Risk Assessment and Control in Financial Networks under Incomplete Information

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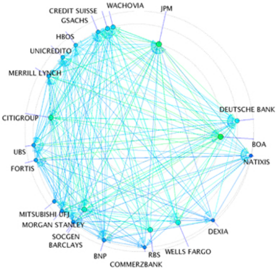
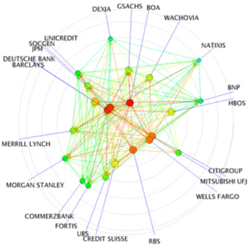
Financial Network

- Interacting with each other through borrowing and lending.
- Interconnecting indirectly through the market by holding similar shares or portfolio.

The positiveness of interconnections: Speed up the transaction process and save cost.





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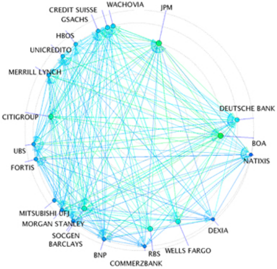
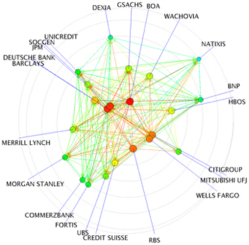
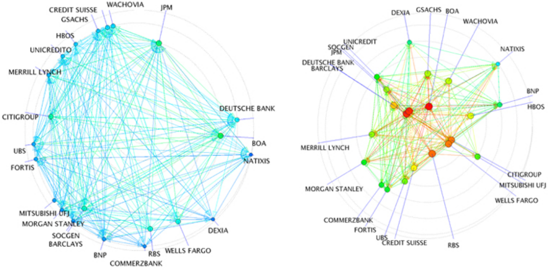




Financial Network

- Interacting with each other through borrowing and lending.
- Interconnecting indirectly through the market by holding similar shares or portfolio.

  The positiveness of interconnections: Speed up the transaction process and save cost, help to diversify the risk.

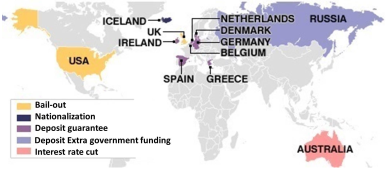


Systemic Risk in Financial Network

- The failure of a single node (caused by some market shock) can be
- quickly spread to the entire network

As evidence: 2008 financial crisis in US

Global effect of financial crisis 2007-2008.



Global action after financial crisis 2007-2008.

Related Works

Eisenberg & Noe

- Systemic Risk in Financial Systems (2001).

Elsinger et al.

- Using Market Information for Banking System Risk Assessment(2005-2006).
- Financial Networks, Cross Holdings, and Limited Liability(2009).
- Network Models and Systemic Risk Assessment (2013).

Allen, F., & Gale, D.

- Financial Contagion (2000).

Acemoglu et al.

- Systemic Risk and Stability in Financial Networks (2013).

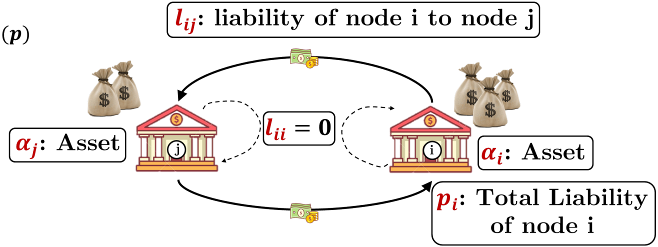
Capponi et al.

- Liability Concentration and Systemic Losses in Financial Networks (2016).

Eisenberge-Noe's (Clearing Agent Model)

$P = diag(p)$

$e = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$



$$\max_x \quad p^T x \longrightarrow \text{Maximize Payments}$$

$$s.t. \quad (P - L^T)x \le \alpha \longrightarrow \text{Limited Liability}$$

$$0 \le x \le e \longrightarrow \text{Absolute Priority}$$

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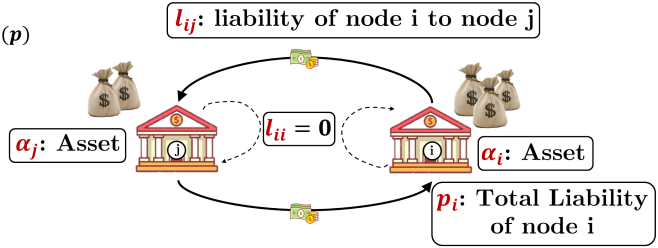

