

**The Ninth  
China-Russia Conference  
on Numerical Algebra  
with Applications**

**(CRCNAA 2023)**

**Nantong University  
Nantong, P.R. China  
July 8–July 13, 2023**

# **The Ninth China-Russia Conference on Numerical Algebra with Applications**

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The Ninth China-Russia Conference on Numerical Algebra with Applications (CRCNAA 2023) will be held at Nantong University, Nantong, Jiangsu Province, P.R. China, during July 8–July 13, 2023 (Registration: July 8, Departure: July 13). It aims at bringing together researchers, scientists, engineers, and graduate students to exchange and stimulate ideas from different disciplines, at discussing the practical challenges encountered and the solutions adopted, and at learning about the recent developments on theory and numerical methods for numerical algebra and scientific computing. The topics of CRCNAA 2023 include, but are not limited to preconditioning and iterative methods for systems of linear and nonlinear equations, large and sparse linear and nonlinear eigenvalue computations, parallel computations, randomized numerical algebra, mathematical modeling, and applications of numerical methods to real-world problems.

This conference is organized by Zhong-Zhi Bai (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing), Yang Cao (Nantong University, Nantong), and Galina V. Muratova (Southern Federal University, Russia).

## **Conference Chairs:**

Zhong-Zhi Bai (Chinese Academy of Sciences, China)

Galina V. Muratova (Southern Federal University, Russia)

## **Invited Plenary Speakers:**

Zhong-Zhi Bai (Chinese Academy of Sciences, China)

Yang Cao (Nantong University, China)

Hua Dai (Nanjing University of Aeronautics and Astronautics, China)

Nikolay I. Gorbenco (Siberian Branch, Russian Academy of Sciences, Russia)

Ken Hayami (National Institute of Informatics, Japan)

Yu-Mei Huang (Lanzhou University, China)

Zheng-Da Huang (Zhejiang University, China)

Mikhail I. Karyakin (Southern Federal University, Russia)

Hao Liu (Nanjing University of Aeronautics and Astronautics, China)

Kang-Ya Lu (Beijing Information Science and Technology University, China)

Galina V. Muratova (Southern Federal University, Russia)

Konstantin A. Nadolin (Southern Federal University, Russia)  
Andrey V. Nasedkin (Southern Federal University, Russia)  
Rostislav D. Nedin (Southern Federal University, Russia)  
Pavel A. Oganesyanyan (Southern Federal University, Russia)  
Zhi-Ru Ren (Central University of Finance and Economics, China)  
Sergey B. Sorokin (Siberian Branch, Russian Academy of Sciences, Russia)  
Xue-Yuan Tan (Nanjing Normal University, China)  
Kirill M. Terekhov (Russian Academy of Sciences, Russia)  
Yuri V. Vassilevski (Russian Academy of Sciences, Russia)  
Yu-Jiang Wu (Lanzhou University, China)  
Jian-Jun Zhang (Shanghai University, China)

### **Invited Participants**

Yin-Xin Bao (Nantong Univ, China)  
Fang Chen (Beijing Inform Sci Tech Univ, China)  
Xiao-Ping Chen (Taizhou Univ, China)  
Ying-Ting Chen (Soochow Univ, China)  
Yu-Juan Chen (Nantong Univ, China)  
Xiao-Xia Guo (Ocean Univ China, China)  
Mei-Qun Jiang (Soochow Univ, China)  
Shang Jiang (Nantong Univ, China)  
Rui Li (Jiaxing Univ, China)  
Tian-Yi Li (Macau Univ, China)  
Xu Li (Lanzhou Univ Tech, China)  
Yu-Hong Ran (Northwest Univ, China)  
Ye-Qin Shao (Nantong Univ, China)  
Qin-Qin Shen (Nantong Univ, China)  
Quan Shi (Nantong Univ, China)  
Lin-Lin Sun (Nantong Univ, China)  
Lu Wang (Hebei Normal Univ, China)  
Lu-Xin Wang (Nantong Univ, China)  
Bo Wu (Ningxia Univ, China)  
Wen-Ting Wu (Beijing Inst Tech, China)  
Xiang-Jian Xu (Nantong Univ, China)  
Zhi-Peng Xu (Nantong Univ, China)  
Geng-Chen Yang (Nantong Univ, China)  
Min-Li Zeng (Putian Univ, China)  
Ju-Li Zhang (Shanghai Univ Engrg Sci, China)  
Chen-Can Zhou (Nantong Univ, China)  
Xiao-Jian Zhou (Nantong Univ, China)  
Mu-Zheng Zhu (Hexi Univ, China)

