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Transceiver Optimization in Full-Duplex Multi-User Networks

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Outline

- Transmit and receive beamforming optimization in half-duplex systems
- Full-duplex (FD) communications
- Transceiver design for multi-user FD systems
- Global solution and efficient suboptimal solution
- Numerical results

Multi-antenna Communications

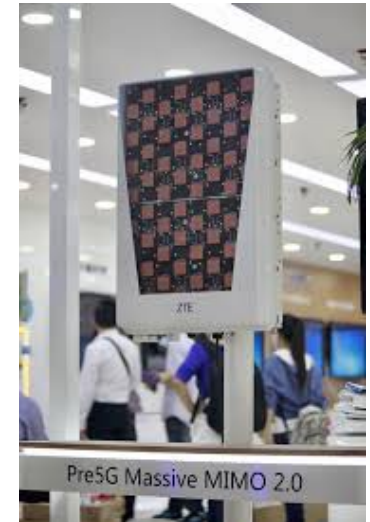
- Key to modern communication systems



Multi-antenna
base station



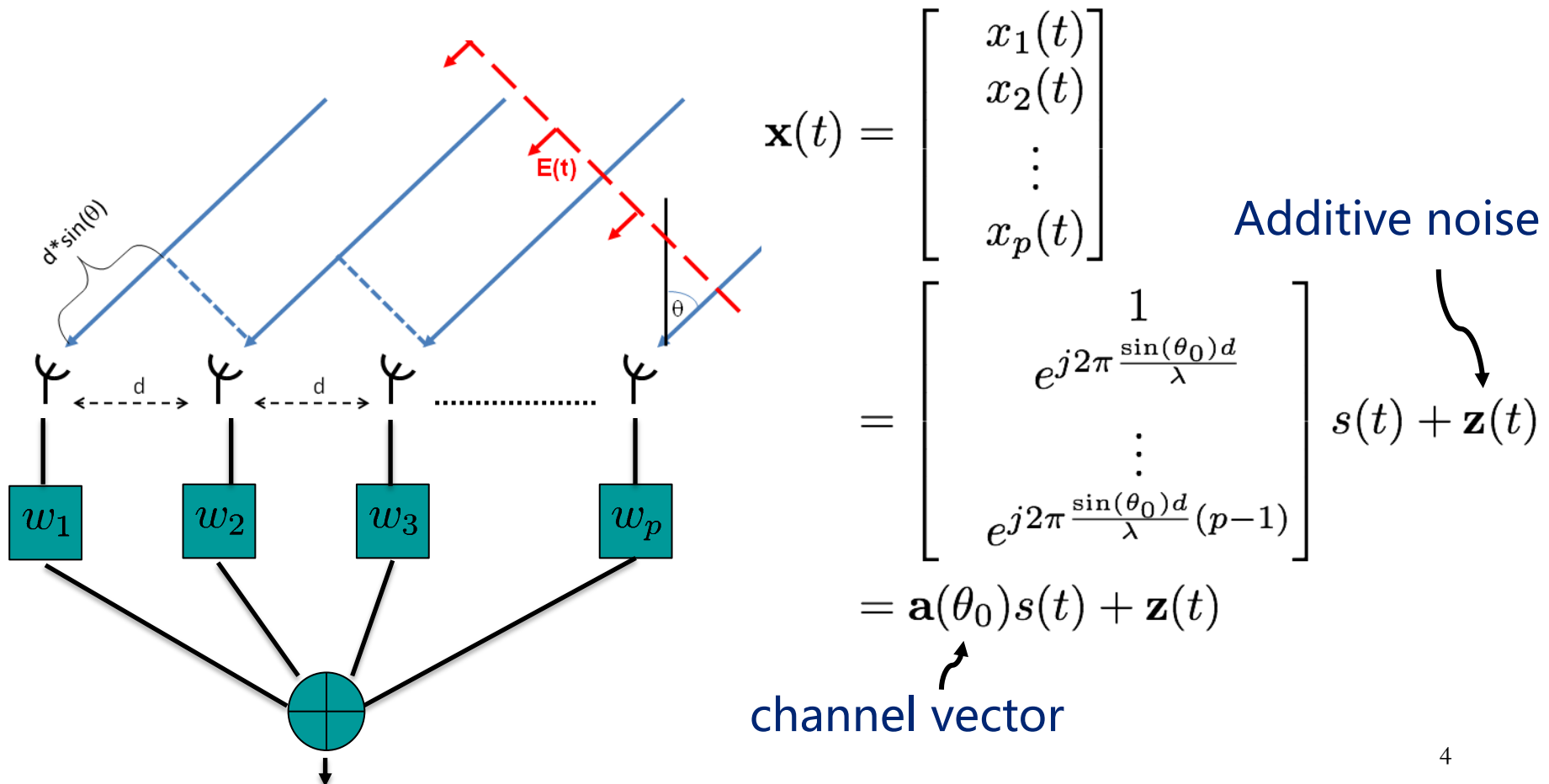
Radar array on Destroyer



Massive MIMO
for 5G

Array Signal Model

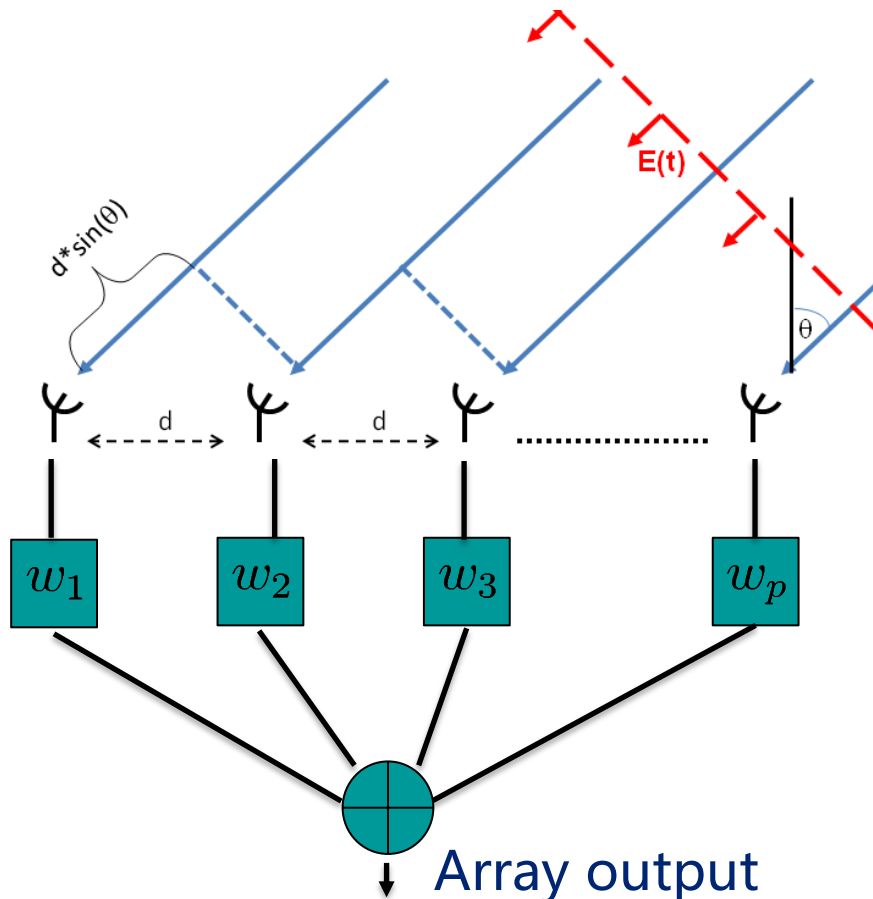
■ Signal reception via antenna array





Array Signal Model

■ Linear processing



Array output

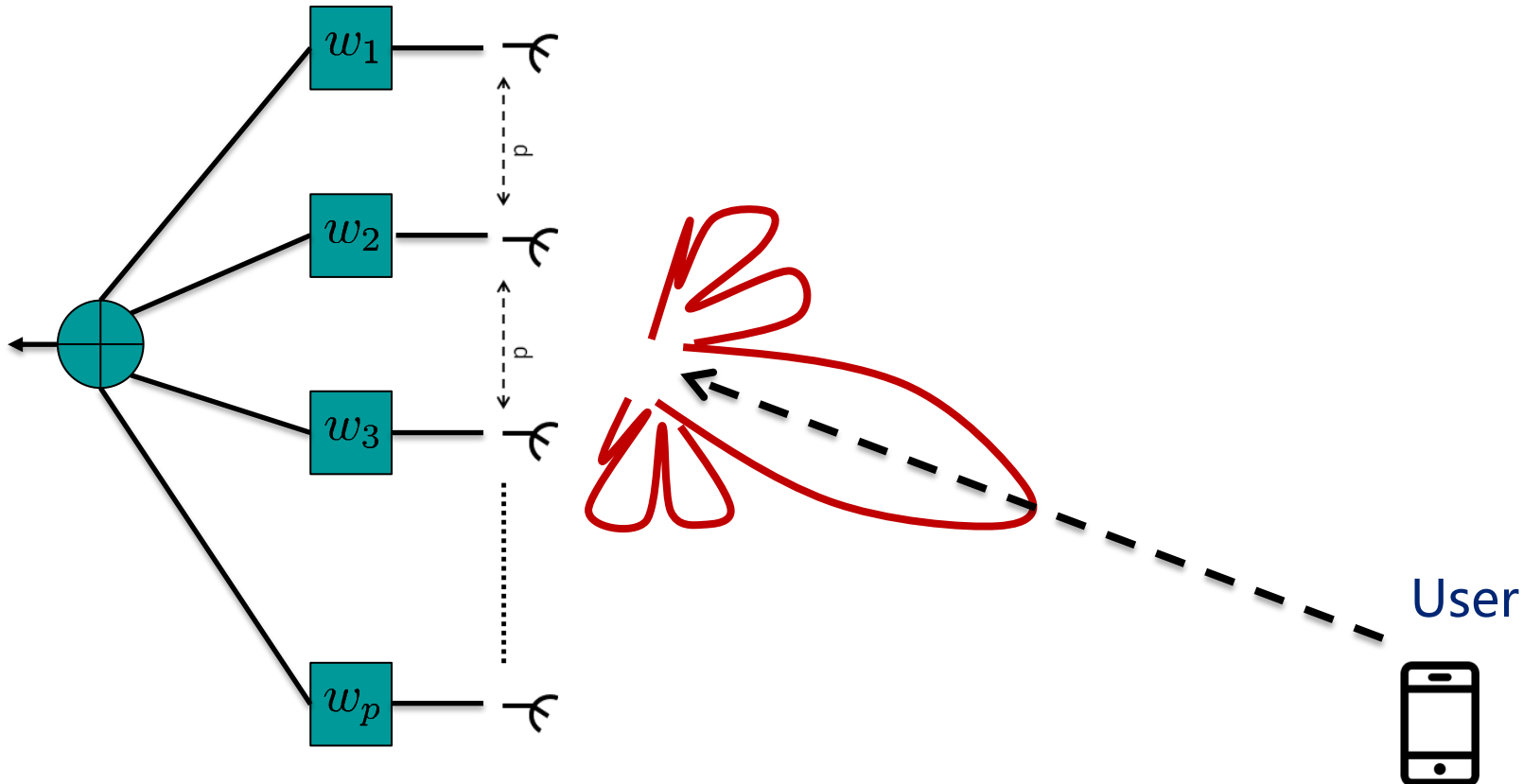
$$= \mathbf{w}^H (\mathbf{a}(\theta_0) s(t) + \mathbf{z}(t))$$