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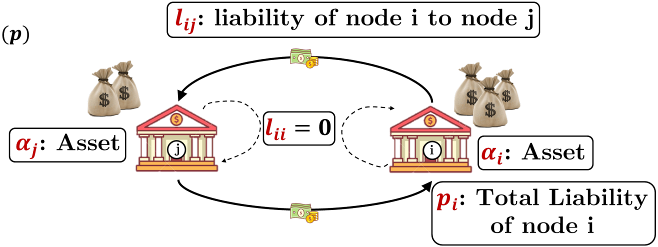

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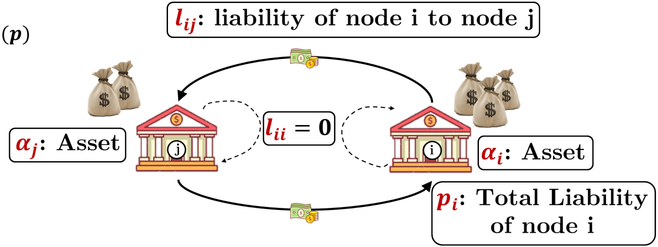

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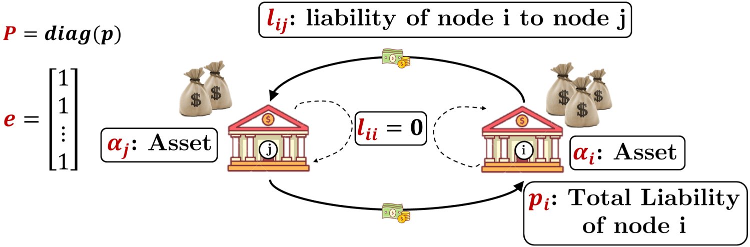


$$\max_x \quad p^T x$$
$$s.t. \quad (P - L^T)x \leq \alpha$$
$$0 \leq x \leq e$$

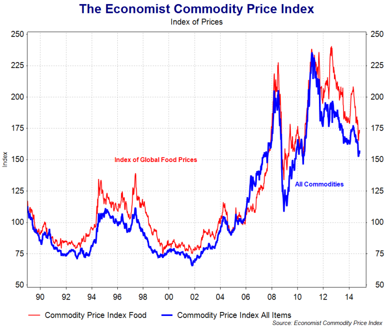
$x_i^* = 1$: Solvent

$x_i^* \in [0, 1)$: Default

$x_i^* < 0$: Bankrupted

The Uncertainties

1. Market Fluctuation
2. Incomplete Information on Interbank Liabilities



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Balance Sheet

As of August 06,2013

Assets	
Cash on Hand	15,000.00
Account Receivable	6,795.00
Inventory	3,000.00
Equipment	1,763.00
Assets	**26,558.00**

Liabilities	
Account Payable	2,200.00
Taxes Payable	883.00
Current Loans Payable	5000.00
Long Terms Loans Payable	1,578.00
Credit Cards Payable	900.00
Other Liabilities	6,117.00
Liabilities	**16,678.00**
Owner's Equity	
Owner's Capital	4,800.00
Retained Earnings	5,080.00
Equity	9,880.00
Liability and Equity	26,558.00

The Uncertainties

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2. Incomplete Information on Interbank Liabilities

$$
\begin{array}{c|c}
 & \sum_j \\
\begin{bmatrix} 0 & l_{12} & l_{13} & \cdots & l_{1n} \\ l_{21} & 0 & l_{23} & \cdots & l_{2n} \\ l_{31} & l_{32} & 0 & \cdots & l_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 0 \end{bmatrix} & \boxed{\begin{matrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{matrix}} \\
\hline
\sum_i \quad \boxed{\begin{matrix} c_1 & c_2 & c_3 & \cdots & c_n \end{matrix}} &
\end{array}
$$

Known
(balance sheet)

This matrix is typically estimated by entropy optimization based on Kullback-Leibler divergence .

