Abstract

The burgeoning field of Finite Element Exterior Calculus (FEEC), which first took the stage in Beijing in 2002, reveals the relationship between and unity among many different families of finite elements used in structural engineering, solid and fluid mechanics, electromagnetics, and other applications. FEEC leads to better understanding of the properties of the finite element methods and new ideas for their implementation, and with it we have been able to address some major problems of long standing in computational engineering. In particular, FEEC reveals the special role of two families of finite element spaces for H(curl) and H(div), which are distinct but inter-related generalizations of standard Lagrange family of finite elements for the Sobolev space H^1. When viewed as spaces of differential forms, these finite element spaces can be constructed in a uniform, dimension-independent way using the Koszul complex. They can be combined in a variety of ways into finite dimensional subcomplexes of the de~Rham complex which can be related to the de~Rham complex via bounded projections which commute with exterior differentiation. From this structure follow the stability properties which make these elements effective in practice for solving a variety of partial differential equations. Moreover this approach leads to the solution of important practical question, such as the construction of natural geometric bases for all the spaces.