Maximum Signal-to-noise ratio (SNR) is achieved when

$$\mathbf{w} = \mathbf{a}(\theta_0)$$

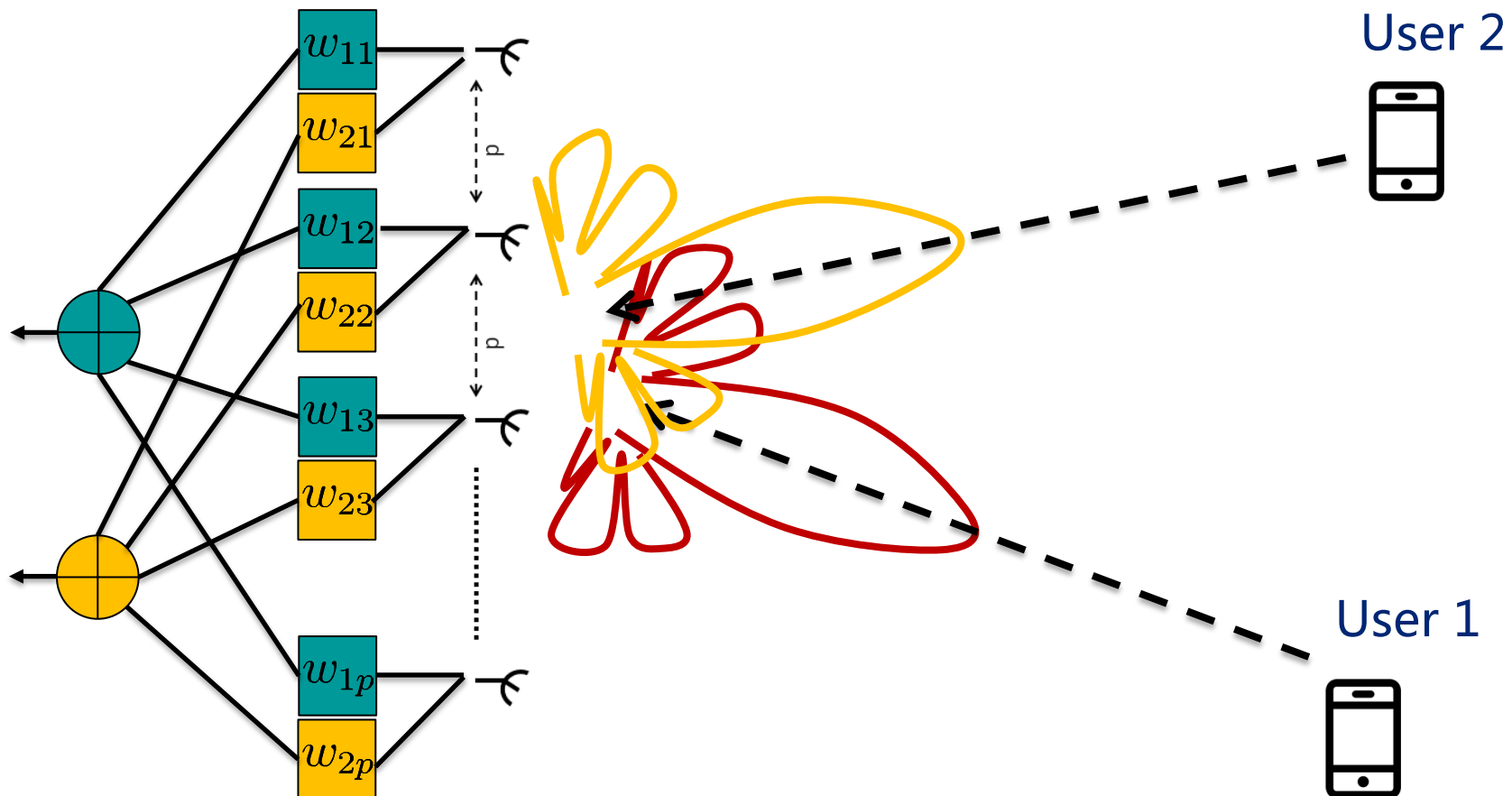
Array Signal Model: **Beamforming**

- Array response: Suppose $\theta_0 = 30^\circ$



Array Signal Model: **Beamforming**

- Array response: Suppose $\theta_1 = 30^\circ$, $\theta_2 = -30^\circ$

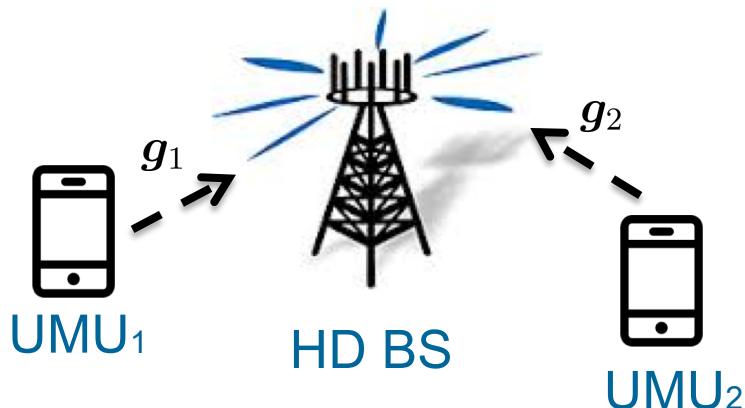


Multi-User Uplink

■ Signal model

Receive beamformer

$\mathbf{v}_1, \mathbf{v}_2$



Received signal at the BS

$$\mathbf{y}^U(t) = \sum_{\ell=1}^L \mathbf{g}_{\ell} \sqrt{p_{\ell}^U} s_{\ell}^U(t) + \mathbf{z}^U(t).$$

Beamforming output

$$\mathbf{v}_{\ell}^H \mathbf{y}^U(t), \ell = 1, \dots, L$$

Signal-to-interference-plus-noise ratio (SINR)

$$\frac{p_{\ell}^U |\mathbf{v}_{\ell}^H \mathbf{g}_{\ell}|^2}{\sum_{j \neq \ell}^L p_j^U |\mathbf{v}_{\ell}^H \mathbf{g}_j|^2 + \sigma_z^2 \|\mathbf{v}_{\ell}\|_2^2}, \ell = 1, \dots, L$$



Optimal Receive Beamforming

- Design problem

$$\begin{aligned} \text{(HUL)} \quad & \min_{\{\mathbf{v}_\ell\}, \{p_\ell^U \geq 0\}} \sum_{\ell=1}^L p_\ell^U \\ \text{s.t.} \quad & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2}{\sum_{j \neq \ell}^L p_j^U \mathbf{v}_\ell^H \mathbf{g}_j \mathbf{g}_j^H \mathbf{v}_\ell + \sigma_z^2 \|\mathbf{v}_\ell\|_2^2} \geq \gamma_\ell^U, \ell \in \mathcal{L}, \end{aligned}$$

where γ_ℓ^U is the target SINR for user ℓ .

- **Non-convex**, but is polynomial time solvable [GSSBO'10].



Optimal Solution Structure

- When the uplink powers $\{p_\ell^U\}$ are fixed, **optimal receive beamforming** are

$$\mathbf{v}_\ell = \frac{\tilde{\mathbf{v}}_\ell}{\|\tilde{\mathbf{v}}_\ell\|}, \quad \tilde{\mathbf{v}}_\ell = \left(\sum_{j=1}^L p_j^U \mathbf{g}_j \mathbf{g}_j^H + \sigma_z^2 \mathbf{I}_{N_t} \right)^{-1} \mathbf{g}_\ell,$$

- The optimal power solution is **unique** and satisfies

$$p_\ell^U = \frac{\rho_\ell^U}{\mathbf{g}_\ell^H \left(\sum_{j=1}^L p_j^U \mathbf{g}_j \mathbf{g}_j^H + \sigma_z^2 \mathbf{I}_{N_t} \right)^{-1} \mathbf{g}_\ell}, \quad \forall \ell \in \mathcal{L}.$$

- **Fixed-point iterations** converges to the optimal power values.



Equivalent as an SDP

- Semidefinite Programming (SDP) reformulation

$$\begin{aligned} \max_{\{p_\ell^U \geq 0\}} \quad & \sum_{\ell=1}^L p_\ell^U \\ \text{s.t.} \quad & \sum_{j=1}^L p_j^U \mathbf{g}_j \mathbf{g}_j^H + \sigma_z^2 \mathbf{I}_{N_t} \succeq \left(\frac{p_\ell^U}{\rho_\ell^U} \right) \mathbf{g}_\ell \mathbf{g}_\ell^H, \quad \forall \ell \in \mathcal{L}, \end{aligned}$$

where $\frac{1}{\rho_\ell^U} \triangleq 1 + 1/\gamma_\ell^U$.

- Thus, (HUL) is polynomial-time solvable.

Multi-User Downlink

■ Signal model

Transmit signal

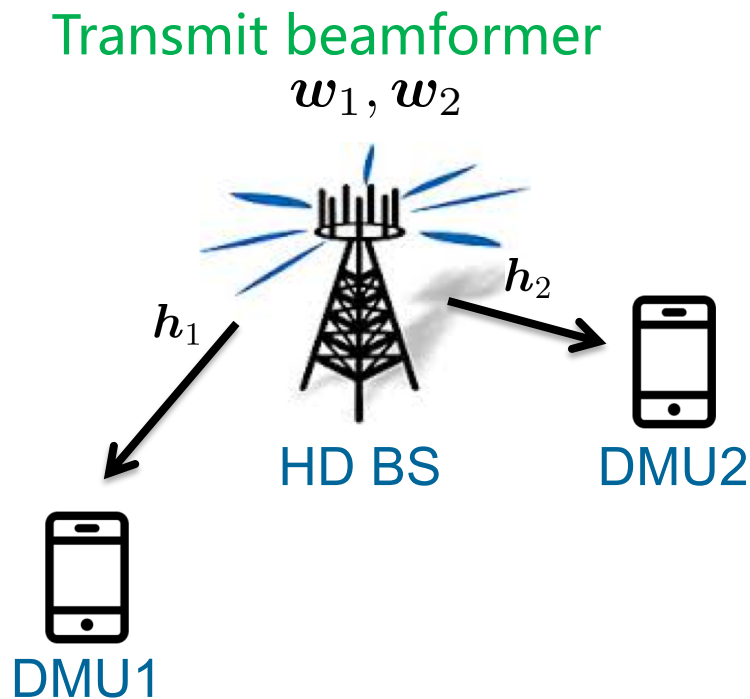
$$\mathbf{x}(t) = \sum_{i=1}^K \mathbf{w}_i s_i^D(t)$$

Received signal at user i

$$y_i^D(t) = \mathbf{h}_i^H \sum_{k=1}^K \mathbf{w}_k s_k^D(t) + z_i^D(t), \quad \forall i \in \mathcal{K}$$

Signal-to-interference-plus-noise ratio (SINR)

$$\frac{|\mathbf{h}_i^H \mathbf{w}_i|^2}{\sum_{k \neq i}^K |\mathbf{h}_i^H \mathbf{w}_k|^2 + \sigma_i^2}, \quad i \in \mathcal{K}$$





Optimal Transmit Beamforming

- Design problem

$$\begin{aligned} \text{(HDL)} \quad & \min_{\{\mathbf{w}_k\}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 \\ \text{s.t.} \quad & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2}{\sum_{k \neq i}^K |\mathbf{w}_k^H \mathbf{h}_i|^2 + \sigma_i^2} \geq \gamma_i^D, \quad I \in \mathcal{K} \end{aligned}$$

where γ_i^D is the target SINR for user i .

- **Non-convex**, but is polynomial time solvable [GSSBO'10].



Equivalent as an SOCP or SDP

- Second-order cone programming (**SOCP**) reformulation (via a proper phase rotation)

$$\begin{aligned} \min_{\{\mathbf{w}_k\}} \quad & \sum_{k=1}^K \|\mathbf{w}_k\|^2 \\ \text{s.t.} \quad & \frac{\mathbf{h}_i^H \mathbf{w}_i}{\sqrt{\rho_i^D}} \geq \sqrt{\sum_{k=1}^K |\mathbf{h}_i \mathbf{w}_k|^2 + \sigma_i^2}, \quad k \in \mathcal{K} \end{aligned}$$

- Moreover, **SDP relaxation** is tight to (HDL)!

$$\mathbf{w}_i \mathbf{w}_i^H \iff \mathbf{W}_i \succeq \mathbf{0}, \text{ rank}(\mathbf{W}_i) = 1$$

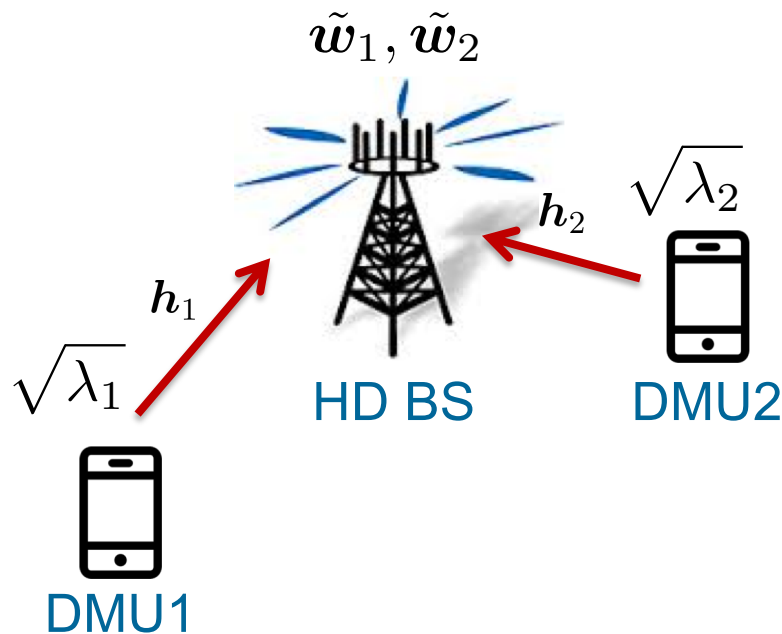
[BO'01] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," Chapter 18 in Handbook of Antennas in Wireless Communications, L. C. Godara, Ed., CRC Press, Aug. 2001.

[GSSBO'10] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming," IEEE Signal Process. Mag., pp. 62–75, May 2010.

Uplink-Downlink Duality

- Via Lagrange duality, (HDL) has a **virtual uplink** problem

Virtual receive beamformer

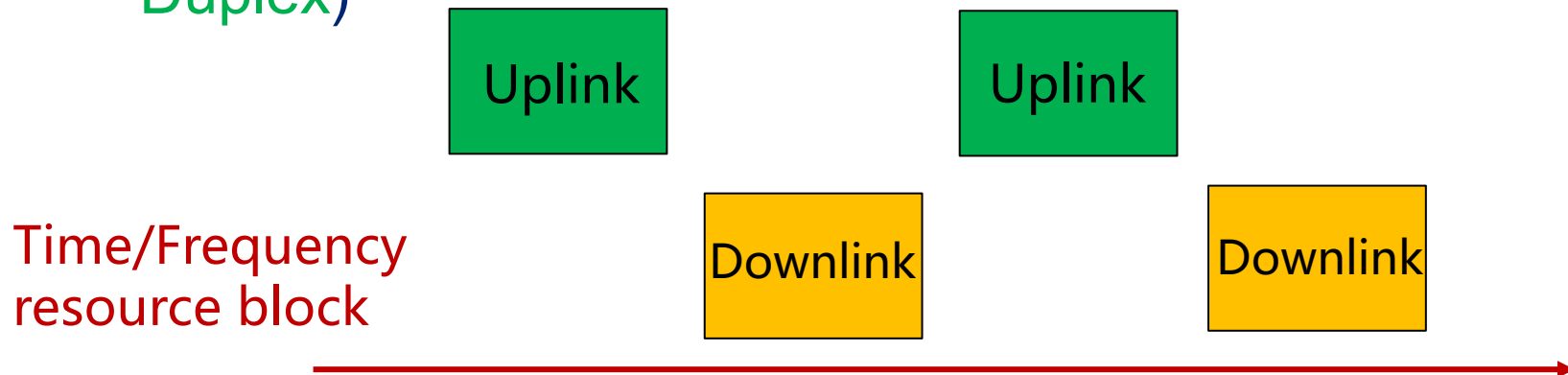


$$\begin{aligned} & \min_{\{\tilde{\mathbf{w}}_i\}, \{\lambda_i \geq 0\}} \sum_{i=1}^K \lambda_i \sigma_i^2 \\ & \text{s.t.} \quad \frac{\lambda_i |\tilde{\mathbf{w}}_i^H \mathbf{h}_i|^2 / \rho_i^D}{\sum_{k=1}^K \lambda_k |\tilde{\mathbf{w}}_i^H \mathbf{h}_k|^2 + \|\tilde{\mathbf{w}}_i\|_2^2} \geq 1, \\ & \quad i \in \mathcal{K} \end{aligned}$$