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The Challenge

Issues in the existing analysis:

- This estimated liability matrix lead to a complete network and underestimate the systemic risk;
- Most of the analysis is done based on small shock assumption;

Questions to be addressed:

- What's the worst-case/best-case structure of the financial network?
- What is the impact of the network structure on the stability of financial system?

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Stability of the Financial Network

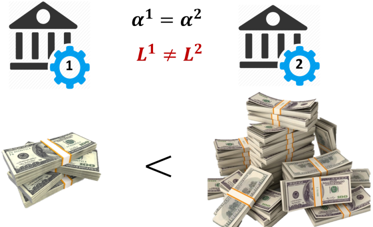
Definition:

Consider two financial systems with the same asset vector (α) with liability matrixes L^1 and L^2 satisfying $e^T L^1 = e^T L^2$, and $L^1 e = L^2 e$. The first system is said to be less stable than the second one if

$$p^T x^*(L^1) < p^T x^*(L^2),$$

where $x^*(L^1)$ and $x^*(L^2)$ denote the optimal solution of (2.1) when $L = L^1$ and $L = L^2$ receptively.

$$\begin{aligned} \max_{x} \quad & p^T x \qquad\qquad (2.1) \\ s.t. \quad & (P - L^T)x \leq \alpha \\ & x \leq e \end{aligned}$$



Early work by Caponi et al. compare two systems that have different total liabilities using the concept of concentration.

Worst-Case Optimization under Uncertain Liability Matrix

$$\begin{aligned} \min_{\Delta L \in \mathcal{U}_L} \max_{x} \quad & p^T x \\ s.t. \quad & (P - L^T - \Delta L^T) x \leq \alpha \\ & x \leq e \end{aligned}$$

$$\begin{aligned} \mathcal{U}_L = \{\Delta L : \; & \Delta L e = \Delta L^T e = 0, \\ & \Delta l_{ii} = 0, \quad \forall i = 1, \cdots, n, \\ & -l_{ij} \leq \Delta l_{ij}, \quad \forall i, j = 1, \cdots, n\}. \end{aligned}$$

The dual of sub problem

$$\begin{aligned} \min_{\Delta L \in \mathcal{U}_L} \min_{\lambda} \quad & (\alpha^T - e^T (P - L))\lambda + e^T \Delta L \lambda \\ & + e^T p \\ s.t. \quad & (P - L - \Delta L)\lambda \leq p \\ & \lambda \geq 0, \end{aligned}$$

$$\begin{aligned} \min_{\lambda, \Delta L} \quad & (\alpha^T - e^T (P - L))\lambda + e^T p \\ s.t. \quad & (P - L - \Delta L)\lambda \leq p \\ & \lambda \geq 0 \\ & \Delta L \in \mathcal{U}_L \end{aligned}$$

The WCLO problem involves some non-convex quadratic constraint.

Stability of the System Under L^+

Theorem 2.1

Suppose that λ^* be the optimal solution of (2.3). If we choose ΔL be the feasible solution of (2.2), then we have

$$f(L^+) \leq f(L),$$

where $f(L)$ and $f(L^+)$ denote the objective value of (2.1) when the liability matrix is L and L^+ respectively.

$$\begin{aligned} &\Delta L \lambda^* \geq (P - L)\lambda^* - p \\ &\Delta L \in \mathcal{U}_L \end{aligned} \tag{2.2}$$

$$\begin{aligned} \min_{\lambda} \quad & (\alpha^T - e^T(P - L))\lambda + e^T p \\ s.t. \quad & (P - L)\lambda \leq p \\ & \lambda \geq 0 \end{aligned} \tag{2.3}$$

The index sets $\mathcal{I}_1$ and $\mathcal{I}_2$

For such a purpose, we need to define the following index sets:

$$\mathcal{I}_1 = \{i : \lambda_i^* > 0\} \rightarrow \text{Default Nodes } (x_i^* < 1)$$

$$\mathcal{I}_2 = \{i : \lambda_i^* = 0\} \rightarrow \text{Solvent Nodes } (x_i^* = 1)$$

$$\begin{aligned} \min_{\lambda,\mu} \quad & \alpha^T \lambda + e^T \mu \\ s.t. \quad & (P - L)\lambda + \mu = p \\ & \lambda \geq 0 \\ & \mu \geq 0. \end{aligned} \tag{2.4}$$

The case when $|\mathcal{I}_1| \leq 1$

Suppose that $|\mathcal{I}_1| = 0$. In this case, changing the liability matrix L will not reduce the stability of the system.