### **Scientific Committee:**

Zhong-Zhi Bai (Chinese Academy of Sciences, China)  
Yu-Juan Chen (Nantong Univ, China)  
Hua Dai (Nanjing Univ Aeron Astron, China)  
Mikhail I. Karyakin (Southern Federal Univ, Russia)  
Galina V. Muratova (Southern Federal Univ, Russia)  
Andrey V. Nasedkin (Southern Federal Univ, Russia)  
Quan Shi (Nantong Univ, China)  
Yuri V. Vassilevski (Russian Academy of Sciences, Russia)  
Yu-Jiang Wu (Lanzhou Univ, China)

### **Organizing Committee:**

Zhong-Zhi Bai (Chinese Academy of Sciences, China)  
Yang Cao (Nantong Univ, China)  
Yu-Juan Chen (Nantong Univ, China)  
Mei-Qun Jiang (Soochow Univ, China)  
Galina V. Muratova (Southern Federal Univ, Russia)  
Qin-Qin Shen (Nantong Univ, China)  
Quan Shi (Nantong Univ, China)  
Lu Wang (Hebei Normal Univ, China)  
Li-Li Zhang (Nantong Univ, China)

### **Important Deadlines:**

May 30, 2023: Abstract Submission  
July 08-13, 2023: Conference  
July 08, 2023: Registration  
July 13, 2023: Departure

### **Conference Place (Registration and Hotel):**

Nantong Jinshi International Hotel  
No. 85, Chongchuan Road, Chongchuan District  
Nantong 226019, Jiangsu Province, P.R. China  
Tel: +86-0513-80989999

# **Program**

## **(In Beijing Time)**

**July 9 (Sunday)**

**14:30--15:00** Opening (Chair: Yang Cao)

**15:00--18:00** Invited Plenary Talks (Chair: Hua Dai)

**15:00--15:45** Mikhail I. Karyakin:

**On Determining the Constitutive Parameters of the Highly Elastic Materials Using Evolutionary Algorithms**

**15:45--16:30** Zhong-Zhi Bai:

**On Convergence Rates of Kaczmarz-Type Methods with Different Selection Rules of Working Rows**

**16:30--17:15** Yuri V. Vassilevski:

**Stable Numerical Schemes for Modelling Hemodynamic Flows in Time-Dependent Domains**

**17:15--18:00** Galina V. Muratova:

**Numerical Solution of Large Generalized Eigenvalue Problems by Shift-and-Invert Lanczos Method**

**18:00--20:30** Dinner

**20:30--22:00** Invited Plenary Talks (Chair: Galina V. Muratova)

**20:30--21:15** Andrey V. Nasedkin:

**About Finite Element Technique of Efficiency Analysis of Ultrasonic Transducers Made of Composite Piezoceramics in Acoustic Medium**

**21:15--22:00** Konstantin A. Nadolin:

**Hydrodynamics and Mass Transfer in Long Natural Streams: Mathematical Models and Computational Perspectives**

**July 10 (Monday)**

- 09:00--12:00** Invited Plenary Talks (Chair: Zhong-Zhi Bai)
- 09:00--09:45** Hua Dai:  
Global Rational Krylov Subspace Method for Solving Large-Scale Lyapunov Equations
- 09:45--10:30** Yang Cao:  
Accelerated Modulus-Based Matrix Splitting Iteration Methods for Mixed-Size Cell Circuits Legalization
- 10:30--11:15** Zhi-Ru Ren:  
Randomized Methods of Matrix Decompositions and Its Applications
- 11:15--12:00** Kang-Ya Lu:  
Optimal Rotated Block-Diagonal Preconditioning for Discretized Optimal Control Problems Constrained with Fractional Time-Dependent Diffusive Equations
- 12:00--15:00** Lunch
- 15:00--18:00** Invited Plenary Talks (Chair: Mikhail I. Karyakin)
- 15:00--15:45** Rostislav D. Nedin:  
Concerning Modeling and Identification of Prestress Fields in Elastic Bodies
- 15:45--16:30** Pavel A. Oganessian:  
Computational Modules for Material Properties Identification in ACELAN-COMPOS Finite Element Package
- 16:30--17:15** Kirill M. Terekhov:  
Linear System Solution Methods for Coupled Multiphysics Problems
- 17:15--18:00** Sergey B. Sorokin:  
An Implicit Iterative Method for Solving an Inverse Boundary Value Problem for an Elliptic Equation
- 18:00--20:30** Dinner