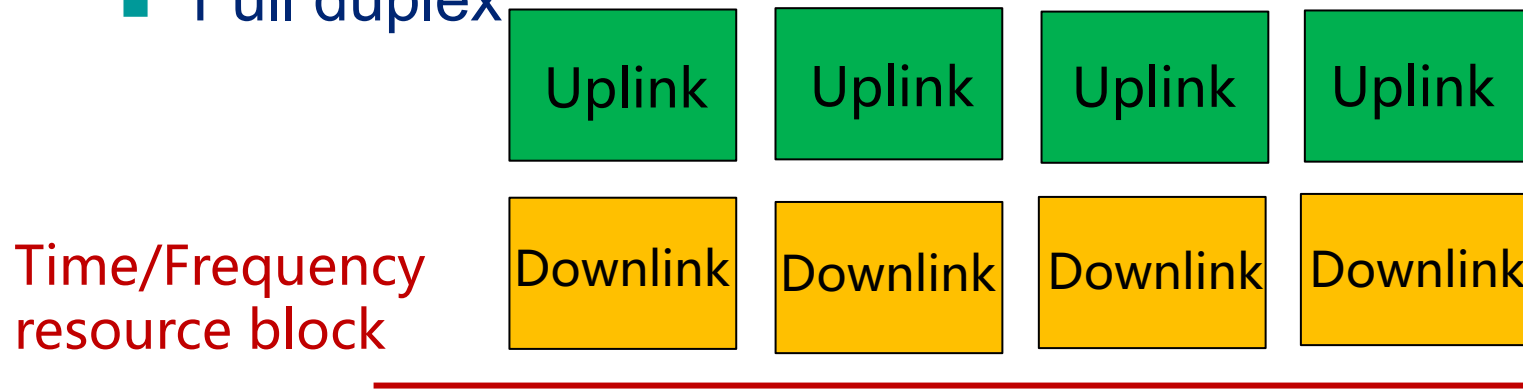


Half Duplex vs Full Duplex

- Half duplex (Time Division Duplex/Frequency Division Duplex)

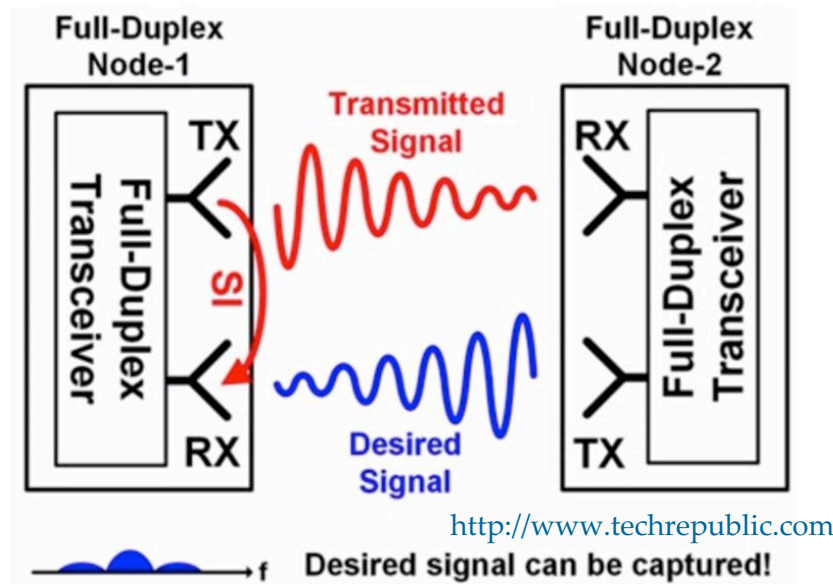


- Full duplex



Full-Duplex (FD) Communications

- Transmitting and receiving signals over the same spectrum and at the same time
- Ideally **double the throughput** comparing to TDD/FDD

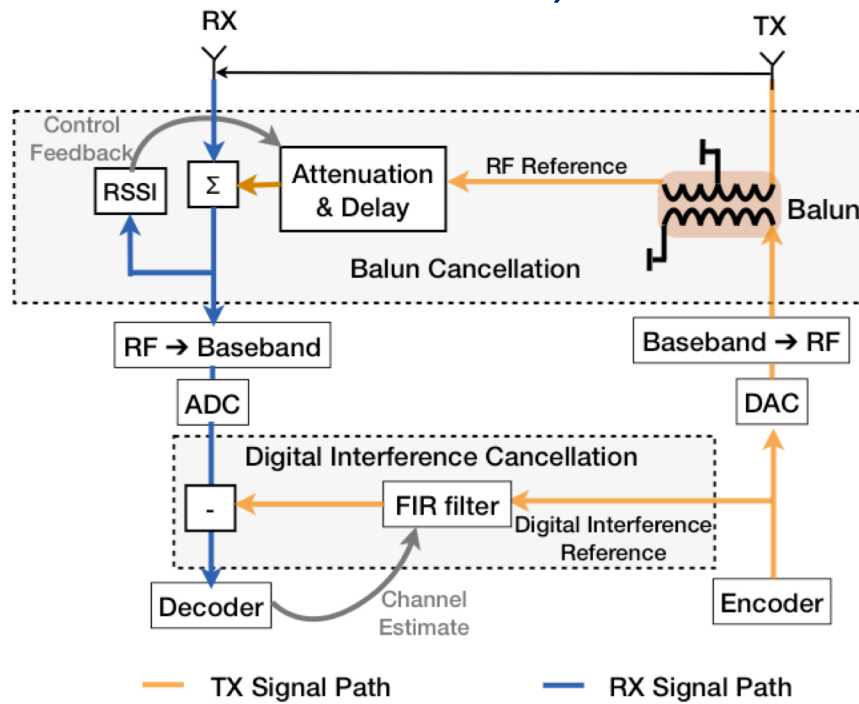


Challenge:

Self-interference (SI) can be 100dB larger than receiver noise and desired signal

Circuit Design

- FD technique has been successfully implemented (Kumi networks and S. Katti in Stanford; H. Krishnaswamy in Columbia Univ.)

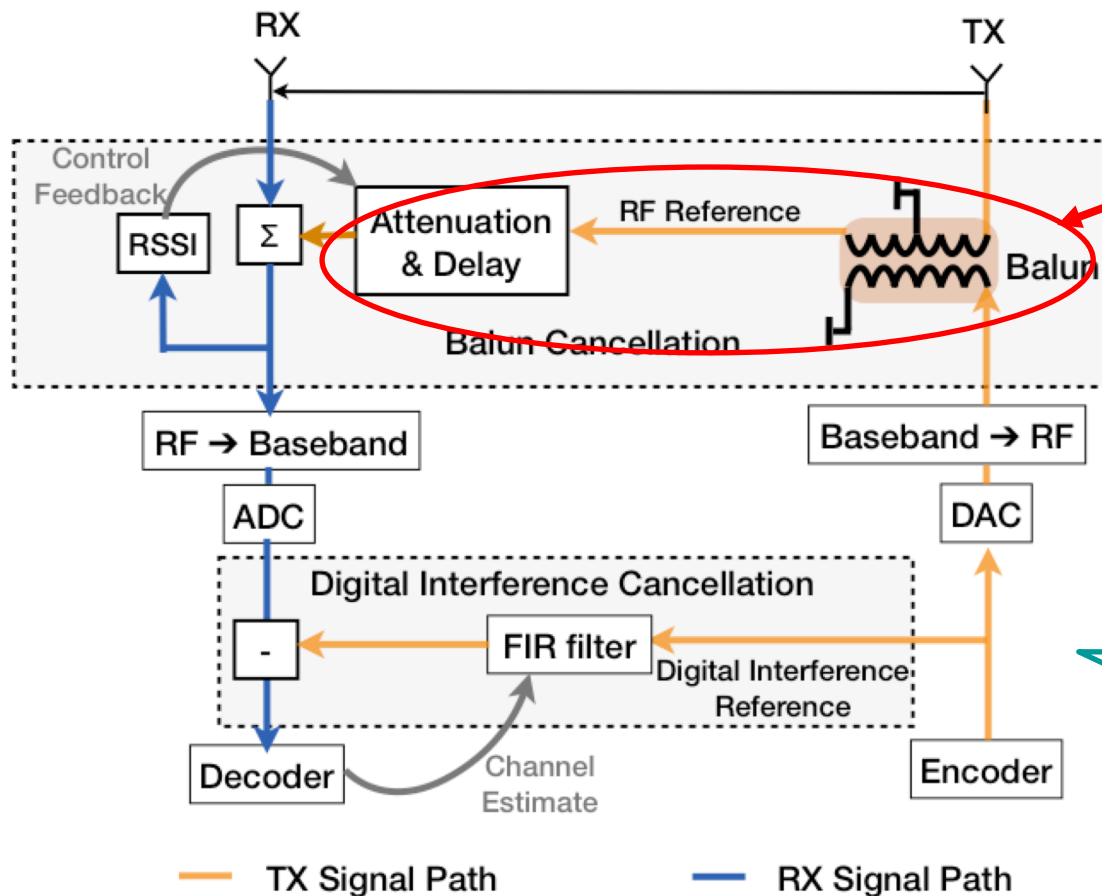


<http://sing.stanford.edu/fullduplex/>

D. Bharadia and S. Katti, "Full-duplex MIMO radios," in Proc. USENIX NSDI, Seattle, 2014, pp. 359–372.

M. Duarte, A. Sabharwal, V. Aggarwal, R. Jana, K. K. Ramakrishnan, C. W. Rice, and N. K. Shankaranarayanan, "Design and characterization of a full-duplex multi-antenna system for WiFi networks," IEEE Trans. Veh. Technol., vol. 63, no. 3, pp. 1160–1177, Mar. 2014.

Circuit Design (cont')



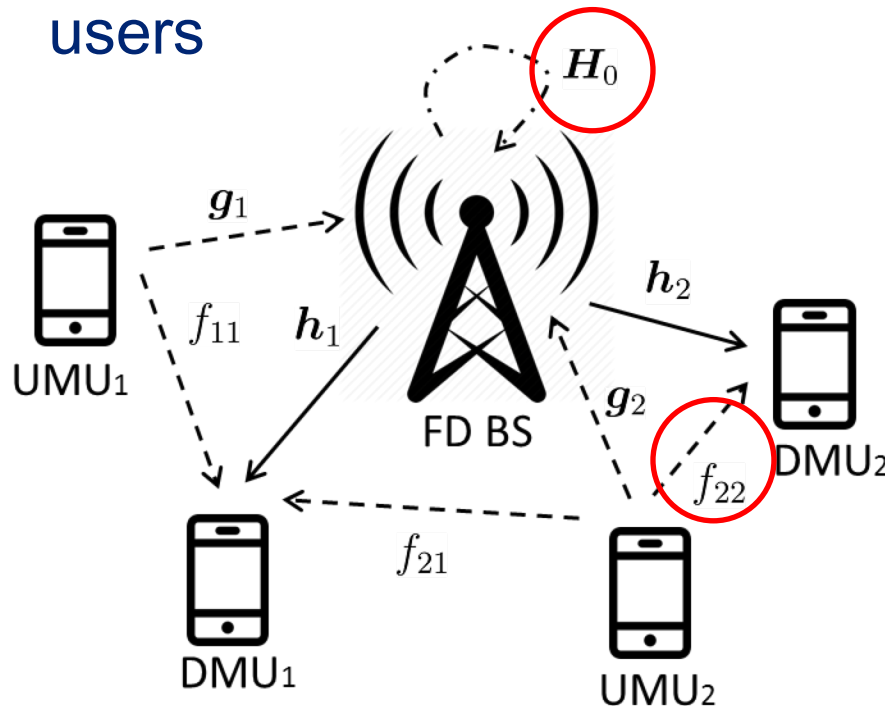
Analog SI
cancellation

Limiting the SI to be within
the dynamic ranges of ADC

Digital SI
cancellation

FD Networking

- FD BS + HD mobile users
- Self-interference
- **Co-channel interference** from uplink users to downlink users



Challenge:
Joint design for
uplink and
downlink
transmissions!

Y. Sun, D. W. K. Ng, J. Zhu, and R. Schober, "Multi-objective optimization for robust power efficient and secure full-duplex wireless communication systems," IEEE Trans. Wireless Commun., vol. 15, no. 8, pp. 5511–5526, Aug. 2016.

A. C. Cirik, O. Taghizadeh, R. Mathar, and T. Ratnarajah, "QoS considerations for full duplex multiuser MIMO systems," IEEE Trans. Commun., vol. 5, no. 1, pp. 36–39, Feb. 2016

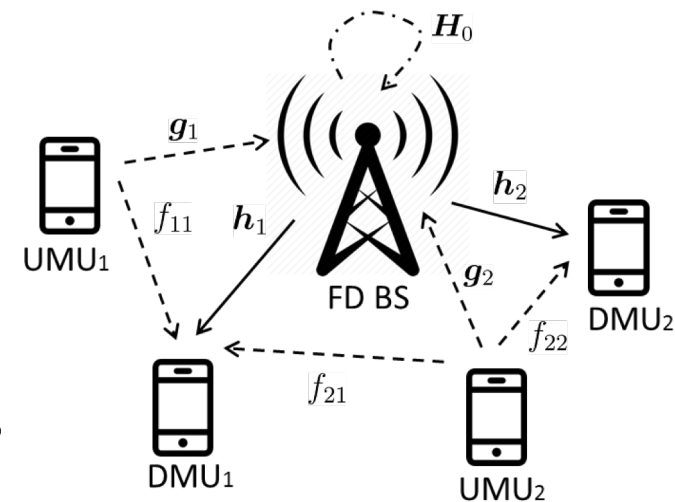
System Model (cont')

- Consider a network with K downlink mobile users (DMUs) and L uplink MUs (UMUs).