Theorem 2.2

Suppose that $|\mathcal{I}_1| = \{1\}$. In this case, the following conclusions hold.

(i) *If $\exists j \in \mathcal{I}_2$ such that problem (2.5) has positive optimal value, then, the stability of the system under updated liability matrix L^+ strictly decreases.*

(ii) *If $\forall j \in \mathcal{I}_2$ problem (2.5) is infeasible, then, the stability of the system under updated liability matrix L^+ remain the same.*

$$\begin{aligned} \Delta L^j = \arg\max \quad & \delta_j \\ s.t. \quad & [(P - L - \Delta L)^T x^*(L)]_j = \alpha_j + \delta_j; \\ & \Delta L \in \mathcal{U}_L, \end{aligned} \tag{2.5}$$

Update Scheme I

For fixed $\lambda = \lambda^*$, the feasible set of problem (2.2) reduces to a polyhedron as follows.

$$\boxed{\begin{array}{l}(P - L - \Delta L)\lambda^* \leq p; \\ \Delta L \in \mathcal{U}_L.\end{array}} \longrightarrow \quad \mathcal{U}_L^*$$

We next develop the following optimization model:

$$\begin{array}{ll} \max\limits_{\Delta L} & e_{\mathcal{I}_1}^T \Delta L \lambda^* \\ s.t. & \Delta L \in \mathcal{U}_L^*; \end{array} \tag{2.6}$$

where $e_{\mathcal{I}_1}$ is a vector such that its i-th element equal 1 for all $i \in \mathcal{I}_1$, and the rest of its elements equal 0.

Theorem 2.3

Let ΔL^* denotes the optimal solution of problem (2.6) and $L^+ = L + \Delta L^*$. If $[\Delta L^* \lambda^*]_i > 0, \forall i \in \mathcal{I}_1$, then the stability of the system strictly decreases.

One Variant of Update Scheme I

We solve problem (2.7) to find ΔL that can reduce the stability of the system and keep the dominance relationship between the payment ratios before/after the update.

Theorem 2.4

Let ΔL^* denotes the optimal solution of problem (2.7) and $L^+ = L + \Delta L^*$. Let $f(L)$ and $f(L^+)$ denote the objective value of (2.1) with liability matrix L and L^+, respectively. If $e_{\mathcal{I}_1}^T \Delta L^* \lambda^* > 0$, then the following holds:

(i) *For all $i \in \mathcal{I}$ we have $\lambda^*(L^+) \geq \lambda^*(L)$, and there exist $i \in \mathcal{I}_1$ such that $\lambda_i^*(L^+) > \lambda_i^*(L)$;*

(ii) *For all $i \in \mathcal{I}$ we have $x_i^*(L^+) \leq x_i^*(L)$ and there exists $i \in \mathcal{I}_1$ such that $x_i^*(L^+) < x_i^*(L)$.*

$$\begin{aligned} \max_{\Delta L} \quad & e_{\mathcal{I}_1}^T \Delta L \lambda^* \\ s.t. \quad & \Delta L \in \mathcal{U}_L^*; \\ & \Delta L \hat{\lambda}^i \geq 0, \quad \forall i \in \mathcal{I}_1. \end{aligned} \tag{2.7}$$

$$\begin{aligned} \min_{\lambda} \quad & (\alpha^T - e^T(P - L))\lambda + p_i \\ s.t. \quad & (P - L)\lambda \leq p_i e_i, \quad \lambda \geq 0; \end{aligned} \tag{2.8}$$

Update Scheme II

For all non-solvent nodes we have the following equation system:

$$p_j \lambda_j - \sum_{k \in \mathcal{I}_1, k \neq j} l_{jk} \lambda_k = p_j, \quad \forall j \in \mathcal{I}_1$$

Assumption 2:

(i) The submatrix $(P - L)_{\mathcal{I}_{11}}$ is diagonally row dominant and with at least one row that is strictly dominant;

(ii) The index set $\mathcal{I}_1$ is precisely the set of default nodes.