**July 11 (Tuesday)**

**09:00--12:00 Invited Plenary Talks (Chair: Zhi-Ru Ren)**

**09:00--09:45 Ken Hayami:**

**GMRES Methods for Tomographic Reconstruction with an Unmatched Back Projector**

**09:45--10:30 Yu-Jiang Wu:**

**A Minimum Residual Modified HSS Iteration Method for a Class of Complex Symmetric Linear Systems**

**10:30--11:15 Yu-Mei Huang:**

**Rank Minimization with Applications to Image Restoration**

**11:15--12:00 Zheng-Da Huang:**

**An Inexact Block Factorization Preconditioner for Incompressible Navier-Stokes Equations**

**12:00--15:00 Lunch**

**15:00--18:00 Invited Plenary Talks (Chair: Yu-Jiang Wu)**

**15:00--15:45 Jian-Jun Zhang:**

**A Block Preconditioner for Double Saddle Point Systems Arising from Liquid Crystal Directors Modeling**

**15:45--16:30 Xue-Yuan Tan:**

**A Block Kaczmarz Algorithm for Solving Large Linear Systems**

**16:30--17:15 Hao Liu:**

**Numerical Methods for Solving Fractional Eigenvalue Problems**

**17:15--17:20 Break**

**17:20--18:00 Contributed Talks (Chair: Yang Cao)**

**17:20--17:40 Wen-Ting Wu:**

**On Convergence of the Partially Randomized Extended Kaczmarz Method**

**17:40--18:00 Lu Wang:**

**On Relaxed Greedy Randomized Augmented Kaczmarz Methods for Solving Large Sparse Inconsistent Linear Systems**

**18:00--20:30 Dinner**

**July 12 (Wednesday)**

**09:00--12:00 Meeting (Chairs: Yang Cao, Yu-Mei Huang)**

**09:00--10:30 Preconditioning and Iterative Techniques with Applications**

**10:30--10:40 Break**

**10:40--12:00 Randomization and Projection Methods**

**12:00--15:00 Lunch**

**15:00--18:00 Discussion (Chairs: Yang Cao, Zheng-Da Huang)**

**15:00--16:30 Fast Solutions of Discretized Partial Differential Equations**

**16:30--16:40 Break**

**16:40--18:00 Optimal Control Problems with PDE Constraints**

**18:00--20:30 Dinner**



# On Convergence Rates of Kaczmarz-Type Methods with Different Selection Rules of Working Rows

Zhong-Zhi Bai

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## Abstract

The Kaczmarz method is a classical while effective iteration method for solving very large-scale consistent systems of linear equations, and the randomized Kaczmarz method is an important and valuable variant of the Kaczmarz method. By theoretically analyzing and numerically experimenting several criteria typically adopted in the non-randomized and the randomized Kaczmarz method for selecting the working row, we derive sharper upper bounds for the convergence rates of some of the correspondingly induced Kaczmarz-type methods including those with respect to the maximal residual, maximal distance, and distance selection rules of the working row, and, for this whole suite of iteration methods consisting of the Kaczmarz methods with respect to the uniform, non-uniform, residual, distance, maximal residual, and maximal distance selection rules of the working row, we reveal their comparable relationships in terms of both mean-squared distance and mean-squared error, and show their computational effectiveness and numerical robustness based upon implementing a large number of test examples. Here the mean-squared distance is defined as the mean-value of the squared Euclidean norm of the current update increment of the iteration, and the mean-squared error is defined as the mean-value of the squared Euclidean norm of the current error that is the difference between the current iterate and the true solution of the target linear system.

# Accelerated Modulus-Based Matrix Splitting Iteration Methods for Mixed-Size Cell Circuits Legalization

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## Abstract

Mixed-size cell circuits dominate in advanced technology node designs, with attendant increases in layout complexity. The introduction of multi-row-height cells requires additional constraints, such as power line alignment, to be considered in the legalization stage, in addition to eliminating overlap while maintaining the results of the global placement stage as much as possible. In this paper, a three-stage legalization methodology is presented. The first is cell preprocessing, namely cell diffusion based on the network flow algorithm. Then the legalization problem is modeled as a quadratic programming problem, which is converted into a linear complementarity problem and solved by accelerated modulus-based matrix splitting iteration methods. Finally, the illegal cells are reshaped. Experimental results reveal that our method achieves 11.8% and 0.4% reduction in total displacement, respectively, over existing two state-of-the-art works, and is 1.407x faster than the modulus-based matrix splitting iteration method. In particular, the proposed approach provides a reference for solving large-scale quadratic programming problems.