- Multiple-antenna BS and single-antenna MUs

- Received signal of the i th DMU:

$$y_i^D(t) = \mathbf{h}_i^H \left(\sum_{k=1}^K \mathbf{w}_k s_k^D(t) + \mathbf{u}_{\text{tx}}(t) \right) + \underbrace{\sum_{j=1}^L f_{ji} \sqrt{p_j^U} s_j^U(t)}_{\text{Uplink-to-downlink interference}} + z_i^D(t), \quad \forall i \in \mathcal{K},$$



Uplink-to-downlink interference

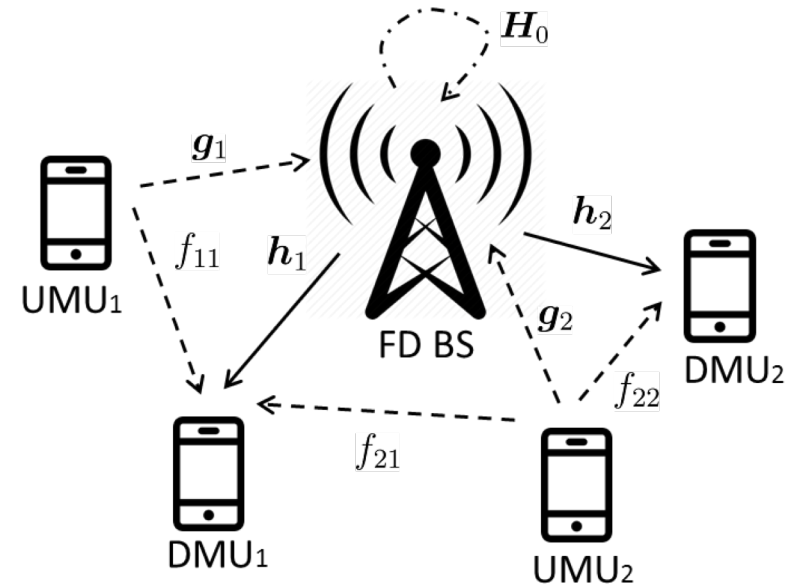
System Model (cont')

- SINR of the i th DMU:

$$\text{SINR}_i^{\text{D}}(\mathbf{W}, \mathbf{p}^{\text{U}}) \geq \gamma_i^{\text{D}}$$

$$\Leftrightarrow \frac{|\mathbf{h}_i^{\text{H}} \mathbf{w}_i|^2 / \rho_i^{\text{D}}}{\sum_{k=1}^K \mathbf{w}_k^{\text{H}} \tilde{\mathbf{H}}_i \mathbf{w}_k + \underbrace{\sum_{j=1}^L p_j^{\text{U}} |f_{ji}|^2}_{\text{Uplink-to-downlink interference}} + \sigma_i^2} \geq 1,$$

Uplink-to-downlink interference



$$\tilde{\mathbf{H}}_i \triangleq \mathbf{h}_i \mathbf{h}_i^{\text{H}} + \beta_1 \text{diag}(|\mathbf{h}_i|^2)$$

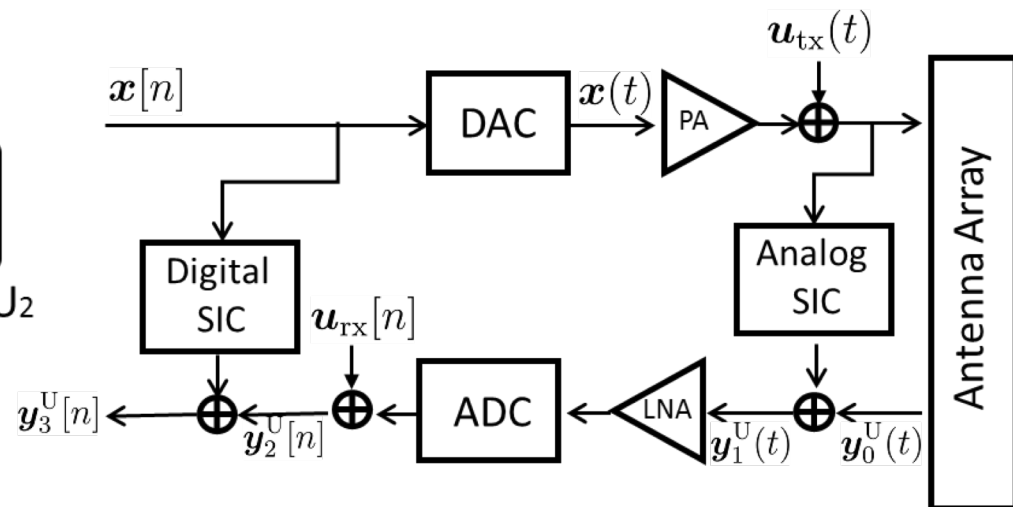
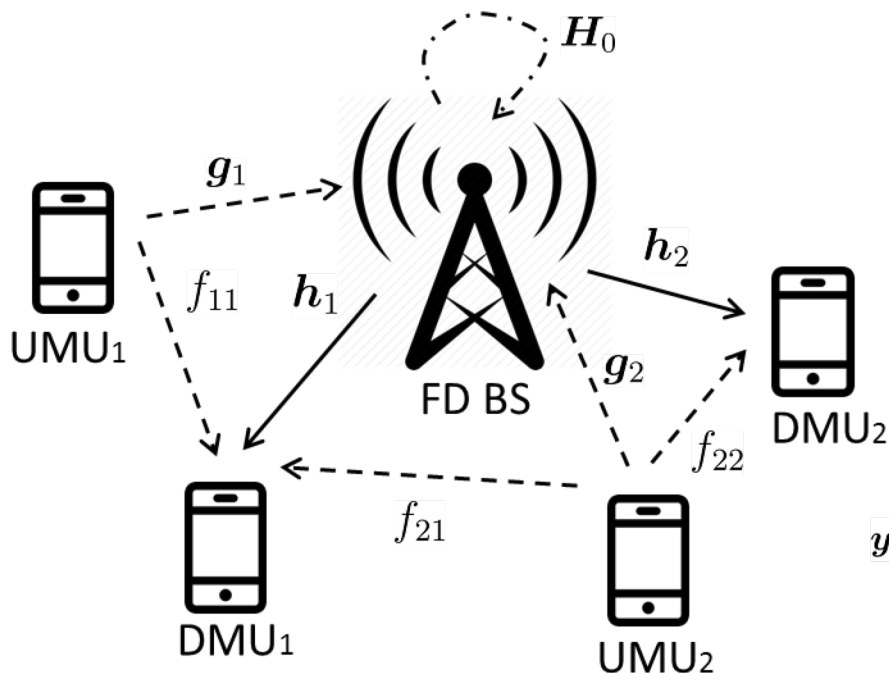
$$\frac{1}{\rho_i^{\text{D}}} \triangleq 1 + 1/\gamma_i^{\text{D}}$$

System Model (cont')

Received signal of the FD BS

$$\mathbf{y}_0^U(t) = \sum_{\ell=1}^L \mathbf{g}_\ell \sqrt{p_\ell^U} s_\ell^U(t) + \underbrace{\mathbf{H}_0(\mathbf{x}(t) + \mathbf{u}_{\text{tx}}(t))}_{\text{Self-interference}} + \mathbf{z}^U(t)$$

Self-interference





System Model (cont')

- Assume $\mathbf{H}_0 = \hat{\mathbf{H}}_0 + \mathbf{\Phi}_0$ where $\hat{\mathbf{H}}_0$ and $\mathbf{\Phi}_0$ are channel estimate and **estimation error**, respectively.
- **Analog SIC** removes $\hat{\mathbf{H}}_0(\mathbf{x}(t) + \mathbf{u}_{\text{tx}}(t))$ from $\mathbf{y}_0^{\text{U}}(t)$.
- **Digital SIC** further scales down the linear interference component and non-linear interference component by δ_1 and δ_2 , respectively.

$$\begin{aligned} \mathbf{y}_3^{\text{U}}[n] = & \sum_{\ell=1}^L \mathbf{g}_{\ell} \sqrt{p_{\ell}^{\text{U}}} s_{\ell}^{\text{U}}[n] + \sqrt{\delta_1} \mathbf{\Phi}_0 \mathbf{x}[n] \\ & + \sqrt{\delta_2} (\mathbf{\Phi}_0 \mathbf{u}_{\text{tx}}[n] + \mathbf{u}_{\text{rx}}[n]) + \mathbf{z}^{\text{U}}[n]. \end{aligned}$$

Residual
self-interference



System Model (cont')

- SINR of the ℓ th UMU:

$$\text{SINR}_\ell^{\text{U}}(\mathbf{W}, \mathbf{p}^{\text{U}}, \mathbf{v}_\ell) \geq \gamma_\ell^{\text{U}}$$

$$\mathbf{R}_{\Phi_0} \triangleq \mathbb{E}[\text{vec}(\Phi_0)\text{vec}^H(\Phi_0)]$$

$$\tilde{\mathbf{G}}_j \triangleq \mathbf{g}_j \mathbf{g}_j^H + \delta_2 \beta_2 \text{diag}(|\mathbf{g}_j|^2)$$

$$\tilde{\sigma}_z^2 \triangleq (1 + \delta_2 \beta_2) \sigma_z^2$$

$$\Leftrightarrow \frac{p_\ell^{\text{U}} |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^{\text{U}}}{\sum_{j=1}^L p_j^{\text{U}} \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \mathbf{v}_\ell^H \Omega(\mathbf{W}, \mathbf{R}_{\Phi_0}) \mathbf{v}_\ell + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1,$$

Residual
self-interference

where $\Omega(\mathbf{W}, \mathbf{R}_{\Phi_0})$ and $\Lambda(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0})$ are quadratic functions of \mathbf{W} and \mathbf{v}_ℓ , respectively.

System Model (cont')

- SINR of the ℓ th UMU:

$$\text{SINR}_\ell^{\text{U}}(\mathbf{W}, \mathbf{p}^{\text{U}}, \mathbf{v}_\ell) \geq \gamma_\ell^{\text{U}}$$

Residual
self-interference

$$\Leftrightarrow \frac{p_\ell^{\text{U}} |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^{\text{U}}}{\sum_{j=1}^L p_j^{\text{U}} \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \mathbf{v}_\ell^H \boldsymbol{\Omega}(\mathbf{W}, \mathbf{R}_{\Phi_0}) \mathbf{v}_\ell + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1,$$
$$\Leftrightarrow \frac{p_\ell^{\text{U}} |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^{\text{U}}}{\sum_{j=1}^L p_j^{\text{U}} \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \sum_{k=1}^K \mathbf{w}_k^H \boldsymbol{\Lambda}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0}) \mathbf{w}_k + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1$$

where $\boldsymbol{\Omega}(\mathbf{W}, \mathbf{R}_{\Phi_0})$ and $\boldsymbol{\Lambda}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0})$ are quadratic functions of \mathbf{W} and \mathbf{v}_ℓ , respectively.



Problem Formulation

■ QoS constrained Power minimization design

$$\begin{aligned} \text{(P)} \quad & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ & \text{s.t.} \quad \text{SINR}_i^D(\mathbf{W}, \mathbf{p}^U) \geq \gamma_i^D, \quad i \in \mathcal{K}, \\ & \quad \text{SINR}_\ell^U(\mathbf{W}, \mathbf{p}^U, \mathbf{v}_\ell) \geq \gamma_\ell^U, \quad \ell \in \mathcal{L}, \end{aligned}$$

■ Challenging to handle

- ✓ non-convex
- ✓ coupled uplink and downlink transmissions
- ✓ results for HD systems no longer applicable



Problem Formulation

■ QoS constrained Power minimization design

$$\begin{aligned} \text{(P)} \quad & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ \text{s.t.} \quad & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \sum_{j=1}^L p_j^U |f_{ji}|^2 + \sigma_i^2} \geq 1, \quad i \in \mathcal{K}, \\ & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \mathbf{v}_\ell^H \boldsymbol{\Omega}(\mathbf{W}, \mathbf{R}_{\Phi_0}) \mathbf{v}_\ell + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L}, \end{aligned}$$

■ Challenging to handle

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Problem Formulation

■ QoS constrained Power minimization design

$$\begin{aligned} \text{(P)} \quad & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ \text{s.t.} \quad & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \sum_{j=1}^L p_j^U |f_{ji}|^2 + \sigma_i^2} \geq 1, \quad i \in \mathcal{K}, \\ & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \mathbf{v}_\ell^H \boldsymbol{\Omega}(\mathbf{W}, \mathbf{R}_{\Phi_0}) \mathbf{v}_\ell + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L}, \end{aligned}$$

■ We will show

- ✓ a polynomial-time solvable case
- ✓ efficient low-complexity algorithm for the general case



A Globally Solvable Case

- Consider a **worst-case SI scenario**: As the CSI error correlation matrix

$$\mathbf{R}_{\Phi_0} \preceq \lambda_{\max}(\mathbf{R}_{\Phi_0}) \mathbf{I}_{N_t}$$

we can bound

$$\Omega(\mathbf{W}, \mathbf{R}_{\Phi_0}) \preceq \xi \sum_{i=1}^K \|\mathbf{w}_i\|_2^2 \mathbf{I}_{N_t}$$

where $\xi \triangleq (\delta_1 + \delta_2\beta_1 + \delta_2\beta_2(1 + \beta_1))\lambda_{\max}(\mathbf{R}_{\Phi_0})$

- Or assume that the **CSI errors** are independent and identically distributed (**i.i.d.**)

$$\mathbf{R}_{\Phi_0} = \sigma_{\Phi_0}^2 \mathbf{I}_{N_t^2}$$



A Globally Solvable Case (cont')

- For such case, (P) is given by

$$\begin{aligned}
 \text{(P1)} \quad & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\
 \text{s.t.} \quad & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \hat{\sigma}_i^2(\mathbf{p}^U)} \geq 1, \quad i \in \mathcal{K}, \\
 & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \xi \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 \|\mathbf{v}_\ell\|_2^2 + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L}.
 \end{aligned}$$

$$\hat{\sigma}_i^2(\mathbf{p}^U) \triangleq \sum_{j=1}^L p_j^U |f_{ji}|^2 + \sigma_i^2$$

Residual

self-interference

- Still not convex
- Turn out to be polynomial-time globally solvable



A UL-DL Decomposable Subproblem

- Consider a problem (P_η)

$$\begin{aligned} \underline{F(\eta)} = & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ \text{s.t. } & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \hat{\sigma}_i^2(\mathbf{p}^U)} \geq 1, \quad i \in \mathcal{K}, \\ & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \underline{\xi \eta \|\mathbf{v}_\ell\|_2^2} + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L}, \end{aligned}$$

- Here η is a **fixed parameter**.



A UL-DL Decomposable Subproblem

■ Key observation 1:

$$F(\eta) = \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U$$

$$\text{s.t. } \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \hat{\sigma}_i^2(\mathbf{p}^U)} \geq 1, \quad i \in \mathcal{K},$$

$$\frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \xi \eta \|\mathbf{v}_\ell\|_2^2 + \tilde{\sigma}_z^2 \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L},$$

Equality must hold.
**Unique solution
determined!**
according to the HD
Uplink design

A UL-DL Decomposable Subproblem

- **Proposition 1:** Suppose that (P_η) is feasible. Then it can be globally solved by solving the two problems **in order**

$$\{\mathbf{v}_\ell(\eta), p_\ell^U(\eta)\}_{\ell=1}^L = \arg \min_{\{\mathbf{v}_\ell\}, \{p_\ell^U \geq 0\}} \sum_{\ell=1}^L p_\ell^U$$
$$\text{s.t. } \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + ((\xi\eta + \tilde{\sigma}_z^2) \|\mathbf{v}_\ell\|_2^2)} \geq 1, \ell \in \mathcal{L},$$

$$\{\mathbf{w}_k(\eta)\}_{k=1}^K = \arg \min_{\{\mathbf{w}_k\}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2$$
$$\text{s.t. } \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \hat{\sigma}_i^2(\mathbf{p}^U(\eta))} \geq 1, i \in \mathcal{K}.$$

A UL-DL Decomposable Subproblem

$$\{\mathbf{v}_\ell(\eta), p_\ell^{\text{U}}(\eta)\}_{\ell=1}^L = \arg \min_{\{\mathbf{v}_\ell\}, \{p_\ell^{\text{U}} \geq 0\}} \sum_{\ell=1}^L p_\ell^{\text{U}}$$

HD **uplink** problem
Polynomial-time
solvable

$$\text{s.t.} \quad \frac{p_\ell^{\text{U}} |\mathbf{v}_\ell^{\text{H}} \mathbf{g}_\ell|^2 / \rho_\ell^{\text{U}}}{\sum_{j=1}^L p_j^{\text{U}} \mathbf{v}_\ell^{\text{H}} \tilde{\mathbf{G}}_j \mathbf{v}_\ell + (\xi\eta + \tilde{\sigma}_z^2) \|\mathbf{v}_\ell\|_2^2} \geq 1, \quad \ell \in \mathcal{L},$$

$$\{\mathbf{w}_k(\eta)\}_{k=1}^K = \arg \min_{\{\mathbf{w}_k\}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2$$

HD **downlink** problem
Polynomial-time
solvable

$$\text{s.t.} \quad \frac{|\mathbf{h}_i^{\text{H}} \mathbf{w}_i|^2 / \rho_i^{\text{D}}}{\sum_{k=1}^K \mathbf{w}_k^{\text{H}} \tilde{\mathbf{H}}_i \mathbf{w}_k + \hat{\sigma}_i^2 (p^{\text{U}}(\eta))} \geq 1, \quad i \in \mathcal{K}.$$



Search of Optimal η

- **Proposition 2:** Suppose that (P1) is feasible.