Consequently we can rewrite problem (2.2) as the following:

$$\begin{aligned} \min_{\Delta L} \quad & (\alpha^T - e^T (P - L - \beta \Delta L)_{\mathcal{I}_1})(P - L - \beta \Delta L)^{-1}_{\mathcal{I}_{11}} p_{\mathcal{I}_1} \\ s.t. \quad & \Delta L \in \mathcal{U}_L. \end{aligned} \tag{2.9}$$

Update Scheme II

$$\begin{array}{ll} \min\limits_{\Delta L} & \overbrace{(\alpha^T - e^T(P-L))_{\mathcal{I}_1}(P-L)^{-1}_{\mathcal{I}_{11}} \Delta L_{\mathcal{I}_{11}} (P-L)^{-1}_{\mathcal{I}_{11}} p_{\mathcal{I}_1}}^{\mathcal{D}_{\Delta L} f(L)} \\ s.t. & \Delta L \in \mathcal{U}_L. \\ & ||\Delta L||_1 \leq 2\min\{l_{ij} > 0, \forall i,j = 1, \cdots, n\}. \end{array} \tag{2.10}$$

If the optimal value of (2.10)→ is negative, we can apply a line search procedure to find β ($L^+ = L + \beta\Delta L$) the objective function value in (2.9) will be reduced.

$$\begin{array}{ll} \min\limits_{\Delta L} & (\alpha^T - e^T(P-L))_{\mathcal{I}_1}(P-L)^{-1}_{\mathcal{I}_{11}} (\Delta L_{\mathcal{I}_{11}} (P-L)^{-1}_{\mathcal{I}_{11}})^2 p_{\mathcal{I}_1} \\ s.t. & \Delta L \in \mathcal{U}_L; \\ & ||\Delta L||_1 \leq 2\min\{l_{ij} > 0, \forall i,j = 1, \cdots, n\}. \end{array}$$

If the solution from (2.10) is meaningless, we can use the second-order Taylor expansion to approximate the objective function in (2.9)

Update Scheme II

$$\begin{array}{lll} \min\limits_{\Delta L} & \overbrace{(\alpha^T - e^T(P-L))_{\mathcal{I}_1}(P-L)^{-1}_{\mathcal{I}_{11}}\Delta L_{\mathcal{I}_{11}}(P-L)^{-1}_{\mathcal{I}_{11}}p_{\mathcal{I}_1}}^{\mathcal{D}_{\Delta L}f(L)} & (2.10) \\ s.t. & \Delta L \in \mathcal{U}_L. & \\ & ||\Delta L||_1 \leq 2\min\{l_{ij} > 0, \forall i, j = 1, \cdots, n\}. & \end{array}$$

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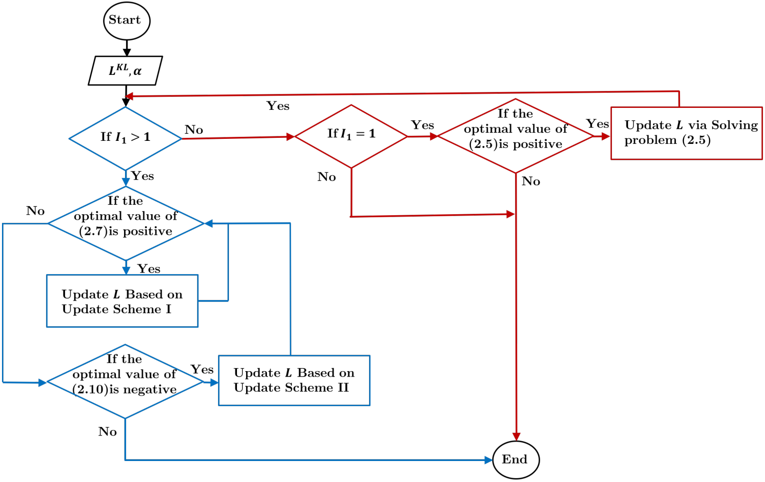
The above second-order approximation model can help to get a local optimal solution!

Generic Algorithm



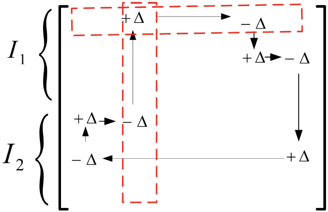
The Characteristics of the Identified Least Stable Network

To illustrate the characteristics of the identified least stable network structure, we use the graph modeling approach.