# Global Rational Krylov Subspace Method for Solving Large-Scale Lyapunov Equations

Hua Dai

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## Abstract

Lyapunov matrix equations play a fundamental role in numerous problems in control theory, stability analysis of dynamical systems, model reduction of linear time-invariant systems and Newton's method to the algebraic Riccati equations. In this talk, we introduce a new global rational Krylov subspace, and present a process for constructing an orthonormal basis of the global rational Krylov subspace. We develop an adaptive selection of shifts parameters for the global rational Krylov subspace, show how to extract low rank approximate solutions to Lyapunov matrix equations, propose a global rational Krylov subspace method for solving large-scale Lyapunov matrix equations and give some theoretical results. Finally, some numerical tests will be reported to illustrate the effectiveness of the proposed method.

(This is a joint work with Song Nie)

# GMRES Methods for Tomographic Reconstruction with an Unmatched Back Projector

Ken Hayami

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## Abstract

Unmatched pairs of forward and back projectors are common in X-ray CT computations for large-scale problems; they are caused by the need for fast algorithms that best utilize the computer hardware, and it is an interesting and challenging task to develop fast and easy-to-use algorithms for these cases. Our approach is to use preconditioned GMRES, in the form of the AB- and BA-GMRES algorithms, to handle the unmatched normal equations associated with an unmatched pair. These algorithms are simple to implement, they rely only on computations with the available forward and back projectors, and they do not require the tuning of any algorithm parameters. We show that these algorithms are equivalent to well-known LSQR and LSMR algorithms in the case of a matched projector. Our numerical experiments demonstrate that AB- and BA-GMRES exhibit a desired semi-convergence behavior that is comparable with LSQR/LSMR and that standard stopping rules work well. Hence, AB- and BA-GMRES are suited for large-scale CT reconstruction problems with noisy data and unmatched projector pairs.

(This is a joint work with Per Christian Hansen and Keiichi Morikuni)

## References

- [1] P.C. Hansen, K. Hayami and K. Morikuni, GMRES methods for tomographic reconstruction with an unmatched back projector, *Journal of Computational and Applied Mathematics*, 413 (2022), 114352.

# Rank Minimization with Applications to Image Restoration

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## Abstract

Rank minimization problem has a wide range of applications in different areas. However, since this problem is NP-hard and non-convex, the frequently used method is to replace the matrix rank minimization with nuclear norm minimization. Nuclear norm is the convex envelope of the matrix rank and it is more computationally tractable. Matrix completion is a special case of the rank minimization problem. In this talk, we consider directly using matrix rank as the regularization term instead of nuclear norm in the cost function for matrix completion problem. The solution is analyzed and obtained by a hard-thresholding operation on the singular values of the observed matrix. Then by exploiting patch-based nonlocal self-similarity scheme, we apply the proposed rank minimization algorithm to remove white Gaussian additive noise in images. We also develop a model for image restoration using the sum of block matching matrices' weighted nuclear norm to be the regularization term in the cost function. An alternating iterative algorithm is designed to solve the proposed model and the convergence analysis of the algorithm is also presented. Numerical experiments show that the proposed method can recover the images much better than the existing regularization methods in terms of both recovered quantities and visual qualities.

# An Inexact Block Factorization Preconditioner for Incompressible Navier-Stokes Equations

Zheng-Da Huang

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## Abstract

Let

$$\begin{pmatrix} A_1 & 0 & B_1^T \\ 0 & A_2 & B_2^T \\ -B_1 & -B_2 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ p \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ -g \end{pmatrix},$$

be the  $3 \times 3$  block structured system of real linear equations related to the numerical solution of incompressible Navier-Stokes equations, where  $A_1$  and  $A_2$  are nonsymmetric positive matrices,  $B_1$  and  $B_2$  full row rank ones, and  $f_1$ ,  $f_2$  and  $g$  known vectors. An inexact block factorization preprocessor is constructed based on an equivalent form of the system. By obtaining an easier matrix similar to the preconditioned matrix, the upper and lower bounds of the real and imaginary parts of the eigenvalues of the preprocessed matrix are estimated based on Bendixson theorem. For uniform and stretchable meshes, numerical experiments show that compared with the existing ones, the constructed preconditioner is superior in the number of iteration steps and CPU time, and can make the GMRES a less dependent on the mesh size and the viscosity coefficient.

