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中国科学院数学与系统科学研究院
Academy of Mathematics and Systems Science
Chinese Academy of Sciences

随机偏微分方程保结构算法
青年研讨会

中国科学院数学与系统科学研究院

2021 年 4 月 9 日 – 4 月 11 日

会议指南

1. 会议方式

腾讯会议

日期	腾讯会议 ID	会议密码
4 月 9 日 （周五）	569 222 639	210409
4 月 10 日（周六）	604 166 912	210410
4 月 11 日（周日）	445 670 955	210411

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3. 友情提示

✧会议期间听众请尽量静音，如有问题可在聊天室留言或报告结束后提问。

✧报告时间 35 分钟，包含提问时间 5 分钟。

资助单位

❉ 国家自然科学基金数学天元基金项目---天元数学高级研讨班项目（No. 12026428）

❉ 中国科学院数学与系统科学研究院

❉ 计算数学与科学工程计算研究所

❉ 科学与工程计算国家重点实验室

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随机偏微分方程保结构算法青年研讨会

4 月 9 日（周五）　腾讯会议会议号：569 222 639，　密码：210409

时间	报告题目	报告人	主持人
09:00-09:10	开幕式		陈楚楚
09:10-09:45	On spectral-expansion-based approximation for solving stochastic elliptic equations and second-order SDEs	曹婉容	
09:45-10:20	Energy regularization approximation for the stochastic logarithmic Schrödinger equation	崔建波	
10:20-10:30	休息		
10:30-11:05	Strong approximations of semilinear monotone SPDEs	刘智慧	
11:05-11:40	Weak convergence rates of an explicit time-stepping scheme for SPDEs with non-globally Lipschitz coefficients	蔡　猛	
休息			
14:00-14:35	Accelerated exponential Euler scheme for stochastic heat equation: convergence rate of densities	陈楚楚	姬利海
14:35-15:10	Finite element methods for nonlinear backward stochastic partial differential equations and their error estimates	杨　旭	
15:10-15:20	休息		
15:20-15:55	Spectral Galerkin method for stochastic Cahn–Hilliard equation	孙丽莹	
15:55-16:30	Large deviations principles of sample paths and invariant measures of numerical methods for parabolic SPDEs	陈子恒	
16:30-17:05	Probabilistic superiority of stochastic symplectic methods via large deviations principles	金典聪	

4 月 10 日（周六）　　腾讯会议会议号：604 166 912，密码：210410

时间	报告题目	报告人	主持人
09:00-09:35	Uncertainty quantification for PDEs with high dimensional random parameters	周 涛	陈楚楚
09:35-10:10	Penalty finite element approximation of the double-diffusive convection in a stochastic climate model	邹广安	
10:10-10:20	休息		
10:20-10:55	Convergence analysis of parareal algorithms for SPDEs	张利英	
10:55-11:30	Hitting probabilities of numerical discretizations for linear stochastic parabolic system	盛德瑞	
休息			
14:00-14:35	Finite time blowup of solutions to SPDEs with Bernstein functions of the Laplacian	刘 暐	张利英
14:35-15:10	Numerical simulation and analysis for the stochastic Navier–Stokes equations	赵文举	
15:10-15:20	休息		
15:20-15:55	A splitting-up method for nonlinear filtering problem with correlated noise	柴世民	
15:55-16:30	Efficient full discretization for nonlinear stochastic wave equations	侯宝慧	
16:30-17:05	A temporal semi-discretization for 2D stochastic incompressible Euler equations	周 滔	

4 月 11 日（周日）　腾讯会议会议号：445 670 955，　密码：210411

时间	报告题目	报告人	主持人
09:00-09:35	A linear implicit time-stepping scheme for the finite element approximation of SPDEs with polynomial nonlinearity	王小捷	姬利海
09:35-10:10	Inverse elastic scattering for a random potential	王 旭	
10:10-10:20	休息		
10:20-10:55	Super-convergence analysis on exponential integrator for stochastic heat equation driven by additive fractional Brownian motion	黄楚荧	
10:55-11:30	Weak intermittency of a fully discrete scheme for stochastic heat equation	党同贺	
休息			
14:00-14:35	Stochastic symplectic methods for stochastic Maxwell equations	姬利海	孙丽莹
14:35-15:10	Structure preserving methods for the stochastic fractional nonlinear Schrödinger equation	刘子源	

报告摘要

Weak convergence rates of an explicit time-stepping scheme for SPDEs with non-globally Lipschitz coefficients

蔡猛

Abstract: A novel approach of weak convergence analysis will be reported for additive noise driven SPDE with non-Lipschitz coefficients. The expected weak convergence rates are successfully recovered for an explicit time-stepping fully discrete scheme. Numerical examples are finally provided to support our theoretical findings. This talk is based on a joint work with Siqing Gan and Xiaojie Wang.