Let $\eta^* \triangleq \sum_{k=1}^K \|\mathbf{w}_k^*\|_2^2$. Then

(1) η^* is unique;

(2) when $\eta = \eta^*$, $\{\mathbf{w}_k(\eta)\}$ of (P_η) is **optimal** to (P1)

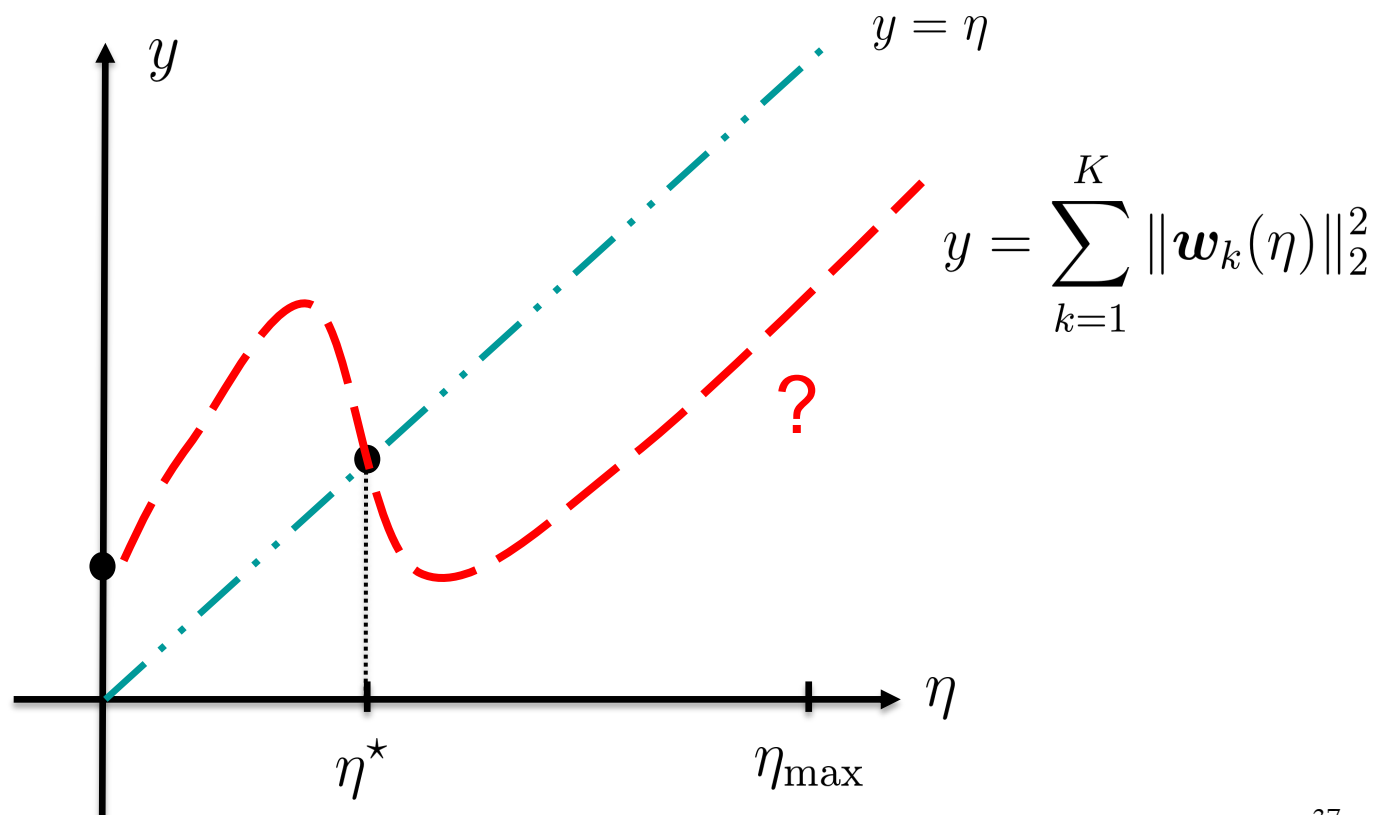
and satisfies $\sum_{k=1}^K \|\mathbf{w}_k(\eta)\|_2^2 = \eta^*$

- Therefore, as long as η^* is obtained, then (P1) can be globally solved.
- How to get η^* ?



Search of Optimal η

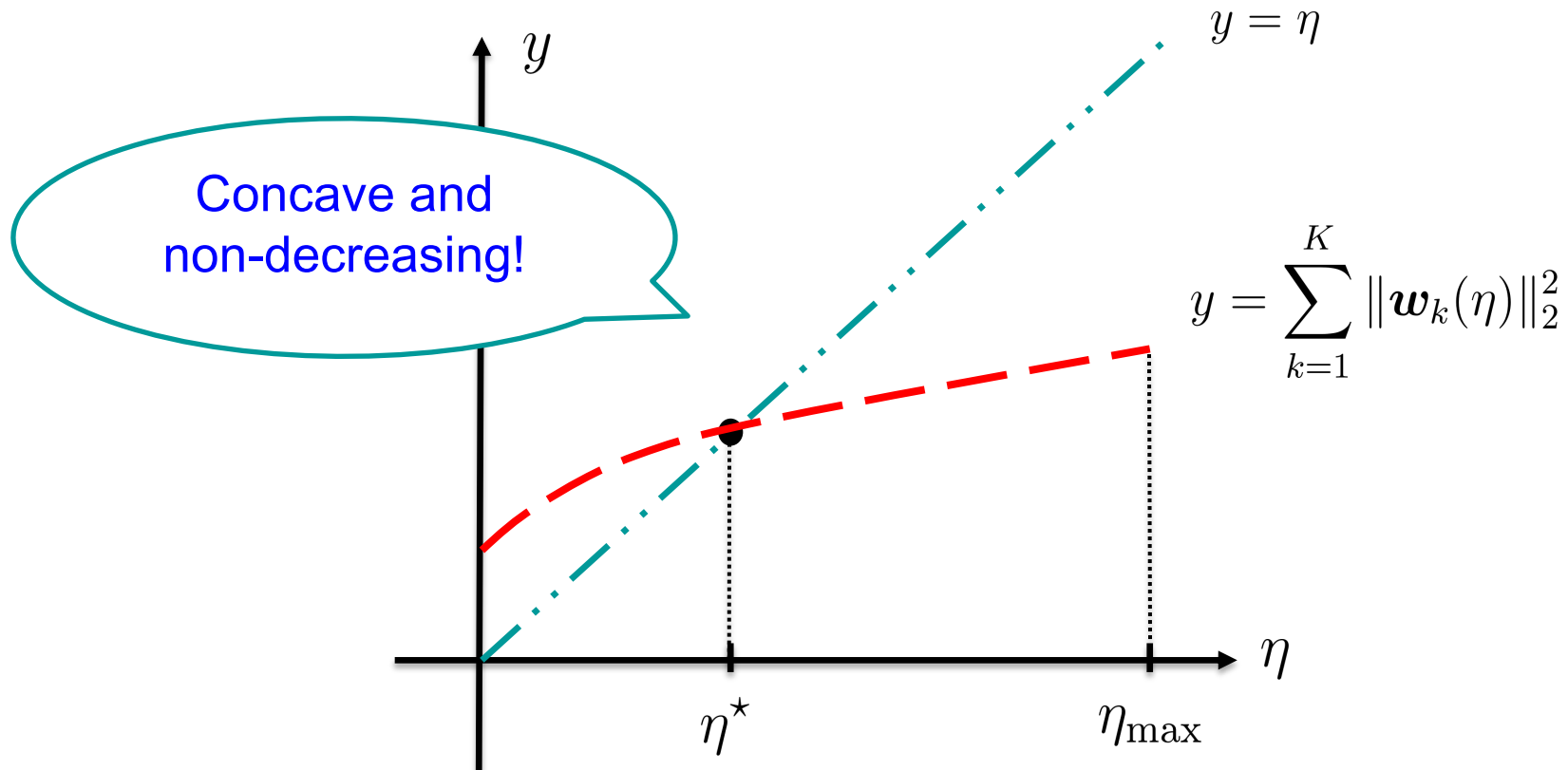
- From Proposition 2, we know





Search of Optimal η

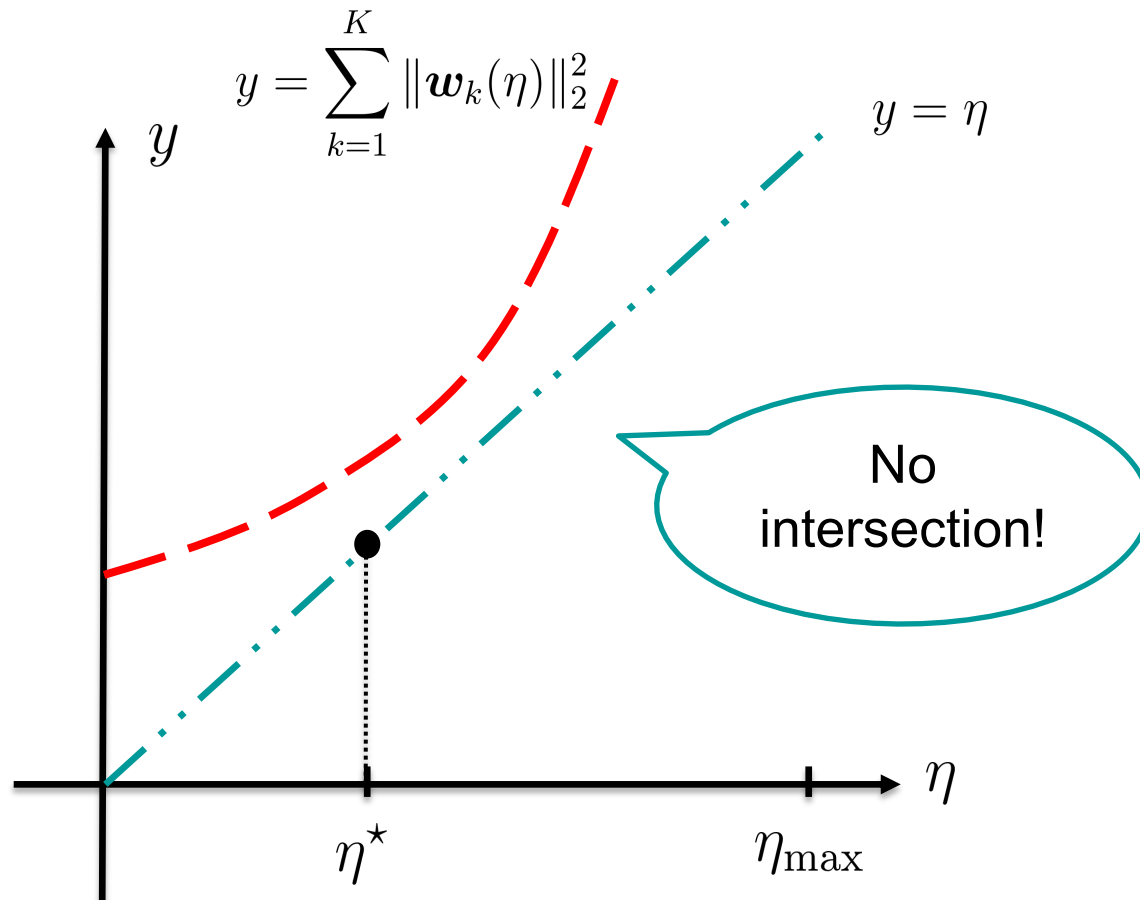
- **Proposition 3:** Suppose that (P1) is feasible.