# On Determining the Constitutive Parameters of the Highly Elastic Materials Using Evolutionary Algorithms

Mikhail I. Karyakin

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## Abstract

In connection with the emergence of an increasing number of new materials with nonlinear properties, researchers are interested in the development of mathematical models that correctly describe their nonlinear behavior, and the classical problem of mechanics of determining the material characteristics does not lose its relevance. Highly elastic materials rubber-like materials and soft biological tissues have pronounced non-linear properties. Due to their ability to undergo large elastic deformations, they are widely used in many areas of mechanical engineering and the medical industry. An important task is to identify the material parameters of models of highly elastic materials by modeling various experiments. Various approaches are used to solve this problem; in this paper, it is studied using evolutionary algorithms.

In the present work, we study several formulations of the problems of determining the material characteristics of nonlinear elastic samples based on classical experiments on uniaxial and biaxial tension. The experiments were modeled using the semi-inverse method of nonlinear elasticity theory. To describe the mechanical properties of materials, commonly used models of compressible nonlinear elastic media were used — the Blatz and Ko material and the Murnaghan material. Analytical expressions for the dependence of the applied load on the elongation ratio are obtained. Based on this, at the first stage, various direct problems were solved, which consisted in constructing loading diagrams for the given parameters of nonlinear elastic models. The obtained results were taken as input data for the inverse problem.

At the second stage, the inverse problem consisting in restoring the parameters of the models used based on loading diagrams was solved in various formulations. The problem is reduced to finding the minimum of the function of deviation of the calculated theoretical data of the loading diagram from the “experimental”; the objective function was introduced using the least squares method. The Squirrel search algorithm based on the feeding behavior model of the flying squirrel population was used as an analysis tool, implemented in this work in the Python environment using the capabilities of the NumPy and SciPy libraries. As an alternative means of analysis, the differential evolution algorithm was used.

The results of solving the inverse problem are compared with the initial data for the direct problem. A fairly reliable determination of the material parameters of models using evolutionary algorithms, including the use of artificial noise of “experimental” data, is shown. The obtained results make it possible to speak about the effectiveness and prospects of using evolutionary algorithms to solve similar problems of identifying the parameters of mathematical models.

(This is a joint work with Sofia A. Egorova)

# Numerical Methods for Solving Fractional Eigenvalue Problems

Hao Liu

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## **Abstract**

In this talk, we explore some recent developments in numerical methods for solving fractional eigenvalue problems. Fractional eigenvalue problems originate from the fields such as anomalous diffusion, electromagnetic wave, viscoelasticity. We briefly introduce the background and application of fractional eigenvalue problems, and give some numerical discretization methods for fractional derivatives. In order to solve these problems effectively, some numerical methods are proposed. We mainly introduce how to construct the preconditioner and analyze the properties of eigenvalues of the preconditioned matrix. This talk also reveals some questions that deserve further study.



# Optimal Rotated Block-Diagonal Preconditioning for Discretized Optimal Control Problems Constrained with Fractional Time-Dependent Diffusive Equations

Kang-Ya Lu

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## Abstract

For a class of optimal control problems constrained with certain time- and space-fractional diffusive equations, by making use of mixed discretizations of temporal finite-difference and spatial finite-element schemes along with Lagrange multiplier approach, we obtain specially structured block two-by-two linear systems. We demonstrate positive definiteness of the coefficient matrices of these discrete linear systems, construct rotated block-diagonal preconditioning matrices, and analyze spectral properties of the corresponding preconditioned matrices. Both theoretical analysis and numerical experiments show that the preconditioned Krylov subspace iteration methods, when incorporated with these rotated block-diagonal preconditioners, can exhibit optimal convergence property in the sense that their convergence rates are independent of both discretization stepsizes and problem parameters, and their computational workloads are linearly proportional with the number of discrete unknowns.

(This is a joint work with Zhong-Zhi Bai)

# Numerical Solution of Large Generalized Eigenvalue Problems by Shift-and-Invert Lanczos Method\*

Galina V. Muratova

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## Abstract

The generalized eigenvalue problem for a symmetric definite matrix pencil obtained from finite element modeling is solved numerically. The mass matrix is singular in the considered problem therefore process proceeds with the semi-inner product defined by this matrix. Shift-and-Invert (SI) Lanczos algorithm is used to find multiple eigenvalues closest to some shift. A preconditioned system of linear algebraic equations is solved by iterative solvers such as SYMMLQ and Algebraic Multigrid method with the HSS smoother at each step of the SI Lanczos algorithm.