We consider an un-directed graph where the non-zero elements of the liability matrix are the vertices, whose nodes in the same column or the same row are connected to each other.

Assumption: All the entries of λ^* are listed in a decreasing order as follows: $\lambda_1^* \geq \cdots \geq \lambda_n^*$.

Definition: A Δ-loop in $\mathcal{U}_L$ is a matrix such that each of its nontrivial row (or column) contains exactly two nonzero elements, one with value $+\Delta$ and another with $-\Delta$.



The Characteristics of the Identified Least Stable Network

For the case that a meaningful cycle cannot be found, the algorithm will fail to reduce the stability of the system.
One possible scenario is when $L_{\mathcal{I}_{12}} = L_{\mathcal{I}_{21}} = 0$, and the off-diagonal elements in sub-matrix $L_{\mathcal{I}_{11}}$ is zero (tridiagonal network structure).

Table 1: Domino Effect of Bankruptcy in a Tridiagonal Financial Network with a Monopoly Node.

Liability Matrix						
Node	1	2	3	4	p	α
1	0	31855	0	0	31855	32698
2	31855	0	18016	0	49871	0
3	0	18016	0	73185	91201	0
4	0	0	73185	0	73185	0
Claims	31855	49871	91201	73185	246112	32698

Optimal Solution				
x_1^*	x_2^*	x_3^*	x_4^*	
1	1	1	1	α
1	0	0	0	$\alpha_{2-} = -31855$

The Best-case Scenario

Theorem 3.1

Suppose that $|\mathcal{I}_1| \geq 1$ and L^+ be the liability matrix that is obtained from modified generic algorithm, then we have

(i) *For all $i = 1, \cdots, n$, we have $x_i^*(L^+) \geq x_i^*(L)$;*

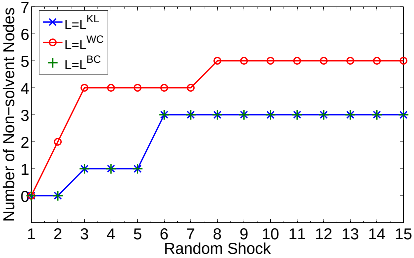
(ii) *There exists $i \in \mathcal{I}_1$ such that $x_i^*(L^+) > x_i^*(L)$.*

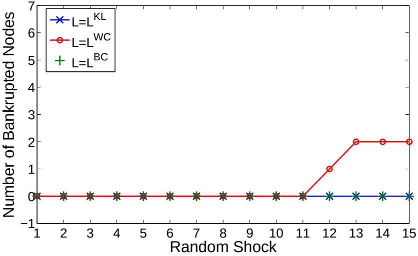
where $x^*(L^+)$ and $x^*(L)$ denote the optimal solution of (2.1) when the liability matrix is L^+ and L.

$$\begin{array}{lll} \max\limits_{\Delta L \in \mathcal{U}_L} & \max\limits_{x} & p^T x \\ & s.t. & (P - L^T - \Delta L^T)x \leq \alpha \\ & & x \leq e \end{array} \qquad \begin{array}{ll} \max\limits_{\Delta L} & e_{\mathcal{I}_1}^T \Delta L^T x^* \\ s.t. & \Delta L \in \mathcal{U}_L^*. \end{array}$$

Contagious Effect of Failure

Example: We consider a complete financial network with 8 banks. The liability matrix is extracted from the liability matrix in [Chen et al.(2016)] (see Table 6 in [Chen et al.(2016)]) by considering the first eight banks in the network which is estimated based on the KL divergence. The asset vector is $\alpha^0 = (160, 10, 50, 50, 1000, 1700, 1570, 1670)^T$ with $e^T\alpha^0 = 6210$.





Contagious Effect of Failure

Table 2: The shadow price when the financial system is subjected to shock vectors s ($e^T s = 0$) under three different network structure (L^{KL}, L^{WC}, L^{BC}).