On spectral-expansion-based approximation for solving stochastic elliptic equations and second-order SDEs

曹婉容

Abstract: In this talk, a spectral-expansion-based approximation is applied to stochastic elliptic equations driven by fractional Brownian motion, as well as the second-order stochastic differential equations (SDEs) with initial values. The consistency between the original problem and the approximated counterpart has been carefully analyzed. The advantage of the algorithm is the flexibility of using several kinds of solvers to solve the resulted equation, which is deterministic with some random parameters. Theoretical analysis and numerical examples show that for stochastic elliptic equations, the proposed full-discrete scheme can reach the optimal strong convergence order, and for second-order SDEs with initial values, the given full-discrete difference scheme has better strong convergence order than the classical Euler-type and Milstein-type schemes.

A splitting-up method for nonlinear filtering problem with correlated noise

柴世民

Abstract: In this talk, we discussed a splitting-up method for nonlinear filtering problem with correlated noise. The Zakai equation for the nonlinear filtering system is obtained. A splitting-up method for time variable and Galerkin method for spatial variable are applied. Finally, numerical examples are provided to show the efficacy of the proposed methods.

Accelerated exponential Euler scheme for stochastic heat equation: convergence rate of densities

陈楚楚

Abstract: In this talk, we study the numerical approximation of the density of the stochastic heat equation driven by space-time white noise via the accelerated exponential Euler scheme. The existence and smoothness of the density of the numerical solution are proved by means of Malliavin calculus. Based on a priori estimates of the numerical solution, we propose a test function-independent weak convergence analysis, which is crucial to show the convergence of the density. The convergence order of the density in uniform convergence topology is shown to be exactly 1/2 in nonlinear drift case and nearly 1 in affine drift case.

Large deviations principles of sample paths and invariant measures of numerical methods for parabolic SPDEs

陈子恒

Abstract: In order to numerically inherit the small noise asymptotics of the parabolic stochastic partial differential equations (SPDEs), we introduce an explicit full-discrete numerical method via the spectral Galerkin method in space and the accelerated exponential Euler method in time. We prove that both spatial semi-discretization and spatio-temporal full discretization satisfy the sample path large deviations, which together with the tail exponential estimates enables us to establish the large deviations principles (LDPs) of invariant measures of these discretizations. Moreover, the asymptotical behavior of the discrete rate functions is analyzed, which shows that the considered discretizations can weakly asymptotically preserve the two kinds of LDPs. To the best of our knowledge, this is the first result on the numerically asymptotical preservations for the LDPs of sample paths and invariant measures of SPDEs.

Energy regularization approximation for the stochastic logarithmic Schrödinger equation

崔建波

Abstract: In this talk, we consider the energy regularization approximation, which avoids the singularity near zero density, for the stochastic logarithmic Schrödinger (SlogS) equation driven by additive noise or multiplicative noise. Then we study several properties of the numerical approximation, including the a priori estimates, the regularized entropy and energy, the stochastic symplectic structure, as well as the strong convergence analysis.

Weak intermittency of a fully discrete scheme for stochastic heat equation

党同贺

Abstract: In this talk, we first show the weak intermittency of the exact solution of the stochastic heat equation with multiplicative noise and periodic boundary condition. In order to inherit numerically the weak intermittency of the original equation, we introduce a fully discrete scheme, whose spatial direction is based on the finite difference method and temporal direction is based on the θ-scheme. Moreover, it is shown that both of the numerical schemes could preserve the sharp exponential order of the second moment of the exact solution.

Efficient full discretization for nonlinear stochastic wave equations

侯宝慧

Abstract: In this talk, we consider the efficient numerical approximation of the nonlinear stochastic wave equation with multiplicative noise. We propose two spatio-temporal fully-discrete schemes, which can preserve the energy evolution law exactly. Numerical experiments are provided to confirm the conservation of energy evolution law of two proposed full discretization.

Super-convergence analysis on exponential integrator for stochastic heat equation driven by additive fractional Brownian motion

黄楚荧

Abstract: In this talk, we will begin with the definition of fractional Brownian motion and relative Malliavin calculus. Then we present references about numerical researches for SDEs and SPDEs driven by additive fractional Brownian motions. In the third and fourth part, we show the regularity analysis and strong convergence analysis on a full discretization for the stochastic heat equation driven by fractional Brownian motion with Hurst parameter larger than half. By utilizing the Malliavin calculus, we overcome the difficulty that the fractional Brownian motion is neither a Markov process nor a semi-martingale and that its increments are not independent, and then obtain the super-convergence result in temporal direction for the exponential integrator.

