenvalues of a supersymmetric tensor (see Cui, Dai and Nie (2014)). This is a highly-cited work. For constrained optimization, jointly with Prof. Liu, he proposed a new primal-dual interior-point relaxation method in 2020, which does not require any primal or dual iterates to be interior-points and hence is prominently different from the existing interior-point methods.

For integer programming, together with his Ph.D. student Weikun Chen, Prof. Dai negatively answered the open question proposed by Nemhauser et al. (1999) by giving an elegant proof for the NP-hardness of computing generally sequentially lifting cover inequalities for the knapsack polytope. A full introduction of this work has been given in the AOR paper by Marc Pfetsch et al. (2020). Recently, Dai also provided a positive answer to the unsolved problem proposed by Wolsey and Yaman (2016) by providing an efficient polynomial-time combinatorial algorithm for their knapsack separation problems. Prof. Dai's group developed the CMIP integer programming solver in 2018, which narrowed the gap in integer programming between China and the world to much extent. Meanwhile, they have successfully solved many difficult practical problems such as rocket trajectory control, AGV dock distribution, base station location and natural gas transportation, and won the only 2018 OR Application Prize awarded by the OR Society of China.

For applied optimization, jointly with collaborators, he proposed efficient convex quadratic semidefinite programming models in estimating the fiber orientation distribution in 2013. Together with his former Ph.D. student Yafeng Liu and others, he worked on fundamental optimization problems for SISO, MISO and MIMO interference channels in wireless communication and published a good number of papers in *Mathematical Programming* and *IEEE T. on Signal Proces.* Their works have also been well cited by tens of IEEE fellows. Specifically, the work where they completely described the computational complexity of the max-min fairness linear transceiver design problem for the multi-user MIMO interference channel and proposed an efficient and convergent algorithm, received Best Paper Award of the ICC2011, a flagship conference of IEEE on Signal Processing.

He has been invited to give talks in many conferences, including Plenary Talk at The 24th International Symposium on Mathematical Programming (ISMP2021; Dai will be the first Chinese Plenary speaker on this series of ISMP), Semi-Plenary talk at the International Conference on Continuous Optimization (ICCOPT2016), Plenary talk at Joint Meeting of American Mathematical Society and Chinese Mathematical Society, Plenary talk at the XII Brazilian Workshop on Continuous Optimization(Foz Do Iguacu, 2018), Memorial Talk at EUROPT Workshop on Advances in Continuous Optimization (Canada, 2017), Plenary talk at Annual Meeting of China Society for Computational Mathematics (Shenyang, 2015), and Plenary talk at Annual Meeting of Operations Society of China (Guangzhou, 2012). In 2013, he was invited as a leading expert to review the nonlinear conjugate gradient method for the Wiley Encyclopedia of ORMS. In 2011, he won Best Paper Award of the International Conference on Communications (ICC2011).

He received Second Grade of National Natural Science Award of China (2006) with Prof. Yuan, which is a remarkable honor. He also won numerous important awards named after famous scientists, including Shiing-Shen Chern Mathematics Prize (2017), Feng Kang Scientific Computing Prize (2015), Xiao Shutie Applied Mathematics Prize (2018) and Zhong Jiaqing Mathematics Prize (1998), and other national honors, including Leading Talents in Science and Technology Innovation (2017), Chinese Natural Science Fund for Distinguished Young Scholars (2011) and Science and Technology Award for Chinese Youths (2007). In 2014, he received the title of Feng Kang Chair Professor in Academy of Mathematics and Systems Science, Chinese Academy of Sciences.