The results of numerical experiments are presented. Considered methods were implemented in ACELAN-COMPOS package which consists of multiple computational modules based on class library that implements finite element method. This library implemented using C# programming language and .NET platform.

(Research has done in collaboration with T.S. Martynova, P.A. Oganessian and V.V. Bavin)

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# Hydrodynamics and Mass Transfer in Long Natural Streams: Mathematical Models and Computational Perspectives

Konstantin A. Nadolin

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## **Abstract**

The reduced mathematical models of long natural streams are analyzed. These models describe hydrodynamics and passive transfer of matter in long natural streams, such as rivers and canals. The technique of obtaining these simplified three-dimensional mathematical models is based on the method of small parameters, which is applied simultaneously to the Reynolds turbulent hydrodynamic equations, supplemented in accordance with the Boussinesq hypothesis, and to the convection-diffusion equation. All equations are written previously in a special dimensionless form. As a result, five different simplified models appear. They consider the peculiarities of the five types of natural streams. These models are much simpler for numerical and analytical analysis than mathematical models based on full three-dimensional equations of hydrodynamics and mass transfer. The results of numerical experiments show that the proposed mathematical models adequately describe the passive propagation of matter in natural streams.

(Research has done in collaboration with I.V. Zhilyaev and K.V. Ambaryan)

# About Finite Element Technique of Efficiency Analysis of Ultrasonic Transducers Made of Composite Piezoceramics in Acoustic Medium\*

Andrey Nasedkin

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## Abstract

The paper discusses the general strategy of finite element analysis of ultrasonic transducers with active elements made of composite piezoceramics. As an example, a focusing multilayer transducer of medical ultrasound, made of porous piezoceramics, is studied. The efficiency of such an emitter is determined by the characteristics of the focal zone under steady-state oscillations or by the amplitudes of pressure waves under non-stationary influences in the surrounding acoustic medium.

A complete study includes the following steps:

- solution of homogenization problems on determining the effective moduli of porous piezoceramics depending on the characteristics of porosity;
- determination of electrically active resonant frequencies, which in piezoelectric instrumentation are called the frequencies of electrical resonances and antiresonances, with the calculation of the electromechanical coupling coefficients;
- building a graph of the amplitude-frequency characteristics of not loaded on the acoustic medium transducer in the vicinity of the main operating resonant frequencies;
- building a graph of the amplitude-frequency characteristics of the transducer, taking into account the external acoustic medium  $t$  in the vicinity of the selected operating resonant frequencies;
- study of the pressure field and focal zone in the external acoustic medium at the operating resonant frequency;
- solution of the transient problem and analysis of pressure waves under impulse actions for the transducer, taking into account the external acoustic medium at the time of action of impulses associated with wavelengths at resonant frequencies;
- possible complication of dynamic problems associated with damping in the transducer and in the acoustic medium, as well as the addition of an external electrical circuit to the transducer.

The features of the numerical solution of problems at each of the listed stages by the finite element method, implemented in the ANSYS and ACELAN packages, are described. In this regard, methods for solving finite element problems in statics, in eigenvalue problems, in problems of steady-state oscillations, and in non-stationary problems are considered. It should be noted that finite element matrices for static problems of piezoelectricity have a saddle structure, while in dynamic problems the degrees of freedom of the electric potential are massless. In this regard, it is possible to use a set of algorithms that work with symmetric saddle matrices, as was implemented in the ACELAN package.

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# Concerning Modeling and Identification of Prestress Fields in Elastic Bodies

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## Abstract

To date, technologies for the manufacture of heterogeneous materials of a complex structure allow to create objects of complex geometry without the use of classical production processes, such as casting, requiring additional production of press forms. For example, for the manufacture of structural elements made of functionally graded composites, high-temperature technologies are often used, as a result of which, after cooling in the samples, prestresses are often formed. Such prestress fields have a significant impact on the dynamic characteristics of structures and destruction criteria. In this regard, the study of the problems of monitoring and identification of prestresses in structural elements within the framework of non-destructive concepts is of greater practical significance.