Optimal dual solution when $L = L^{KL}$

λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	λ_6^*	λ_7^*	λ_8^*	
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	α^1
2.54	2.53	2.42	2.39	0.0	0.0	0.0	0.0	α^2

Optimal dual solution when $L = L^{WC}$

λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	λ_6^*	λ_7^*	λ_8^*	
36.58	35.58	0.0	0.0	0.0	0.0	0.0	0.0	α^1
47.80	46.80	11.22	9.07	0.0	0.0	0.0	0.0	α^2

Optimal dual solution when $L = L^{BC}$

λ_1^*	λ_2^*	λ_3^*	λ_4^*	λ_5^*	λ_6^*	λ_7^*	λ_8^*	
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	α^1
2.39	2.32	2.31	2.26	0.0	0.0	0.0	0.0	α^2

Note that asset vector can be obtained as $\alpha^i = \alpha^{i-1} + s, i = 1, 2$, where $s = (-200, 0, 0, 0, 0, 0, 100, 100)^T$.

The Performance of the New Policy

Example: We consider the system consisting of the banking sectors in eight European countries for December 2009. The data matrix and asset vector is extracted from Capponi et al. (2015).

Liability Matrix $L(0)$

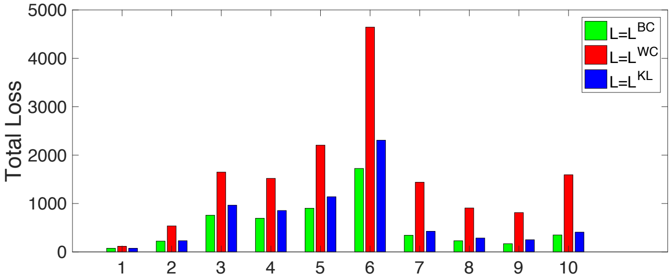
Node	1	2	3	4	5	6	7	8	p	$\alpha(0)$
1	0	500.62	341.62	409.36	189.95	231.97	36.22	10.43	1720.2	1483.7
2	172.97	0	292.94	51.02	176.58	36.35	20.52	4.62	755	-141.71
3	239.17	195.64	0	50.42	92.73	20.6	32.57	8.08	639.21	-335
4	114.14	237.98	219.64	0	119.73	30.23	26.56	28.08	776.36	457.58
5	96.69	155.65	150.57	22.82	0	15.47	28.11	11.39	480.7	-110.07
6	187.51	183.76	60.33	15.66	30.82	0	64.5	21.52	564.1	308.94
7	30.72	40.68	301.37	9.42	131.55	6.11	0	1.17	521.02	386.4
8	24.26	47.38	44.74	86.08	12.41	5.43	3.14	0	223.44	184.15
c	865.46	1361.7	1411.2	644.78	753.77	346.16	211.62	85.29	5680	2234

Optimal Solution

x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*
1	1	1	1	1	1	1	1

The Performance of the New Policy

Example:
We set time horizon $T = 10$, and time steps $\Delta t = 1$.
$s_i \in \mathcal{N}(0, \sigma^2) \rightarrow \sigma_i = 0.25 p_i, \forall i = 1, \cdots, n.$



Total loss : $\sum_i p_i(1 - x_i^*)$

Summary

Contribution:

- Identify the worst-case and best case scenario with incomplete liability matrix;
- Identify the contagion effect of the risk;
- Identify the mitigation policy to mitigate the systemic risk.

Future Work:

- Studying the vulnerability of financial networks considering liquidation process;
- Studying the resilience of financial networks under new regulations; New strategies to stablize the financial crisis (fast recovery).

References I

- Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2013). Systemic risk and stability in financial networks. *American Economic Rev.* **105**(2): 564–608.
- Allen, F., and Gale, D. (2000). Financial contagion. *Journal of Political Economy.* **108**(1):1–33.
- Chen, N., Liu, X., and Yao, D. D. (2014). An optimization view of financial systemic risk modeling: network effect and the market liquidity effect. *Oper. Res.* **64**(5):1089–1108.
- Eisenberg, L., and Noe, T. H. (2001). Systemic risk in financial systems.*Management Sci.* **47**(2):236–249.
- Elsinger, H., Lehar, A., and Summer, M. (2005). Using market information for banking system risk assessment. *International Journal of Central Banking2,* 137–165.
- Elsinger, H., Lehar, A., and Summer, M. (2006). Risk assessment for banking systems. *Management Sci.* **52**(9):1301–1314.
- Elsinger, H. (2009). Financial networks, cross holdings, and limited liability. *Oesterreichische National Bank.*

Thank You!