Search of Optimal η

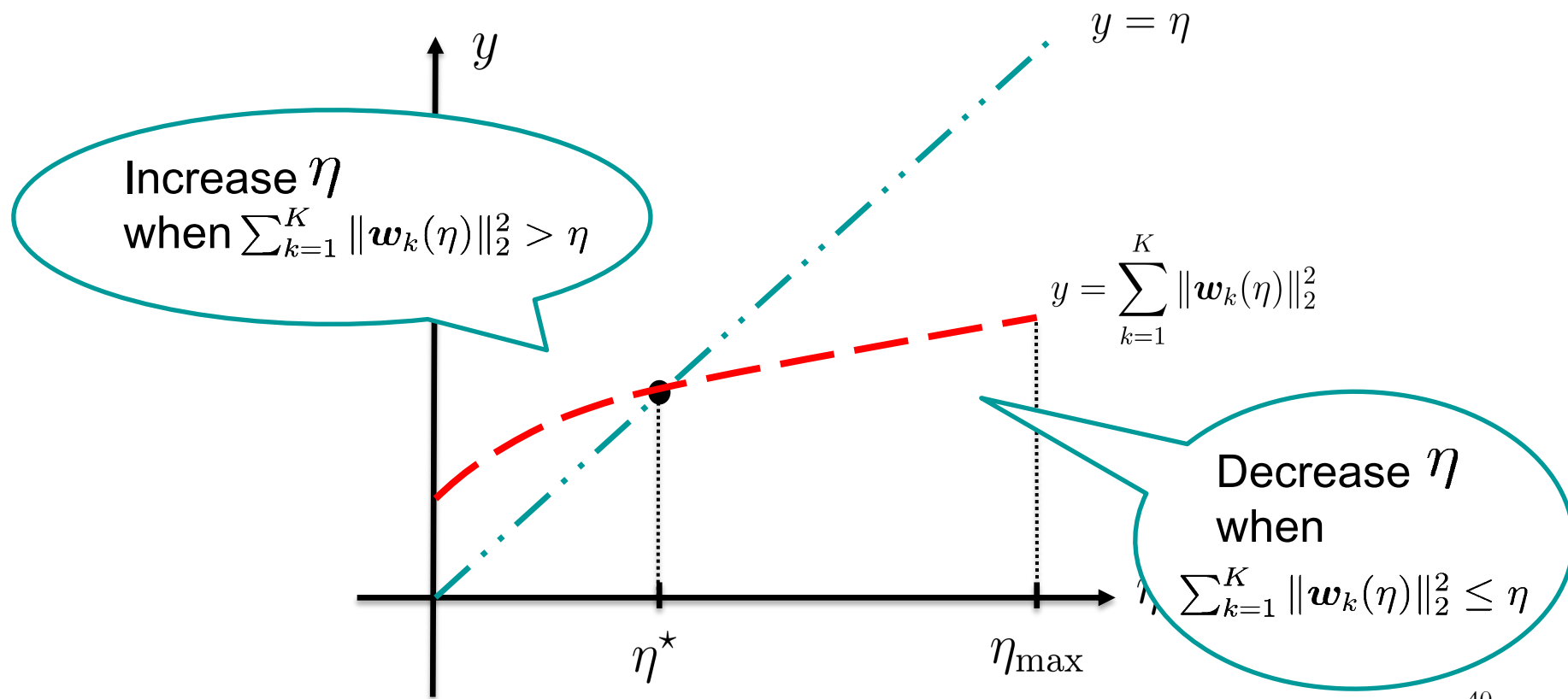
- **Proposition 3:** (P1) is infeasible if and only if





Search of Optimal η

- Proposition 3 suggests a **bisection search** strategy





(P1) Is Polynomial-time Solvable

- Since bisection search of η is polynomial-time solvable, and (P_η) is polynomial-time solvable, we conclude:

Theorem 1: Suppose that (P1) is feasible. Then (P1) is polynomial-time solvable.

- **Insight:** SI is the major bottleneck for the transceiver optimization of the FD networks.

As long as the residual SI is negligible, the FD problem can be as easy as the HD problems



Alternating Optimization

- Recall our original problem (P)

$$\begin{aligned} \text{(P)} \quad & \min_{\substack{\{\mathbf{w}_k\}, \{\mathbf{v}_\ell\}, \\ \{p_\ell^U \geq 0\}}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ & \text{s.t.} \quad \text{SINR}_i^D(\mathbf{W}, \mathbf{p}^U) \geq \gamma_i^D, \quad i \in \mathcal{K}, \\ & \quad \text{SINR}_\ell^U(\mathbf{W}, \mathbf{p}^U, \mathbf{v}_\ell) \geq \gamma_\ell^U, \quad \ell \in \mathcal{L}, \end{aligned}$$

- Analysis above implies that it is unlikely to globally solve (P) in a convex fashion.
- We employ an **AO method** to achieve a **KKT solution** of (P)

Alternating Optimization

- Suppose that **UL beamformers** $\{\mathbf{v}_\ell\}$ are fixed:

$$\begin{aligned} \min_{\{\mathbf{w}_k\}, \{p_\ell^U \geq 0\}} \quad & \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\ \text{s.t.} \quad & \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \sum_{j=1}^L p_j^U |f_{ji}|^2 + \sigma_i^2} \geq 1, \quad \forall i \in \mathcal{K}, \\ & \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \underbrace{\sum_{k=1}^K \mathbf{w}_k^H \boldsymbol{\Lambda}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0}) \mathbf{w}_k}_{\text{Residual self-interference}} + \tilde{\sigma}_z^2} \geq 1, \quad \forall \ell \in \mathcal{L}. \end{aligned}$$

- Apply **change of variables** $q_\ell = \sqrt{p_\ell^U}$, $\ell \in \mathcal{L}$



Alternating Optimization

- Suppose that UL beamformers $\{\mathbf{v}_\ell\}$ are fixed:

$$\begin{aligned} \min_{\{\mathbf{w}_k\}, \{q_\ell \geq 0\}} \quad & \sum_{k=1}^K \|\mathbf{w}_k\|^2 + \sum_{\ell=1}^L q_\ell^2 \\ \text{s.t.} \quad & \frac{\mathbf{h}_i^H \mathbf{w}_i}{\sqrt{\rho_i^D}} \geq \sqrt{\sum_{k=1}^K \|\tilde{\mathbf{H}}_i^{\frac{1}{2}} \mathbf{w}_k\|_2^2 + \sum_{j=1}^L q_j^2 |f_{ji}|^2 + \sigma_i^2}, \quad k \in \mathcal{K}, \\ & \frac{q_\ell |\mathbf{v}_\ell^H \mathbf{g}_\ell|}{\sqrt{\rho_\ell^U}} \geq \sqrt{\sum_{j=1}^L q_j^2 \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \sum_{k=1}^K \|\Lambda^{\frac{1}{2}}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0}) \mathbf{w}_k\|_2^2 + \tilde{\sigma}_z^2}, \quad \forall \ell \in \mathcal{L}. \end{aligned}$$

- Apply change of variables $q_\ell = \sqrt{p_\ell^U}$, $\ell \in \mathcal{L}$
- Equivalent to a **SOCP**.



Alternating Optimization

- When $\{\mathbf{w}_k\}$ and $\{p_\ell^U\}$ are fixed, $\{\mathbf{v}_\ell\}$ are updated by the maximum SINR solution

$$\mathbf{v}_\ell = \frac{\mathbf{M}_\ell^{-1} \mathbf{g}_\ell}{\|\mathbf{M}_\ell^{-1} \mathbf{g}_\ell\|_2}$$

where $\mathbf{M}_\ell \triangleq \sum_{j=1}^L p_j^U \tilde{\mathbf{G}}_j + \Omega(\mathbf{W}, \mathbf{R}_{\Phi_0}) + \tilde{\sigma}_z^2 \mathbf{I}_{N_t}$.

- **Theorem 2:** AO strategy yields a KKT solution of (P) as the iteration number goes to infinity.



Extended Uplink-Downlink Duality

- Complexity of SOCP: $\log(1/\epsilon)\mathcal{O}(N_t^{3.5}K^{4.5})$ if $K = L$
- The classical UDD in HD systems can be extended to the FD system for solving (fixed $\{\mathbf{v}_\ell\}$)

$$\begin{aligned}
 & \min_{\{\mathbf{w}_k\}, \{p_\ell^U \geq 0\}} \sum_{k=1}^K \|\mathbf{w}_k\|_2^2 + \sum_{\ell=1}^L p_\ell^U \\
 & \text{s.t.} \quad \frac{|\mathbf{h}_i^H \mathbf{w}_i|^2 / \rho_i^D}{\sum_{k=1}^K \mathbf{w}_k^H \tilde{\mathbf{H}}_i \mathbf{w}_k + \sum_{j=1}^L p_j^U |f_{ji}|^2 + \sigma_i^2} \geq 1, \quad \forall i \in \mathcal{K}, \\
 & \quad \frac{p_\ell^U |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L p_j^U \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \sum_{k=1}^K \mathbf{w}_k^H \boldsymbol{\Lambda}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0}) \mathbf{w}_k + \tilde{\sigma}_z^2} \geq 1, \quad \forall \ell \in \mathcal{L}.
 \end{aligned}$$

F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," IEEE J. Sel. Areas Commun., vol. 16, no. 8, pp. 1437–1449, Oct. 1998.

M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," vol. 53, no. 1, pp. 18–28, Jan. 2004.



Extended Uplink-Downlink Duality

■ Virtual problem:

$$\begin{aligned} (\{\tilde{\mathbf{w}}_k^*\}, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) &= \arg \min_{\{\tilde{\mathbf{w}}_k\}, \boldsymbol{\lambda}, \boldsymbol{\mu} \succeq \mathbf{0}} \boldsymbol{\lambda}^T \boldsymbol{\sigma}^2 + \tilde{\sigma}_z^2 \boldsymbol{\mu}^T \mathbf{1} \\ \text{s.t. } &\frac{\lambda_k |\tilde{\mathbf{w}}_k^H \mathbf{h}_k|^2 / \rho_k^D}{\tilde{\mathbf{w}}_k^H (\sum_{i=1}^K \lambda_i \tilde{\mathbf{H}}_i + \sum_{\ell=1}^L \mu_\ell \boldsymbol{\Lambda}(\mathbf{v}_\ell, \mathbf{R}_{\Phi_0}) + \mathbf{I}_{N_t}) \tilde{\mathbf{w}}_k} \geq 1, \quad k \in \mathcal{K}, \\ &\frac{\mu_\ell |\mathbf{v}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^U}{\sum_{j=1}^L \mu_j \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \sum_{k=1}^K \lambda_k |f_{\ell k}|^2 + 1} \geq 1, \quad \ell \in \mathcal{L}, \end{aligned}$$

F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," IEEE J. Sel. Areas Commun., vol. 16, no. 8, pp. 1437–1449, Oct. 1998.

M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," vol. 53, no. 1, pp. 18–28, Jan. 2004.



Extended Uplink-Downlink Duality

- Fixed-point equations:

$$\lambda_k = \mathcal{F}_k^{\text{U}}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \triangleq \frac{1}{\mathbf{h}_k^H \mathbf{Q}^{-1}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \mathbf{h}_k / \rho_k^{\text{D}}}, \quad \forall k \in \mathcal{K},$$
$$\mu_\ell = \mathcal{F}_\ell^{\text{D}}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \triangleq \frac{\sum_{j=1}^L \mu_j \mathbf{v}_\ell^H \tilde{\mathbf{G}}_j \mathbf{v}_\ell + \sum_{k=1}^K \lambda_k |f_{\ell k}|^2 + b_\ell}{|\tilde{\mathbf{v}}_\ell^H \mathbf{g}_\ell|^2 / \rho_\ell^{\text{U}}}, \quad \forall \ell \in \mathcal{L}.$$

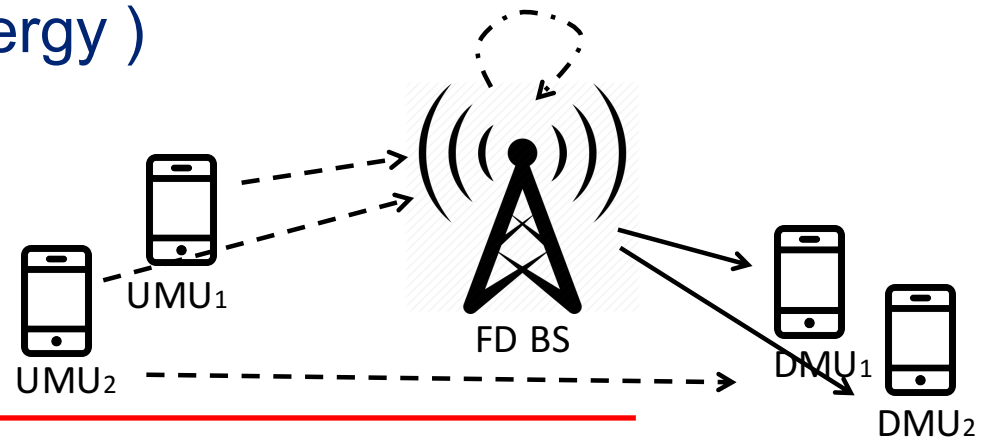
- A **fixed-point algorithm** can be developed and efficiently implemented analogous to the HD system.
- Complexity: $\log(1/\epsilon) \mathcal{O}(N_t^3 + K^2)$ if $K = L$ and the FP iterations have a linear convergence rate

F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," IEEE J. Sel. Areas Commun., vol. 16, no. 8, pp. 1437–1449, Oct. 1998.

M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," vol. 53, no. 1, pp. 18–28, Jan. 2004.

Simulation Setting

- The BS has 10 antennas ($N_t = 10$)
- The path loss between the MUs and the BS is set to -80 dB and that between the UMUs and DMUs is set to -83 dB.
- The SI channel has a -10 dB path loss.
- Analog SIC uses LMMSE channel estimation ($E > 0$ denotes the training energy)

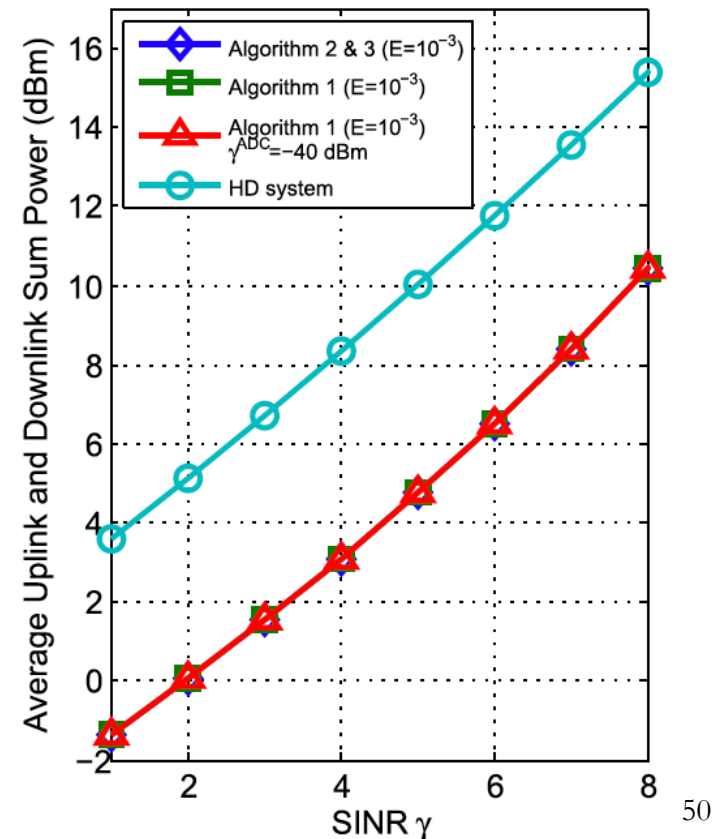
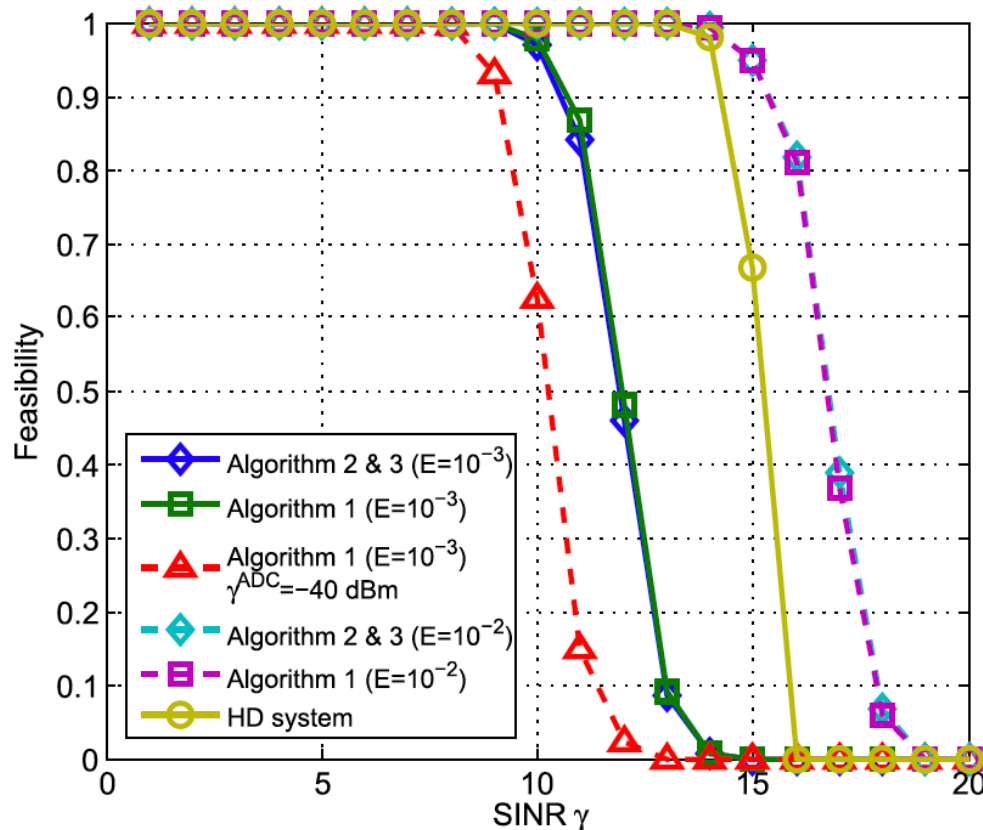