The present research is devoted to the issues of modeling and identifying inhomogeneous prestress fields in elastic bodies within steady-state motion. Based on the linearized model, a general problem statement and its variation formulation are presented. Inverse coefficient problems on the identification of prestress fields in the presence of additional data of two types is considered: 1) the displacement field is set everywhere in the area for one oscillation frequency; 2) the field is set on some area parts in a certain frequency range. In the first formulation, the problem is linear, and in the second one it is non-linear and requires special approaches to the study. For a preliminary analysis, the concept of sensitivity is introduced using the Fréchet derivative, and the effect of the desired prestress characteristics on the dynamic response is estimated. A technique is proposed for determining the sensitivity characteristics of given boundary functions with respect to prestress, which made it possible to identify the most effective ways of applying a dynamic load and sounding frequency ranges. To study the inverse problems of prestress reconstruction in the second formulation, iterative-regularization approaches based on the linearized reciprocity relation are proposed, making it possible to build an iterative process for determining corrections for the prestress tensor components with respect to some chosen initial state. Some specific inverse problems on the reconstruction of the initial stress-strain state of various types in plates and cylinders have been studied; computational experiments were carried out, recommendations for choosing the most efficient modes of acoustic sounding were formulated.

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# Computational Modules for Material Properties Identification in ACELAN-COMPOS Finite Element Package

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## Abstract

Computational mechanics plays a crucial role in material properties identification as it enables the design and simulation of complex materials at multiple scales. With the help of numerical simulations, it is possible to investigate the behavior of materials under various loading conditions and to determine their properties, which is essential for the development of new materials and for improving the performance of existing ones.

In this study, we analyze the implementation of the finite element method for material properties identification that was used in the ACELAN-COMPOS package. The model of the composite material is constructed on multiple levels: the representative volume of the composite concerning different types of material distribution; the discretization technique used for numerical analysis; the sparse matrix storage formats and numerical methods used for solving emerging systems of linear equations; material models, and post-processing techniques. The goal of the study is to provide detailed descriptions of the design solutions and implementations that were used to build the ACELAN-COMPOS package as a modular system. The differences between general finite element method approaches and specific cases arising from materials properties identification problems are discussed.

Composites with 3-0, 3-3, 3-1, and 2-2 connectivity and corresponding mesh generation techniques are discussed, including the case for porous materials. Multi-level models with different types of connectivity are also discussed. The results of numerical analyses of studied composites and devices built with them are presented. All methods described in the study were implemented as modules in C# and Ruby languages, and the overall architecture of the resulting software is also discussed.

# Randomized Methods of Matrix Decompositions and Its Applications

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## **Abstract**

Matrix decompositions are fundamental tools in the area of applied mathematics, statistical computing, and machine learning. In particular, low-rank matrix decompositions are vital and widely used for data analysis, dimensionality reduction and data compression. Massive datasets, however, pose a computational challenge for traditional algorithms, placing significant constraints on both memory and processing power. Recently, the powerful concept of randomness has been introduced as a strategy to ease the computational load. In this talk, we will discuss randomized algorithms of some basic matrix decompositions and its applications.

# An Implicit Iterative Method for Solving an Inverse Boundary Value Problem for an Elliptic Equation

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## Abstract

The Cauchy problem for an elliptic equation is one of the classical ill-posed problems. In general cases (variable coefficients and complex geometry of the computational domain), numerical methods for solving such a problem are based on the fundamental results in the field of ill-posed problems. One of the most commonly used methods is undoubtedly Tikhonov's regularization method that is based on reducing the original ill-posed problem to the minimization problem for the corresponding functional. The search for a minimum of the functional is carried out, as a rule, by applying iterative algorithms. In the numerical solution of the Cauchy problem for elliptic equations, one of the widely used approaches is to reduce it to an inverse boundary value problem [1]. The ill-posedness of the original problem affects the properties of the operator of the discrete analog of the equation, namely, it has a very large conditionality. In turn, the consequence of the operator's large conditionality is that the application of explicit iterative methods for finding a solution of the operator equation with given accuracy requires an unacceptably large number of iterations (proportional to the value of conditionality). In the majority of cases, fundamentally new results in solving the operator equations by the iterative methods are achieved by applying the implicit processes. In this article, an implicit gradient two-layer iterative method is constructed and numerically investigated for the solving the inverse boundary value problem for elliptic equations. As a preconditioner an economical direct method of solving the Cauchy problem for the Laplace equation in a rectangular domain is used [2-3].

## References

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- [3] S.B. Sorokin, An efficient direct method for numerically solving the Cauchy problem for Laplace's equation, *Numerical Analysis and Applications*, 12 (2019), 87-103.