Stochastic symplectic methods for stochastic Maxwell equations

姬利海

Abstract: In this talk, we will introduce some stochastic symplectic methods for stochastic Maxwell equations with multiplicative or additive noise. In particular, a stochastic symplectic RK semidiscretization and an operator splitting method are presented and the mean-square convergence orders of the corresponding numerical methods are given.

Probabilistic superiority of stochastic symplectic methods via large deviations principles

金典聪

Abstract: In this talk, we study the superiority of stochastic symplectic methods by means of the large deviations principles (LDPs). The LDPs of some observables of test equations are given in the finite dimensional and infinite dimensional cases, respectively. We show that the considered a class of stochastic symplectic methods can (weakly) asymptotically preserve the LDPs of observables associated with the original equations, but the considered nonsymplectic methods can not. Our results indicate that stochastic symplectic methods can preserve the asymptotical behaviors of probabilities of rare events of stochastic Hamiltonian systems (SHSs), and provide an effective approach to approximating the large deviations rate functions associated with SHSs.

Finite time blowup of solutions to SPDEs with Bernstein functions of the Laplacian

刘 暐

Abstract: The blowup in finite time of solutions to SPDEs

$$\partial_t u_t(x) = -\phi(-\Delta)u_t(x) + \sigma\big(u_t(x)\big)\dot{\xi}(t,x), \quad t > 0, x \in \mathbb{R}^a$$

is investigated, where $\dot{\xi}$ could be either a white noise or a colored noise and $\phi: (0,\infty) \to (0,\infty)$is a Bernstein function. The sufficient conditions on $\sigma, \dot{\xi}$ and the initial value that imply the non-existence of the global solution are discussed. The results in this paper generalise the existing works on cases of the Laplacian and the fractional Laplacian by P. Chow (2011) and M. Foondun, W. Liu and E. Nane (2019), respectively.

Strong approximations of semilinear monotone SPDEs

刘智慧

Abstract: In this talk, I will present several results on strong approximations of semilinear Monotone SPDEs, including the stochastic Allen–Cahn equation, driven by additive white noise or multiplicative trace-class noise.

Structure preserving methods for the stochastic fractional nonlinear Schrödinger equation

刘子源

Abstract: In this talk we introduce two structure preserving methods for the stochastic space-fractional nonlinear Schrödinger equation with multiplicative noise. After the proofs for the mass and generalized stochastic multi-symplectic conservation law for the equation, a generalized multi-symplectic method and an efficient mass-conserving splitting method are presented. Numerical experiments and comparison between these two methods are also performed to verify our theoretical analysis.

Hitting probabilities of numerical discretizations for linear stochastic parabolic system

盛德瑞

Abstract: This talk focuses on the influences of standard semi-discretizations on hitting probabilities of linear stochastic parabolic system driven by space-time white noises. We give lower and upper bounds for hitting probabilities of the associated numerical solutions of both temporal and spatial semi-discretizations in terms of Bessel–Riesz capacity and Hausdorff measure, respectively. Moreover, the critical dimensions of both temporal and spatial semi-discretizations turn out to be half of those of the exact solution. This reveals that for a large class of Borel sets A, the probability of the event that the paths of the numerical solution hit A cannot converge to that of the exact solution.

Spectral Galerkin method for stochastic Cahn–Hilliard equation

孙丽莹

Abstract: In this talk, we consider the spectral Galerkin method for stochastic Cahn–Hilliard equation driven by additive noise. Based on optimal regularity estimates of both exact and numerical solutions, we prove that the proposed numerical method is strongly convergent with a sharp convergence rate in a negative Sobolev space. By means of the semigroup theory and interpolation inequality, we deduce the spatial optimal convergence rate of the considered numerical method in strong convergence sense.

A linear implicit time-stepping scheme for the finite element approximation of SPDEs with polynomial nonlinearity

王小捷

Abstract: This talk consists of two parts. The first one aims to improve existing regularity results for parabolic stochastic partial differential equations (SPDEs) with polynomial nonlinearity. The second part is devoted to strong convergence analysis of a linear implicit time-stepping scheme for the finite element approximation of SPDEs with polynomial nonlinearity. Strong convergence rates for the fully discrete scheme are successfully recovered both in space and in time.