D. Bharadia, E. McMillin, and S. Katti, "Full duplex radios," in Proc. ACM SIGCOMM, pp. 375–386, 2013.

E. Bjornson and B. Ottersten, "A framework for training-based estimation in arbitrarily correlated Rician MIMO channels with Rician disturbance," IEEE Trans. Signal Process., vol. 58, no. 3, pp. 1807–1820, March 2010.

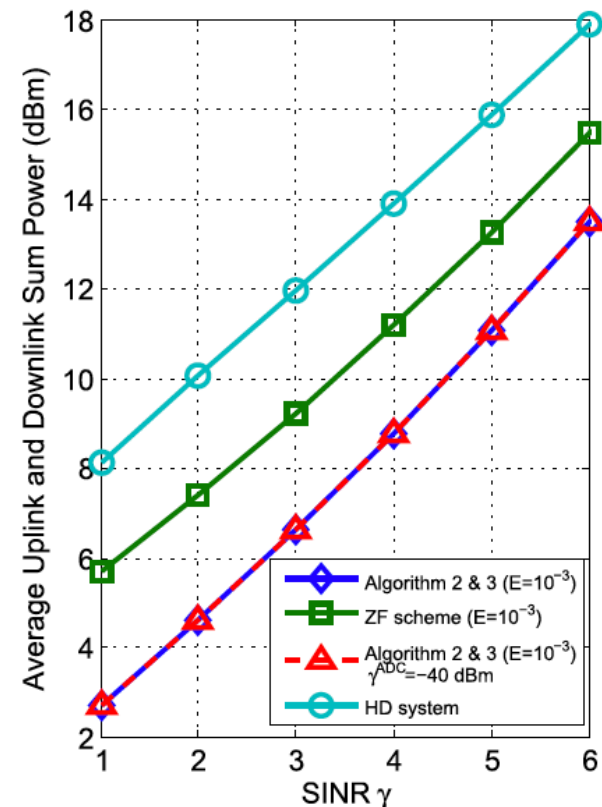
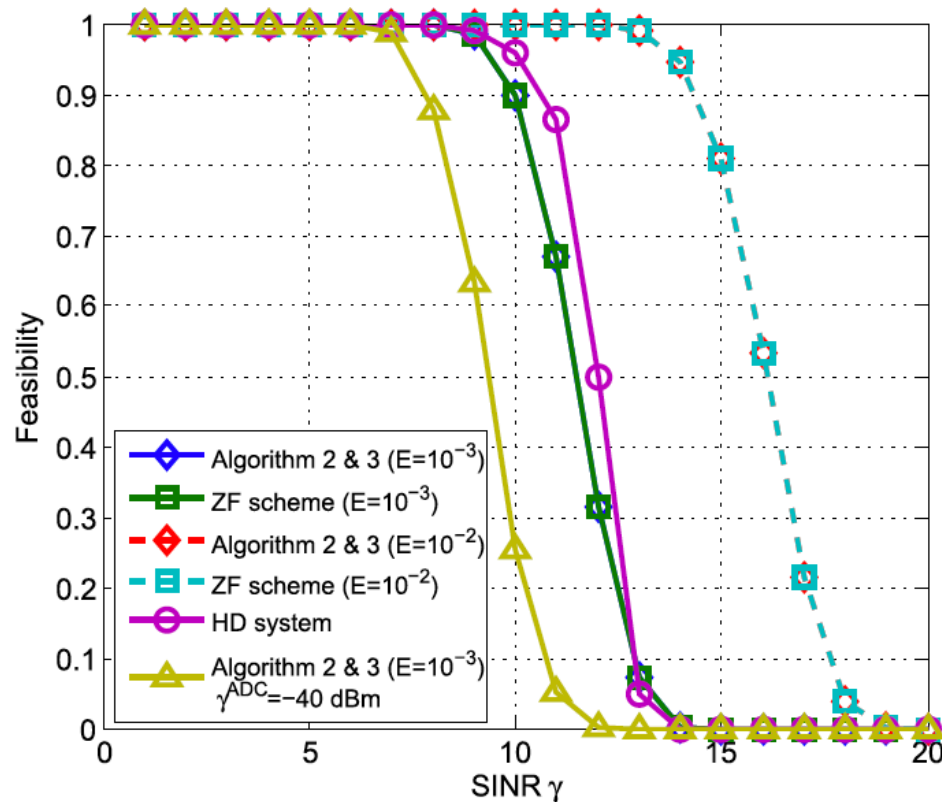
Simulation Results

- (P1) $N_t = 10, K = L = 5$
- Bisection (Algorithm 1) vs AO method (Algorithm 2&3)



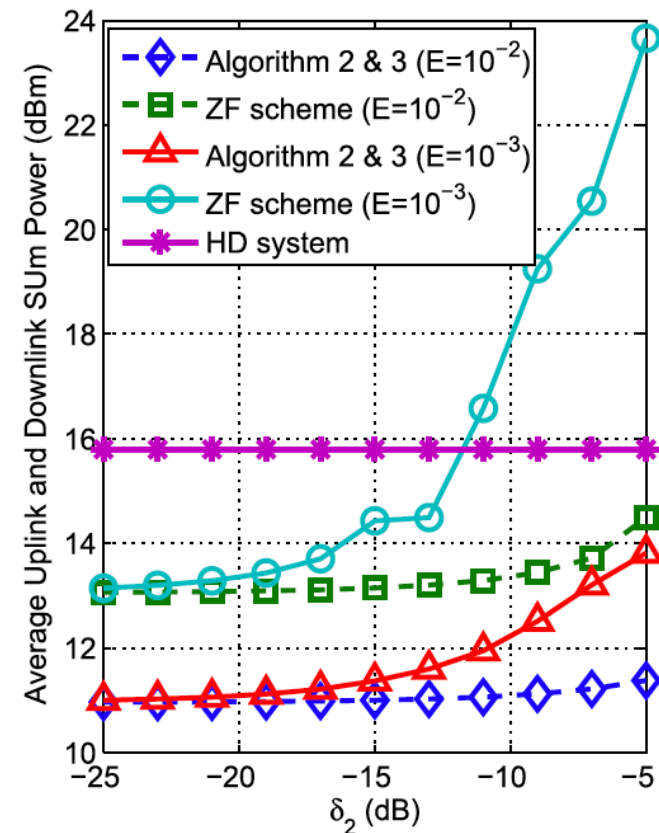
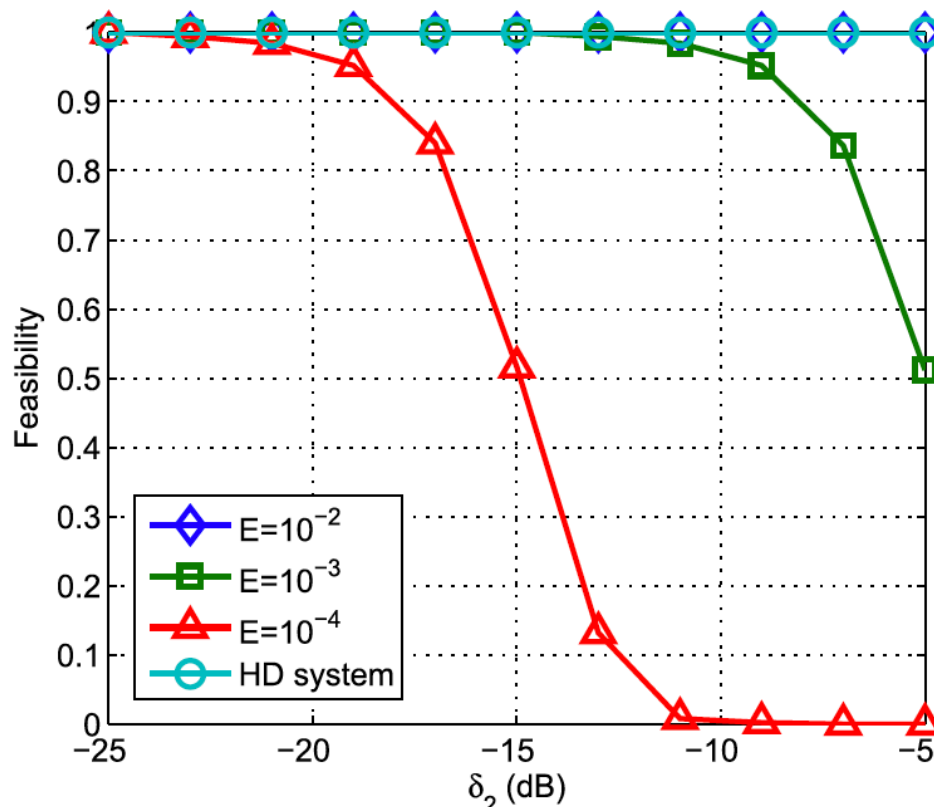
Simulation Results

- (P) $N_t = 10, K = L = 5$
- AO method (Algorithm 2&3) vs ZF scheme



Simulation Results

- (P) $N_t = 10, K = L = 8, \gamma = 5\text{dB}, \delta_1 = \delta_2 10^{-3}$
- AO method (Algorithm 2&3) vs ZF scheme





Concluding Remarks

- The transceiver design for the FD network is **challenging** due to **the SI** and **uplink-to-downlink co-channel interference**.
- We have shown a special instance of (P) (i.e., (P1)) for which the global optimal solution can be obtained in polynomial time.
- An AO algorithm is also proposed for obtaining an efficient suboptimal solution.
- **Both theoretical analysis for (P1) and numerical results indicate that the SI is the fundamental bottleneck of the FD system.** Once SI can be sufficiently mitigated, the FD system greatly outperforms the HD system.



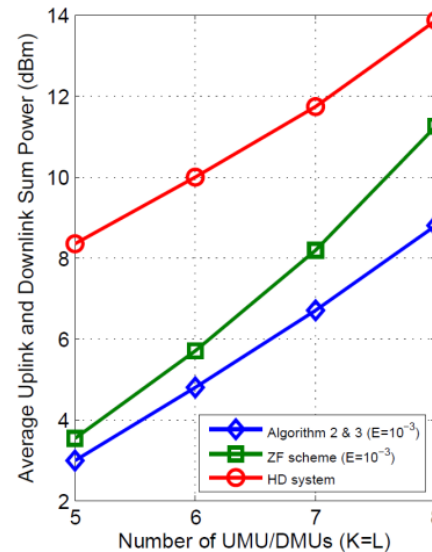
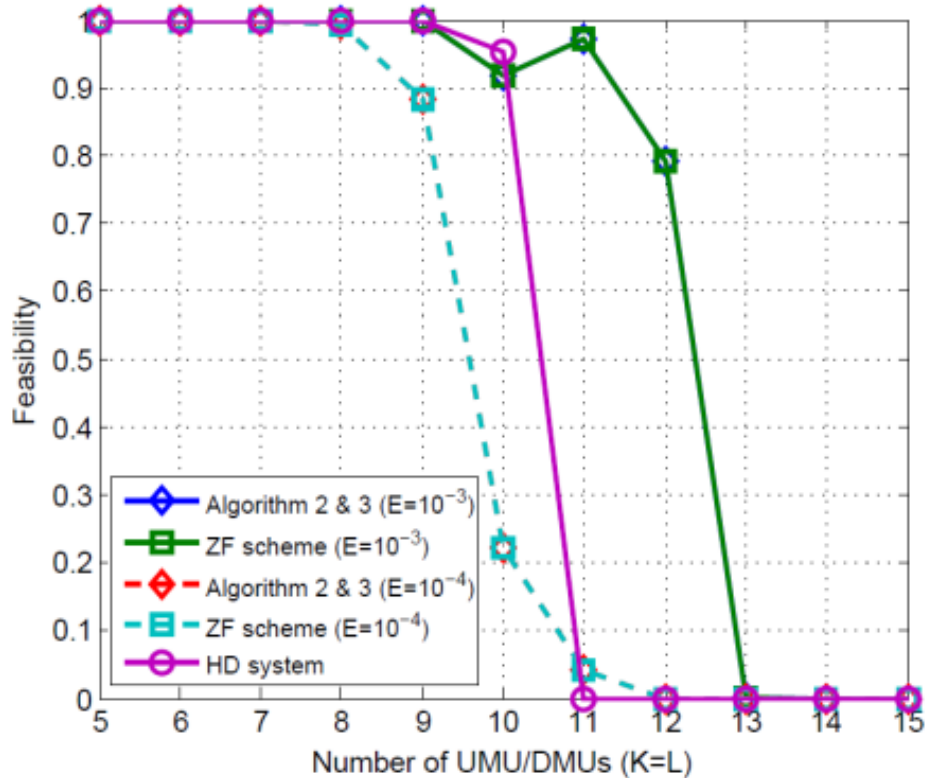
Thank You!!!