# A Block Kaczmarz Algorithm for Solving Large Linear Systems

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## Abstract

The Kaczmarz algorithm, proposed by the Polish mathematician Stefan Kaczmarz, is a simple iterative scheme with row projection for solving large linear systems. Since the second half of the 20th century, this method has been widely applied in many fields. In this paper we present a block Kaczmarz method which at each step, selects three rows of the coefficient matrix and projects the current iterate onto the solution space of the hyperplane formed by these target rows. Moreover, based on the greedy randomized Kaczmarz algorithm, we develop a greedy block randomized Kaczmarz method. Convergence analysis and numerical testing show that these algorithms can be competitive with some state-of-the-art Kaczmarz-type methods in speed and efficiency.

# Linear System Solution Methods for Coupled Multiphysics Problems

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## Abstract

This work is dedicated to the solution methods for complex coupled multiphysics problems. We commence with classical approaches, such as the algebraic multigrid method for pure elliptic problems and its extension to systems of equations: the constrained pressure residual method for multiphase filtration systems and the Bramble-Pasciack method for saddle-point Stokes systems. We demonstrate that the bootstrap adaptive multigrid method is able to directly address systems arising from multiphase filtration problems and show preliminary results of the block algebraic multigrid method applied to systems arising from collocated finite-volume discretization of saddle-point problems of various physics, such as rigid body elasticity, Biot equations, Navier-Stokes equations, and Maxwell equations. At last, we demonstrate some results with the parallel multilevel Crout incomplete factorization method of INMOST platform [1].

## References

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# Stable Numerical Schemes for Modelling Hemodynamic Flows in Time-Dependent Domains

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## Abstract

We present a unified numerical approach to finite-element modelling of incompressible flows in time-dependent domains [1]. The approach features relatively large (independent of mesh size) time steps, solution of one linear system per time step, and relatively coarse computational meshes in space. The approach is monolithic and allows standard  $P_2 - P_1$  (Taylor–Hood) finite element spaces. It is applicable to the Navier-Stokes equations in time-dependent domains, the fluid-structure interaction (FSI) problems, and the fluid-porous structure interaction (FPSI) problems. The properties of the schemes are shown on several benchmarks and hemodynamic applications.

## References

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# On Relaxed Greedy Randomized Augmented Kaczmarz Methods for Solving Large Sparse Inconsistent Linear Systems

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## Abstract

For solving large-scale sparse inconsistent linear systems by iteration methods, we introduce a relaxation parameter in the probability criterion of the greedy randomized augmented Kaczmarz (GRAK) method, obtaining a class of relaxed greedy randomized augmented Kaczmarz (RGRAK) methods. The RGRAK method can be also derived directly by applying the relaxed greedy randomized augmented Kaczmarz method to an equivalent consistent augmented linear system. We prove the convergence of this method and estimate an upper bound for its convergence rate. Theoretical analysis and numerical experiments show that the RGRAK method can outperform the GRAK method if the relaxation parameter is chosen appropriately.

# On Convergence of the Partially Randomized Extended Kaczmarz Method

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## Abstract

To complete the convergence theory of the partially randomized extended Kaczmarz method for solving large inconsistent systems of linear equations, we give its convergence theorem whether the coefficient matrix is of full rank or not, tall or flat. This convergence theorem also modifies the existing upper bound for the expected solution error of the partially randomized extended Kaczmarz method when the coefficient matrix is tall and of full column rank. Numerical experiments show that the partially randomized extended Kaczmarz method is convergent when the tall or flat coefficient matrix is rank deficient, and can also converge faster than the randomized extended Kaczmarz method.

# A Minimum Residual Modified HSS Iteration Method for a Class of Complex Symmetric Linear Systems

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## **Abstract**

Based on the classical modified HSS iteration method (MHSS) for solving a class of complex symmetric systems of linear equations, we recall the application of the minimum residual technique to the MHSS iteration scheme and provide the proposition of the minimum residual MHSS, or MRMHSS iteration method. It has two more iteration parameters with it, and their parameters can be automatically computed. Then, some theoretical properties of the MRMHSS iteration method are carefully studied. Finally, we use several examples to test its performance by comparing its numerical results with some other existing iteration methods.

# A Block Preconditioner for Double Saddle Point Systems Arising from Liquid Crystal Directors Modeling

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## **Abstract**

We propose and investigate a new block preconditioner for a class of double saddle point problems arising from liquid crystal directors modeling using finite element scheme. Spectral properties of the preconditioned matrix are analyzed. Numerical experiments are provided to evaluate the performance of preconditioned iteration methods using the proposed preconditioner.