From multigrid solvers to general multiscale computation

Numerical methods for solving scientific problems tend to be extremely costly, for several general reasons that will be explained. Model studies have shown that each of these reasons can in principle be removed by multiscale (e.g., multigrid) algorithms. These algorithms employ separate processing at each scale of the physical space, combined with interscale iterative interactions, in ways which use finer scales very sparingly. Having been developed first and well known as fast PDE solvers, multiscale techniques have subsequently been developed for many other types of computational tasks, including: inverse PDE problems; highly indefinite equations; highly disordered and grid-free systems (gauge fields, finite elements, etc.); fast computation of large determinants, integral transforms and many-body long-range interactions; integro-differential equations; many-eigenfunction problems (for electronic structures, wave propagation, etc.); fast Monte-Carlo sampling in statistical physics; molecular dynamics of macromolecules and fluids; global and discretestate minimization of functionals fraught with multiscale attraction basins; data mining and large graph problems; image segmentation and recognition; tomography (medical imaging reconstruction); and more.