Inverse elastic scattering for a random potential

王旭

Abstract: In this talk, an inverse scattering problem for the time-harmonic elastic wave equation with a rough potential will be introduced. Interpreted as a distribution, the potential is assumed to be a microlocally isotropic generalized Gaussian random field with the covariance operator being described by a classical pseudo-differential operator. The goal is to determine the principal symbol of the covariance operator from the scattered wave measured in a bounded domain which has a positive distance from the domain of the potential. For such a rough potential, the well-posedness of the direct scattering problem in the distribution sense is established by studying an equivalent Lippmann– Schwinger integral equation. For the inverse scattering problem, it is shown with probability one that the principal symbol of the covariance operator can be uniquely determined by the amplitude of the scattered waves averaged over the frequency band from a single realization of the random potential.

Finite element methods for nonlinear backward stochastic partial differential equations and their error estimates

杨旭

Abstract: In this work, we consider numerical approximation of a class of nonlinear backward stochastic partial differential equations (BSPDEs). By using finite element methods in the physical space domain and the Euler method in the time domain, we propose a spatial finite element semi-discrete scheme and a spatio-temporal full discrete scheme for solving the BSPDEs. Errors of the schemes are rigorously analyzed and theoretical error estimates with convergence rates are obtained.

Convergence analysis of parareal algorithms for SPDEs

张利英

Abstract: A parareal algorithm based on an exponential scheme is proposed for the stochastic Schrödinger equation with weak damping and additive noise. It proceeds as a two-level temporal parallelizable integrator with the exponential θ-scheme as the integrator on the coarse grid. The proposed algorithm in the linear case increases the convergence order from one to k for $\theta \neq 1/2$. In particular, the convergence order increases to 2k when $\theta = 1$ due to the symmetry of the algorithm. The convergence condition for longtime simulation is also established for the proposed algorithm in the nonlinear case, which indicates the superiority of implicit schemes. Numerical experiments are dedicated to illustrating the convergence order of the algorithm for different choices of θ, as well as the error for different iterated number k.

Numerical simulation and analysis for the stochastic Navier–Stokes equations

赵文举

Abstract: For given autocovariance function, we systematically constructed the representation of the random noise, i.e., colored or white noise in time and space. For each kind of temporal-spatial noises, we present detailed definitions and discussions of the noises and their properties. Further we present a Martingale regularization method for the stochastic Navier–Stokes equations with additive noise, where the original system is split into two equivalent parts, the linear stochastic Stokes equations with Martingale solution and the stochastic modified Navier–Stokes equations with relatively-higher regularities. The stability and convergence of numerical scheme for the pathwise modified Navier–Stokes equations are proved. We then apply the discretized colored/white forcings to facilitate numerical experiments in the context of finite element discretizations and compare the efficiency and regularity features of the system resulting from the experiments. The proposed techniques are useful for general settings of partial differential equations with colored or white forcing.

Uncertainty quantification for PDEs with high dimensional random parameters

周涛

摘要: 报告将简单介绍 UQ 中的高维逼近问题。从经典的多项式逼近结果，到近期比较流行的标准化流逼近方法。

A temporal semi-discretization for 2D stochastic incompressible Euler equations

周滔

Abstract: In this talk, we will propose and analyse a splitting semi-implicit Euler method in temporal direction for stochastic incompressible Euler equations on 2D torus driven by an additive noise. By a Galerkin approximation and the fixed point technique, we establish the unique solvability of the proposed method. Based on the regularity estimates of both exact and numerical solutions, we prove that the pathwise convergence order is nearly 1/2 and the convergence order in probability is almost 1. This is joint work with Prof. Jialin Hong and Derui Sheng.

Penalty finite element approximation of the double-diffusive convection in a stochastic climate model

邹广安

Abstract: In this talk, we develop and analyze a fully discrete penalty finite element method for a simplified stochastic climate model driven by multiplicative noise. The model is comprised of the stochastic Navier–Stokes equations coupled with the double-diffusive convection equations for heat and salinity. We establish strong convergence rates for the full discrete scheme based on an Euler–Maruyama scheme in time discretization. This is done with the crucial help of the Holder continuity in time and stability analysis of the random variables with respect to H^1-norm. It also relies on the fact that the application of the Ito formula and Burkholder–Davis–Gundy (B-D-G) inequality, which are very helpful for us to settle the random forcing. For the regularity estimation of the pressure term, we introduce the Helmholtz–Hodge–Leray (H-H-L) decomposition. It should be pointed out that the idea of introducing sample subsets, which avoids situations where the Gronwall lemma can't be employed in the fully discrete scheme due to the discontinuity of the solution. What's more, numerical experiments are provided to validate theoretical results and to demonstrate the efficiency of the proposed method. Numerical results also clearly exhibit that the presence of stochastic forcing can evidently affect the velocity, temperature and salinity in a random medium.

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