www.thebodytransformation.com

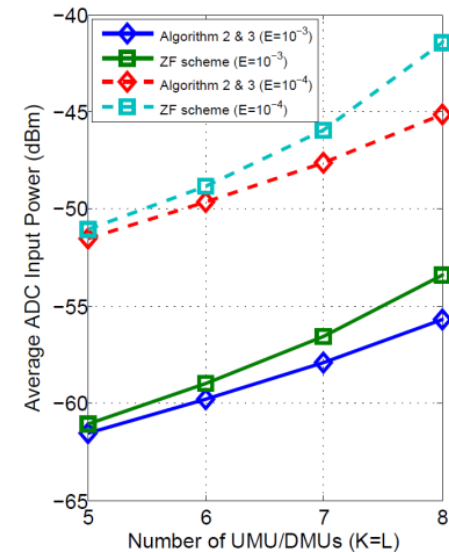


Simulation Results

- (P) $N_t = 10, \gamma = 5, K = L$
- AO method (Algorithm 2&3) vs ZF scheme



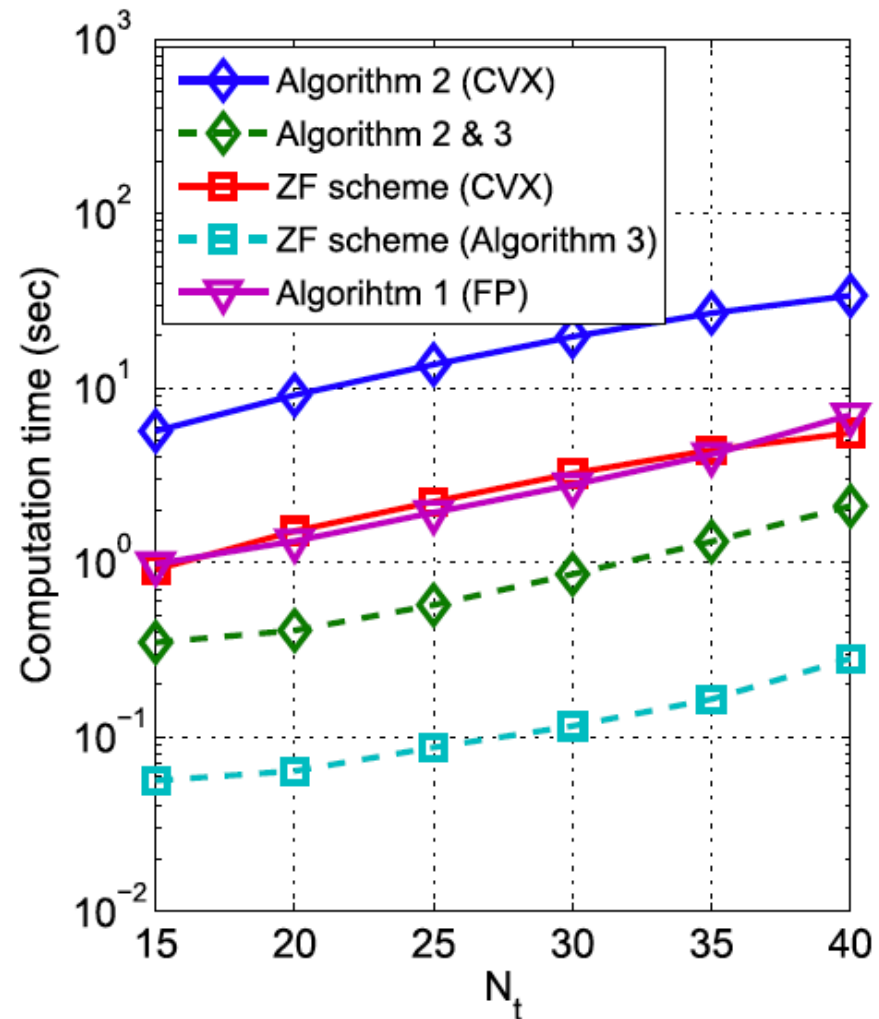
(b) Sum power



(c) ADC input signal power

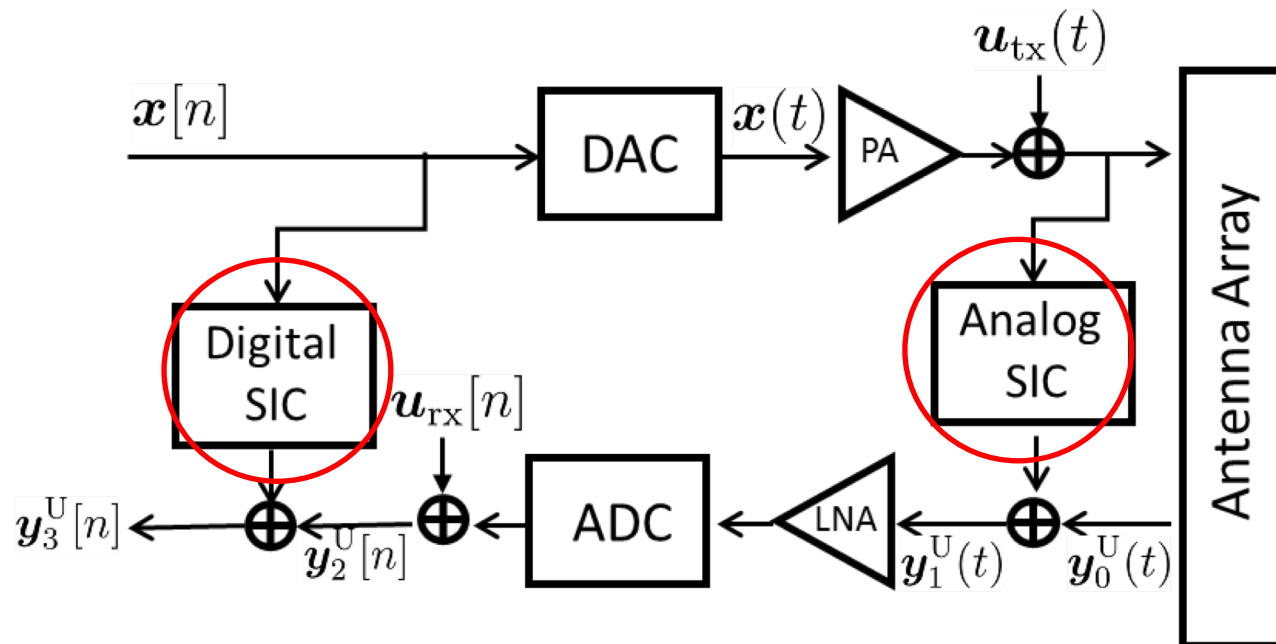
Simulation Results

- (P1) $N_t = 10, K = L = 5$
- Computation time



System Model

- Based on [Sabharwal'14], consider a block diagram of a FD BS

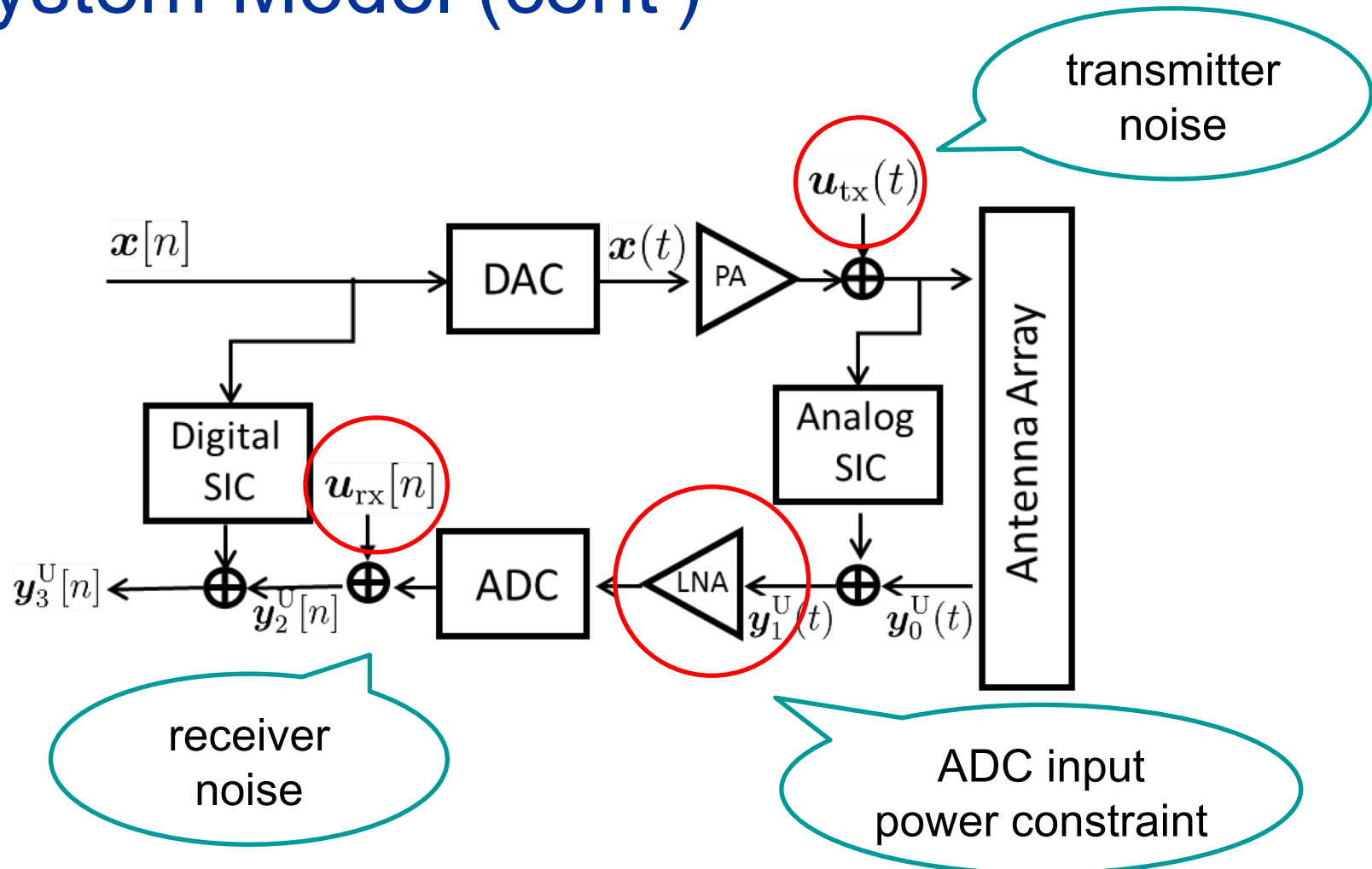


B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex bidirectional MIMO: Achievable rates under limited dynamic range," IEEE Trans. Signal Process., vol. 60, no. 7, pp. 3702–3713, July 2012.

D. Bharadia and S. Katti, "Full-duplex MIMO radios," in Proc. USENIX NSDI, Seattle, 2014, pp. 359–372.



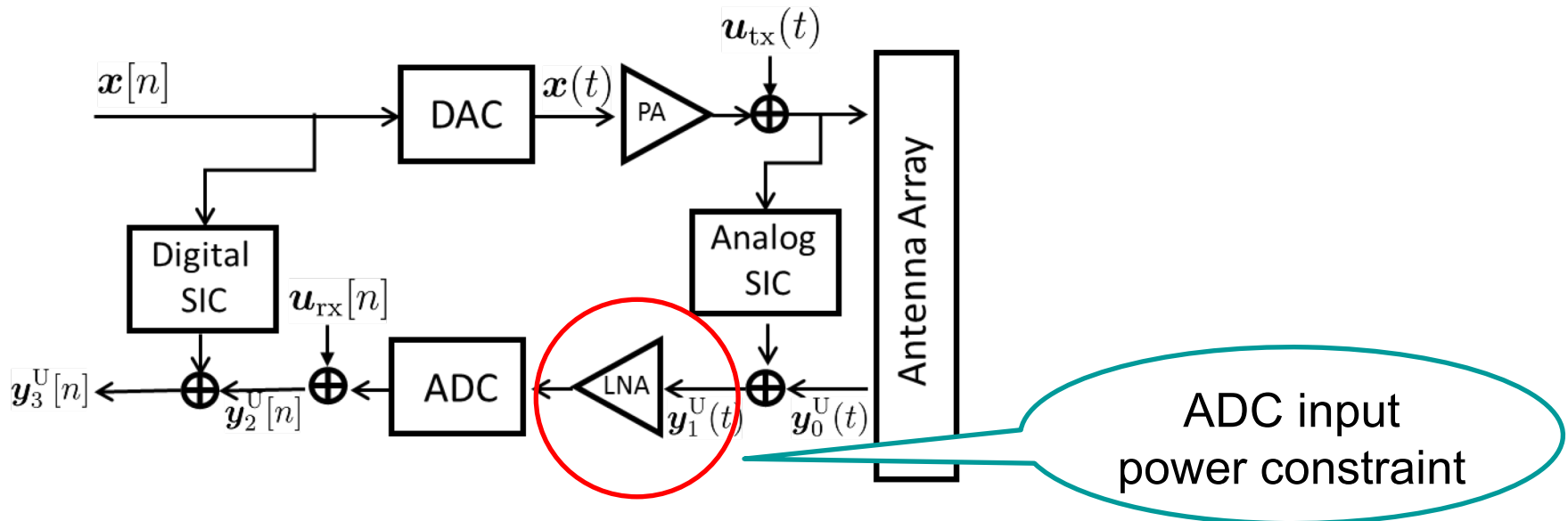
System Model (cont')



B. P. Day, A. R. Margetts, D. W. Bliss, and P. Schniter, "Full-duplex bidirectional MIMO: Achievable rates under limited dynamic range," IEEE Trans. Signal Process., vol. 60, no. 7, pp. 3702–3713, July 2012.

D. Bharadia and S. Katti, "Full-duplex MIMO radios," in Proc. USENIX NSDI, Seattle, 2014, pp. 359–372.

System Model (cont')



■ Constraint on ADC input signal power:

$$\sum_{k=1}^K \mathbf{w}_k^H \Upsilon_n(\{\mathbf{R}_{\Phi_0, m}\}) \mathbf{w}_k + \sum_{j=1}^L p_j^U |\mathbf{e}_n^T \mathbf{g}_j|^2 + \sigma_z^2 \leq \gamma^{\text{ADC}}$$

$$\forall n = 1, \dots, N_t.$$



Search of Optimal η

- Proved based on the solution structures of the **HD** uplink and downlink design problems

